

# Satellite Dynamics and Space Missions: Theory and Applications of Celestial Mechanics

## Abstracts list

### Main Lectures

**Sylvio Ferraz-Mello** (Univ. de Sao Paulo, Brasil)

*Tidal evolution of close-in satellites and exoplanets*

SUMMARY:

1. Elastic and anelastic tides.
2. The theory of Darwin re-visited.
3. The Mignard-Darwin approach.
4. The creep tide theory.
5. Synchronization of the rotation of planetary satellites.
6. Star-exoplanet dynamic interactions.
7. Energy dissipation.
8. Angular momentum transfer via Resonances.

RATIONALE: the new space missions are obtaining data on the rotation and on the shape of large natural satellites and for the first time allowing to obtain clues about their interiors (like the internal Oceans in Titan and Europa and the geysers on Enceladus). The classical Darwin's theory accounts for tidal evolution in gaseous bodies but is not able to give a better knowledge of tidal evolution of stiff bodies. Modern theories (Efroimsky, Ferraz-Mello, Correia) are being built with the hopes of circumventing these difficulties.

**Antonio Giorgilli** (Università di Milano, Italy)

*Perturbation methods in Celestial Mechanics*

The lectures are concerned with the old-standing problem of stability of a planetary system. The aim is to present some relevant phases of the development of our knowledge on this subject.

Specific topics will be:

1. A brief historical account of the origin of the problem of stability, from Kepler to the end of the XIX century, including the theorem of Poincaré on non integrability.
2. The theorem of Kolmogorov on persistence of invariant tori.

3. The problem of stability of an equilibrium and of an invariant torus, through the theory of Poincaré and Birkhoff.
4. The long time stability, with the general formulation of the theorem of Nekhoroshev on exponential stability and the concept of superexponential stability.
5. A short discussion of some recent results based on computer assisted methods.

See the webpage

<http://www.mat.unimi.it/users/antonio/SDSM-2017/SDSM-2017.html>

for the slides of the lectures.

**Anne Lemaitre** ( Université de Namur, Belgium)

*Space Debris: From LEO to GEO*

The presentation will focus on the dynamics of space debris in the Earth environment, with a celestial mechanics and theoretical point of view, and not with an operational perspective. Concretely it will be divided in 6 parts:

1. I shall present the Earth space junk, with the description and the evolution of the debris population, and describe the main forces acting on them, their relative importance and the main regions of interest (Low, Medium and Geostationary Orbits, later called LEO, MEO and GEO).
2. The resonances are present at several levels : gravitational resonances, for MEO and GEO, but also lunar-solar resonances, and secondary resonances involving the Sun. A classical Hamiltonian approach is proposed for GEO or MEO regions, with different associated toys models.
3. Some numerical integrations, their limits, their characteristics, symplectic or not, for short or long time integrations will be presented, commented and compared. Chaotic indicators (MEGNO in particular) are connected, and allow to put the stability of some regions into perspective.
4. The solar radiation pressure will be investigated with more details, without or with shadowing effects especially in the GEO region.
5. For the LEO, the atmospheric drag plays an important role on the dynamics, dependent on the ballistic coefficient. Some comparisons will be presented, concerning the solar activity and the consequences on the reentry times.
6. A few words about the rotation of the debris, the explosions and collisions mechanisms, and the possibility to simulate those events in a synthetic population will conclude the course.

**Andrea Milani** (Università di Pisa, Italy)

*Space missions for minor-body science*

A short theoretical introduction, many case studies, discussion of goals and results (when available).

1. Mission design and implementation: a difficult process Case A: ROSETTA, cometary mission (Deep Impact).
2. Case B: MORO, a proposed lunar mission (Clementine, GRAIL) Case C: DON QUIXOTE, a proposed deflection experiment (AIM-AIDA-DART).
3. Case D: DAWN, rendez-vous asteroids mission (PIAZZI, Kontiki, CONTOUR, NEAR) Asteroid families, proper elements, and asteroid ground truth.
4. Case E: Osiris-REX, asteroid sample return (HAYABUSA I and II) Next missions: LUCY, PSYCHE, Trojan and main belt asteroid missions.

**Josep M. Mondelo** (Univ. Autnoma de Barcelona, Spain)

*Design of libration point trajectories using invariant manifolds*

The goal of the course is to provide the necessary techniques to be able to compute a libration point trajectory that satisfies certain requirements. The contents of the lectures will include:

1. Dynamical systems concepts ("What is a manifold? And an invariant manifold?").
2. Numerical computation of periodic orbits and invariant tori.
3. Numerical computation of invariant manifolds of fixed points, periodic orbits and invariant tori.
4. Differential correction and numerical continuation.
5. Some semianalytical techniques: Lindstedt-Poincar and the parameterization method.
6. Transfer trajectories using invariant manifolds.
7. Computation of homoclinic and heteroclinic connections.
8. Connecting trajectories around different libration points.

**Daniel J. Scheeres** (University of Colorado, USA)

*The Mechanics of Asteroid Exploration: OSIRIS-REx*

This lecture will focus on the orbital mechanics and design of suitable spacecraft trajectories about small solar system bodies. These systems are characterized by spinning, non-spherical mass distributions with strong perturbations from solar radiation pressure. As a focus for this talk the close proximity orbit design of the OSIRIS-REx mission will be detailed and discussed.

#### *Evolution of Rubble Pile Bodies*

Small asteroidal bodies in the solar system are now thought to primarily consist of rubble piles, size distributions of competent components resting on each other. This simple morphology can be related to several observed aspects of small asteroids, including the formation of binary systems, asteroid pairs and super-critical rotating rubble piles. This talk will discuss some current thinking and hypotheses about small asteroids that comes out of this simple model, and identifies upcoming space experiments that will shed light on the physical nature of these bodies.

## Lectures

**Massimiliano Vasile** (University of Strathclyde, UK)

#### *Uncertainty Quantification in Astrodynamics*

This lecture will provide an overview of methods and techniques for uncertainty quantification with specific application to astrodynamics. The lecture will focus mainly on two aspects of uncertainty quantification, uncertainty modelling and uncertainty propagation, with some examples of conditioning. The lecture will cover both intrusive and non-intrusive methods for uncertainty quantification and will provide the attendees with some elements of both probability and imprecise probability models. The theory will be accompanied with some examples of applications in astrodynamics.

#### *Multi-Objective Optimal Control*

This lecture is an introduction to methods and applications of multi-objective optimal control. The lecture will start by presenting some fundamental concepts of multi and many-objective optimisation: Pareto efficiency, Pareto set and Pareto front, scalarisation methods, Karush-Kuhn-Tucker optimality conditions. It will then proceed by introducing the extension of Pontryagin's maximum principle to the case of multiple objectives. Finally it will present some numerical methods for the solution of multi and many-objective optimal control problems together with some examples of applications in aerospace.

## Laboratory Session

**Anna Maria Cherubini** (Università del Salento, Italy)

During this session we will propose ideas for preparing a scientific talk, and discuss how to structure a presentation and communicate results in a clear and effective way.

Examples, and counterexamples, will be proposed, possibly with the help of the public.

## Contributed Talks

### 1) **Andoh Michael Afful** (RMIT University, Australia)

*Multiple Model Estimation for Space Object Characterisation*

As the number of space debris object increases, timely orbit determination and prediction along with object characterisation has become crucial for space situational awareness. In this paper, an algorithm to characterise space objects are proposed based on the multiple model estimation. The motivation for this study is as a result of employing adaptive estimation techniques to determine unknown model parameters such as size shape and surface characteristics while estimating position, velocity, attitude and attitude rates of a space object. The multiple model technique will employ UKF estimation approach to infer the unknown state vector compromised of position and respective temporal rates. Estimates of the likelihood of each hypothesis given the available measurements will be provided from the estimation approach. The performance of this strategy is assessed via simulated scenarios.

### 2) **Christoph Burger** (University of Vienna, Austria)

*Realistic modeling of collisional water transfer and loss during planet formation*

Even most state-of-the-art N-body simulations of late-stage planet formation still assume perfectly inelastic merging once two bodies collide. This strong simplification of the actual collision physics falsifies results in general, and particularly for volatile constituents like water, which are in addition found preferentially at or close to the surface. Our aim is to investigate how the outcome of individual collisions can influence the overall picture of water transfer and loss during (terrestrial) planet formation, believed to be one of the most important factors for planetary habitability. High resolution Smooth Particle Hydrodynamics (SPH) simulations enable us to study the principal outcome as well as the detailed fate of different materials in collisions, covering all outcome regimes from low-velocity, almost perfect merging, to fast and highly erosive, as well as hit-and-run encounters. In addition to the impact velocity other parameters like the impact angle, the colliding bodies' mass ratio and also the total colliding mass play a crucial role, especially if transitions, like from accretion/erosion to hit-and-run, or from subsonic to supersonic impacts, become important.

### 3) **Josué Cardoso dos Santos** (UNESP - Sao Paulo State University, Brasil)

*Study of the roto-translational dynamics using intermediaries: numerical experiments*

The present work deals with the roto-translational motion of an axisymmetric rigid body, considering this body under the influence of a central gravitational field. Following Ferrándiz and Sansaturio a Hamiltonian formalism is considered

based on the total angular momentum and the canonical variables associated, and the model is shaped as a perturbation of the Keplerian motion plus the free-rotation of a rigid body, where the elimination of the nodes is used to reduce two degrees of freedom of the problem. Despite this, the system of equations of motion is non-integrable. The concept of intermediary Hamiltonian is then used to propose a simplification of the system which is integrable in terms of elliptical integrals. An alternative procedure proposed by one of the authors may be followed identifying a second model, where now the canonical transformation known as elimination of the parallax is used before the process of building of a second intermediary Hamiltonian. This second intermediary becomes integrable in terms of elementary functions. As part of the full two-body dynamics, different values of the parameters: relative masses of the bodies and the shape of the rigid body, are considered to visualize possibilities of applications to problems such as binary asteroids and artificial satellite attitude propagation. Numerical experiments are made to compare the order of approximation of the two intermediary Hamiltonians with respect to the full model under the MacCullagh approximation. In all the cases considered in the numerical simulations, both models present good precision after hundreds of orbits for a system with a rigid body in an eccentric relative orbit, with some advantage for the model with elimination of the parallax. Joint work with Sebastin Ferrer and Daniel J. Scheeres.

**4) Valerie Chopovda** (Massey University, New Zeland)

*Planar periodic orbits arising from the Schubart-like periodic orbit in the Caledonian four-body problem*

We consider the special case of the planar four-body problem where the system is symmetric and consists of four equal masses. The bodies are initially collinear with transverse velocities. Starting from a planar Schubart-like orbit found by Sweatman in 2014, we generate a family of related symmetric periodic orbits. The stability of the family of orbits is studied by applying various perturbations to the orbits. Joint work with Winston Sweatman and Robert McKibbin, Institute of Natural and Mathematical Sciences, Massey University, Auckland

**5) Vincenzo Di Pierri** (Università di Pisa, Italy)

*Testing alternative theories of gravity with the BepiColombo Radio Science Experiment*

The parameterized post-Newtonian (PPN) formalism is a general metric theory of gravity depending upon 10 parameters, called post-Newtonian parameters. In particular, experiments within the Solar System can be described within the PN limit, that is the regime of slow motion and weak fields, which includes all the symmetrical metric theories of gravity, with vanishing torsion tensor. In this work we introduce torsion theories, which are metric theories with non-vanishing torsion tensor, in their PPN expansion. In this framework we derived the equations of motion of a massive body including torsion, describing the dynamical effects by means of some torsion parameters to be added

to the standard PN parameters. We will show how the torsion parameters can be estimated with the ESA-BepiColombo radio science experiment (RSE). To this aim, we implemented the torsion contribution to the dynamics in the ORBIT14 software, which is an orbit determination code specifically developed for the Bepi-Colombo RSE by the Celestial Mechanics Group of the University of Pisa. The software allows for both the simulation of radio observables (range and range-rate) and for the determination of a large number of parameters concerning fundamental physics and Mercury geophysics by means of a global least squares fit. We will show the results of a set of simulations performed with ORBIT14, with the aim of estimating the torsion parameters. We will also discuss the issues arising from correlations between PN and torsion parameters. Joint work with Giacomo Tommei, Giulia Schettino, Andrea Milani.

**6) Alessandro Fiocchi** (Università di Roma 'La Sapienza', Italy)

*Analysis and design of reflector antennas for deep space and radioastronomical missions*

The main focus will be on depicting the steps towards designing a functional receptacle system suitable for deep space communication, doing so some great examples of reflector antennas will be carefully studied such as ESA's Cerebros and NASA's Goldstone stations. Furthermore Japan's interest in space exploration has brought some noteworthy results regarding satellite transmission technology, consequently papers from JAXA scientific professionals will be examined thoroughly although they will not be the main subject of this presentation. Deep space communications tackle great engineering problems, usually dealing with low power radiation and hardly ever with high signal-to-noise ratios (SNR), narrow bandwidth, low capacity transmission channels, short visibility periods. Evaluating the free-space path loss (FSPL) for example has several layers of complexity; at first glance, the formula is straightforward and gives a good rounded up value of the attenuation given by the Hertzian channel.  $FSPL=(4\pi d\lambda)^2$ . Nevertheless, the only degree of freedom is the frequency and altering that parameter has an incredible impact on the overall link budget evaluation, as evidenced below in section 1.3. An example of FSPL values relative to BepiColombo is hereby reported for reference:  $FSPL_{min}=287,41$  dB and  $FSPL_{max}=290,82$  dB. The former is derived for a signal using the bottom-end of Ka band ( $\lambda_{max}=0,0111$  m) and travelling the shortest distance possible  $d_{min}=91,69?109$  m. The almost 4 dB difference to the latter is due to the higher frequency and the longer path that have been taken into account for a worst case evaluation ( $\lambda_{min}=0,0075$  m and  $d_{max}=207,51?109$  m). It is interesting to notice that a single signal suffers from various impairments since it contains multiple wavelengths within it, and not just a  $\lambda_{min}$  or a  $\lambda_{max}$ . Nevertheless the factor that represents analytically the medium (or constitutive relation) can be linear as well as a complex dyadic, causing the signal to struggle with attenuation, scattering, shifts of phase, polarization damages and much more; some loopholes exist that help minimize these destructive effects, such as choosing the optimal frequency of operation. Communications is not the only field of application in which antennas fit perfectly, passive systems have

been widely used as "listener" or radio waves, the so called radiotelescopes; they played a huge role in radio astronomical research. Results of simulations run thanks to the GRASP software from TICRA will be presented in order to verify and compare the results reported.

**7) Stefan Frey** (Politecnico di Milano, Italy)

*Density-based method for the evolution of a cloud of fragments in elliptical orbit*

Clouds of fragments in highly eccentric orbits (HEOs), such as the geosynchronous transfer orbit, are subject to the effect of different perturbations due to large changes in altitude over a course of a single orbit. Atmospheric drag and Earth's oblateness are the dominant perturbations at low altitudes. At high altitudes, perturbations from third bodies and solar radiation pressure become relevant, especially for small fragments typically characterised by large area-to-mass ratios. During their orbital lifetime, these fragments interact with the population of the low Earth orbit (LEO), and pose a threat to operational spacecraft. In this work, the evolution of the fragment cloud is modelled with a density-based approach where the cloud of objects is treated as a continuum fluid and propagated using semi-analytical techniques coupled with the continuity equation. Such an approach, applied to the LEO case, was demonstrated to be efficient and accurate compared to results obtained from propagating the trajectory of each single fragment with numerical techniques. The perturbations taken into consideration in this preliminary work are air drag and Earth's oblateness. The collision risk with objects in LEO is assessed with two different methods which are compared; one method assumes the kinetic gas theory, the other geometrical considerations. The short-, mid- and long-term evolution of the cloud is analysed and the repercussions on objects residing in LEO are discussed. Joint work with Camilla Colombo

**8) Pol Gurri** (Universitat Politècnica de Catalunya, Spain)

*Mass and eccentricity constraints on the planetary debris orbiting the white dwarf WD 1145+017*

Being the first of its kind, the white dwarf WD 1145+017 exhibits a complex system of disintegrating debris which offers a unique opportunity to study its disruption process in real time. Even with plenty of transit observations there are no clear constraints on the masses or eccentricities of such debris. Using  $N$ -body simulations we show that masses greater than  $\simeq 10^{20}$  kg (a tenth of the mass of Ceres) or orbits that are not nearly circular (eccentricity  $> 10^{-3}$ ) dramatically increase the chances of the system becoming unstable within two years, which would contrast with the observational data over this timespan. We also provide a direct comparison between transit phase shifts detected in the observations and by our numerical simulations.

**9) Davide Menzio** (Politecnico di Milano, Italy)

*Flyby design in the circular-restricted free body problem*

Flyby has always been particularly appealing to mission designers since it gives the possibility to alter the spacecraft trajectory at no cost in term of pro-



pellant. Its exploitation has opened up unattainable interplanetary deep space and outer planets missions, new launch windows opportunities and new ways to achieve capture, making flyby a priceless resource in mission analysis. The implementation of a flyby in the design of interplanetary trajectories consists generally in a problem of phasing with the encountered planet and energy gain. Over the years, these two aspects have always been tackled separately solving the former one with Lambert algorithm and the latter one with the Tisserand Map. Both methods represent an essential tool to achieve an optimal design, although a comprehensive and simultaneous solution has not been implemented yet since each theory, formulated in a specific gravitational model, is not directly resolvable in the other environment. The aim of this paper consists in solving the phasing condition in the circular restricted body problem by mean of an energy-based approach. This function is demanded to the Tisserand parameter which allows to satisfy directly the sufficient conditions of relative incoming and outgoing velocities equal in magnitude and provides an estimation of the periapsis. The pursuit of the possible trajectories in the infinite solution space is performed through a gradient-based approach which constrains the Jacobi constant to the flight time and to the periapsis, legacy of the phasing and the energy respectively. In the end, the minimum delta-v represents the criterion of selection of the optimal solution. Limits of the applicability of such method are discussed and new formulation of the Tisserand parameter is speculated which would in principle augment its application without restrictions. Joint work with Camilla Colombo.

**10) Cezary Olszowiec** (Imperial College, London, United Kingdom)

*Superheteroclinic orbits, heteroclinic bifurcations and shadowing of the scattering maps*

Although the system which we investigate originates from game theory, the problems which we tackle and dynamical behavior we try to explain, might arise as well in the celestial mechanics models. We consider a Rock-Scissors-Paper game between two agents X, Y. The interaction matrices depend on two parameters  $\epsilon_X, \epsilon_Y \in (-1, 1)$  and the dynamics are described by the coupled replicator equations. We provide the description of naturally appearing heteroclinic network and investigate asymptotic and chaotic behavior in its neighbourhood. We find that the infinite switching happening near the network cannot be described by the full-shift on two symbols and its form strongly depends on the parameter values. In the system we observe different bifurcation scenarios: e.g. transition from order to chaos (through Hamiltonian case where invariant tori and chaos might be observed), loss of one dimension of the local stable manifold of the heteroclinic subcycle or disappearance and appearance of the local stable and unstable manifolds of the different subcycles at the same time. As well we investigate numerically the existence of the heteroclinic connection between different subcycles (i.e. a superheteroclinic orbit, compare with [2]) and its bifurcation to the different heteroclinic connections (forward and backward) from the hyperbolic fixed point to the subcycles. We describe a very peculiar closed loop of scattering maps (between certain invariant manifolds) that ap-

pear in this model and prove that it is possible to shadow this loop in the whole phase space. We investigate the form of these scattering maps (depending on the parameter values) and its consequences for the dynamics (e.g. existence of blender).

This is a joint work with M.Capinski and D.Turaev.

[1] C.Olszowiec "Complex behavior in cyclic competition bimatrix games"

[2] L.P.Shilnikov, D.Turaev "Super-homoclinic orbits and multi-pulse homoclinic loops in Hamiltonian systems with discrete symmetries"

**11 Fabrizio Païta** (Università di Roma 'Tor Vergata', Italy)

*The dynamics of the Laplace resonance*

In this presentation we consider the three body resonance characterizing the dynamics of the Jovian moons Io, Europa and Ganymede, which is commonly known as "Laplace" resonance. Focusing on its history over short time scales, we derive a conservative, planar Hamiltonian that approximates it. Its validity is confirmed through comparison with a Cartesian formulation of the model and an appropriate set of ephemerides. Furthermore, after a suitable canonical change of coordinates, we simplify even further and retain only the purely resonant terms. This model has exactly one *stable* equilibrium, while the Laplace resonance appears as a periodic orbit. By implementing a normal form around the equilibrium, we obtain a one degree of freedom Hamiltonian where the equilibrium itself appears as an elliptic fixed point bounded by a separatrix, beyond which rotating solutions exist. We show that the Laplace state is close to one of these solutions, and we identify approximate initial conditions leading to the two families.

**12 Matteo Romano** (Politecnico di Milano, Italy)

*Planetary protection analysis for interplanetary missions*

Uncertain orbit determination and modelling of the effects of manoeuvres during interplanetary missions may cause spacecraft or launcher stages to have close encounter or impact with celestial bodies within 50 or 100 years, potentially putting humans or biological research at risk. Planetary protection requirements have been created to set an acceptable limit to the impact probability estimated during the mission design phases. The representation of the spacecraft state may in some cases become unreliable due to accumulating numerical errors during the integration. This problem is usually approached by using high order integration methods, but better long term accuracy can be obtained with more efficient propagation methods. Impact probability is usually estimated via Monte Carlo Simulation, which however is a computationally expensive method, thus making more efficient sampling necessary to obtain robust estimations with a reduced number of input samples and integration time. This work proposes a different approach to improve numerical propagation and probability sampling in planetary protection analysis, using the SNAPPshot tool suite for the verification of the compliance to planetary protection requirements, developed at Southampton University in the framework of an ESA study, as a starting point. In our work, the numerical solution is made more accurate and qualitatively closer to

the actual evolution of the dynamic system by introducing symplectic integration methods and additional energy-preserving schemes. Line Sampling is used to estimate the impact probability more efficiently, by solving a lower number of one-dimensional integrals along an "important direction" crossing the impact region of the uncertainty domain. These two approaches to planetary protection analysis will be explained, and applied to different interplanetary mission cases to compare their performance in terms of accuracy and computational cost. Some study cases are chosen among the ones already considered in [1], to perform a comparison with some reference results. Joint work with Camilla Colombo.

[1] Letizia, Van den Eynde, Colombo, "SNAPPshot. ESA planetary protection compliance verification software. Final report", ESA 2016

**13) Badam Singh Kushvah** (Indian Inst. of Technology, Dhanbad, India)  
*Normalization Theory with Applications in Celestial Mechanics*

The method of normalization provides a way to finding simplest dynamical system using suitable coordinate transformations which are found by solving a sequence of linear problems. The normalization may be considered as a local method in the sense that the sequence of coordinate transforms are performed in the neighbourhood of the known solution. The structure of the normal form is determined entirely by the nature of the linear part of the vector field. In view of this, we discuss the basic theory of normalization of vector field using various approaches including Poincare's, Jordan's, Chua and Kokubo's, and Takens and Ushiki's normal forms. We perform normalization of linear and nonlinear vector fields. The symbolic computation is done using Lie transformations method of normal form for non-linear Hamiltonian systems. At the last we apply the theory for normalization of the Hamiltonian in restricted three body problem (RTBP).