Celestial and Spaceflight
Mechanics Laboratory

The Mechanics of Rubble Pile Bodies

D.J. Scheeres
Department of Aerospace Engineering Sciences
The University of Colorado Boulder

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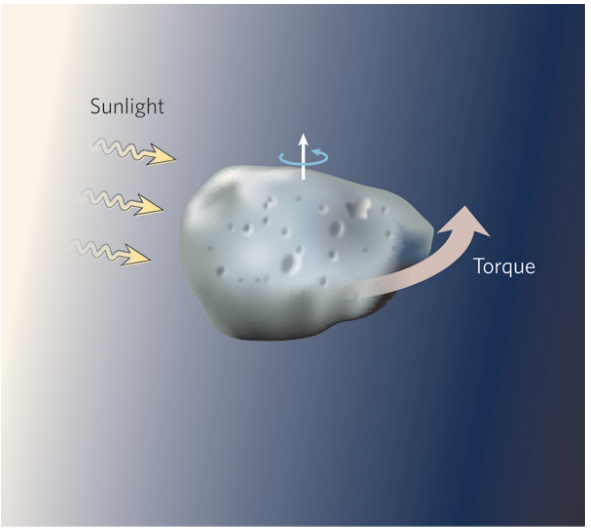
Rubble Pile Asteroids

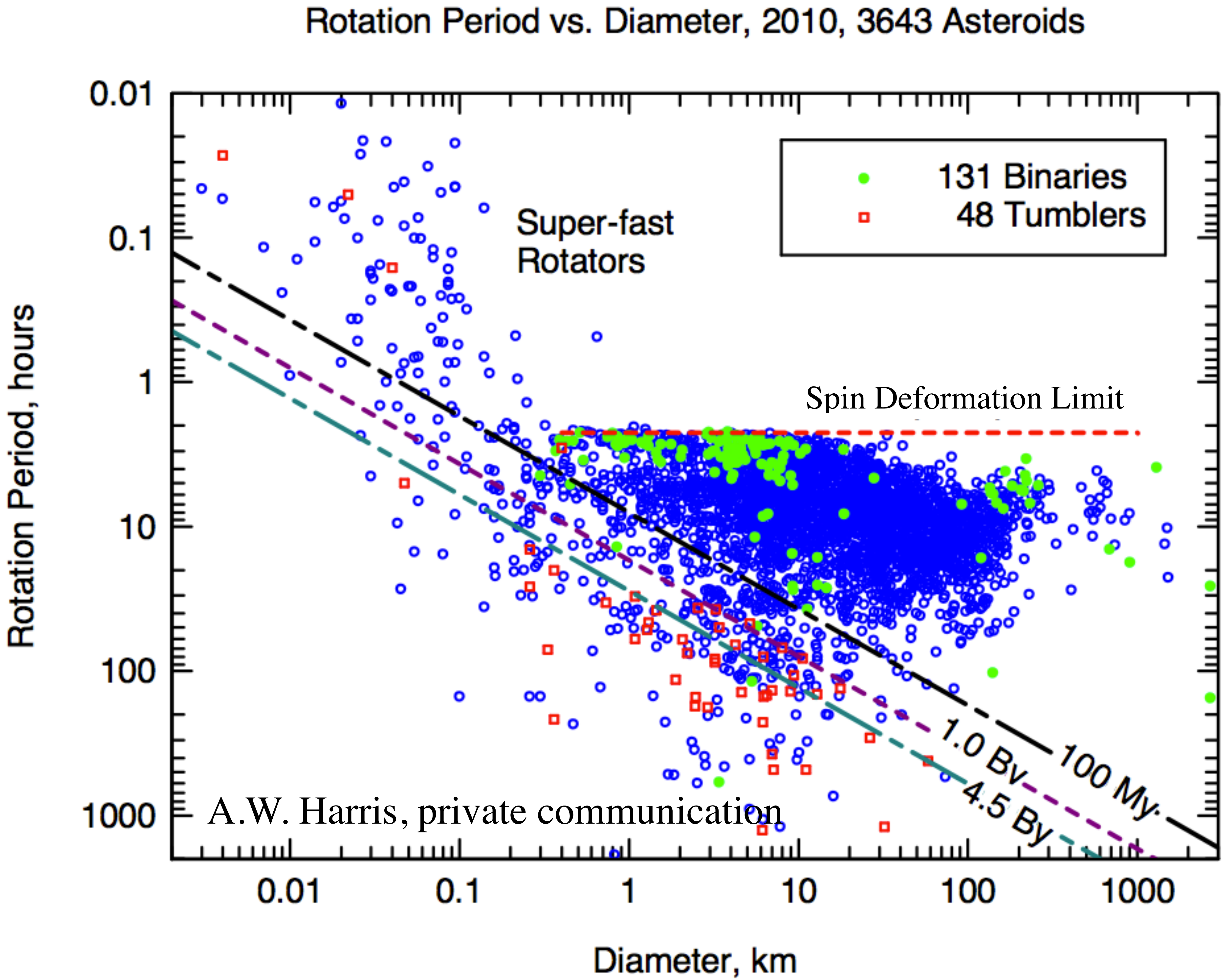
- Most asteroids are thought to be “rubble piles”
  - Consisting of a size distribution of dust, pebbles and boulders
  - Spin periods ranging down to ~2 hours for bodies > 500 m
  - Spin periods going much shorter for bodies < 500 m
  - Holding themselves together with gravity, and perhaps van der Waals forces!
- This talk focuses on “small asteroids” of size less than ~ 10 km
  - Susceptible to the YORP effect:
    - Sunlight causes them to spin up and/or down
    - Can undergo extreme variations in their spin rate over their lifetime
  - What happens when their spin rates get large



- Fundamental Question:

How does celestial mechanics influence rubble pile evolution?

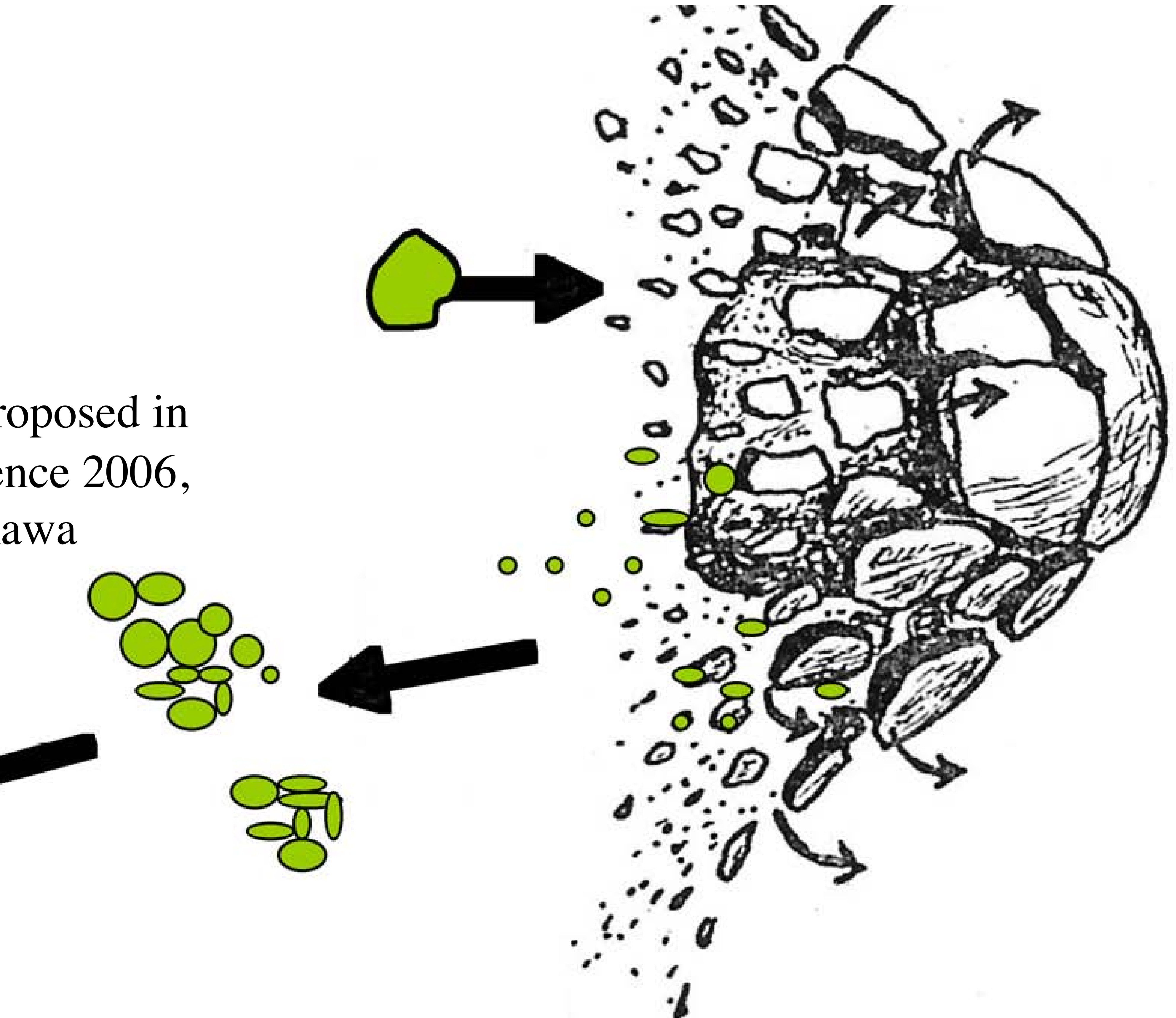
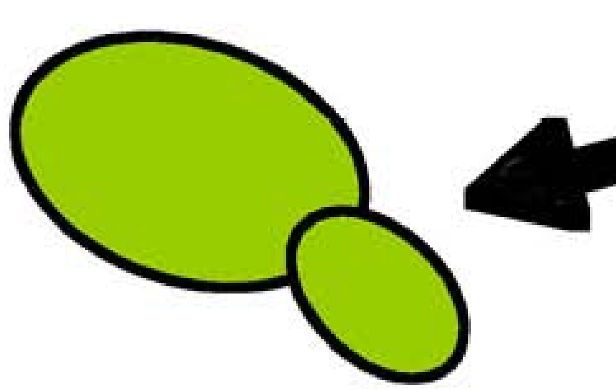
- Answer: Extremely significant at all points of their lives!



Rotation Period vs. Diameter, 2010, 3643 Asteroids
131 Binaries
48 Tumblers
Super-fast Rotators
Spin Deformation Limit
1.0 By
100 My
4.5 By
A.W. Harris, private communication
Rotation Period, hours
0.01
0.1
1
10
100
1000
Diameter, km
0.01
0.1
1
10
100
1000

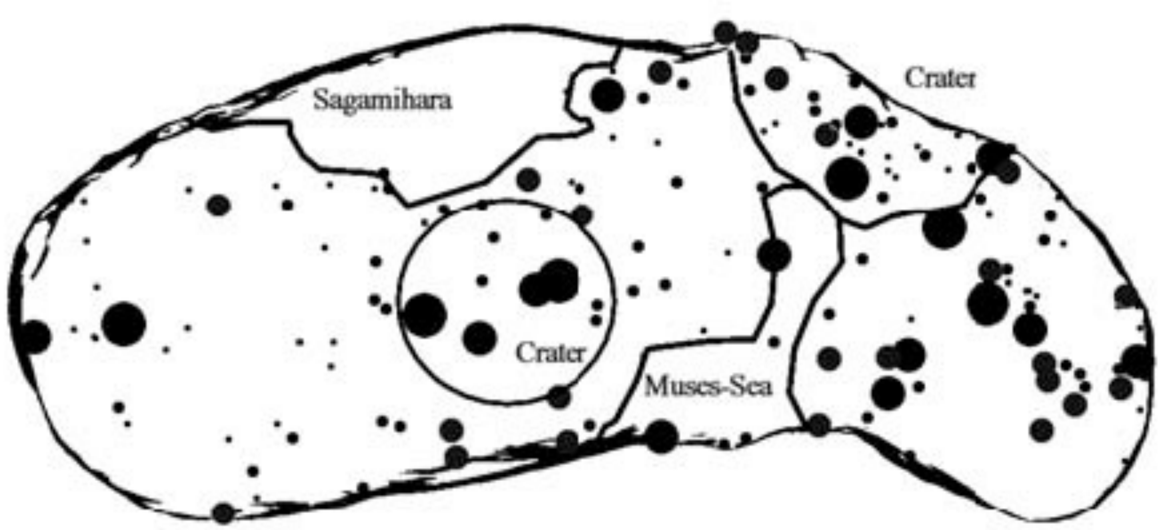
How do Rubble Piles Form?

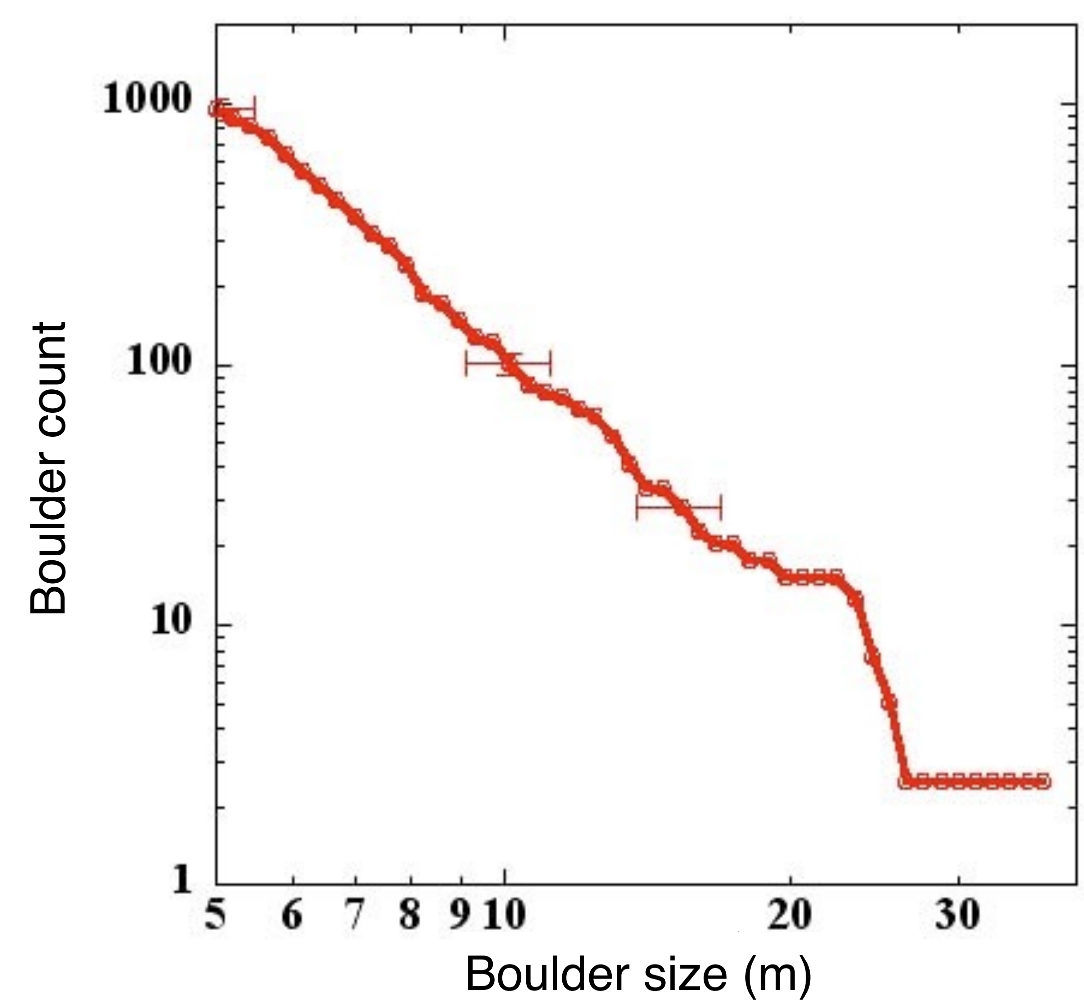
Formation model proposed in Fujiwara et al., Science 2006, consistent with Itokawa

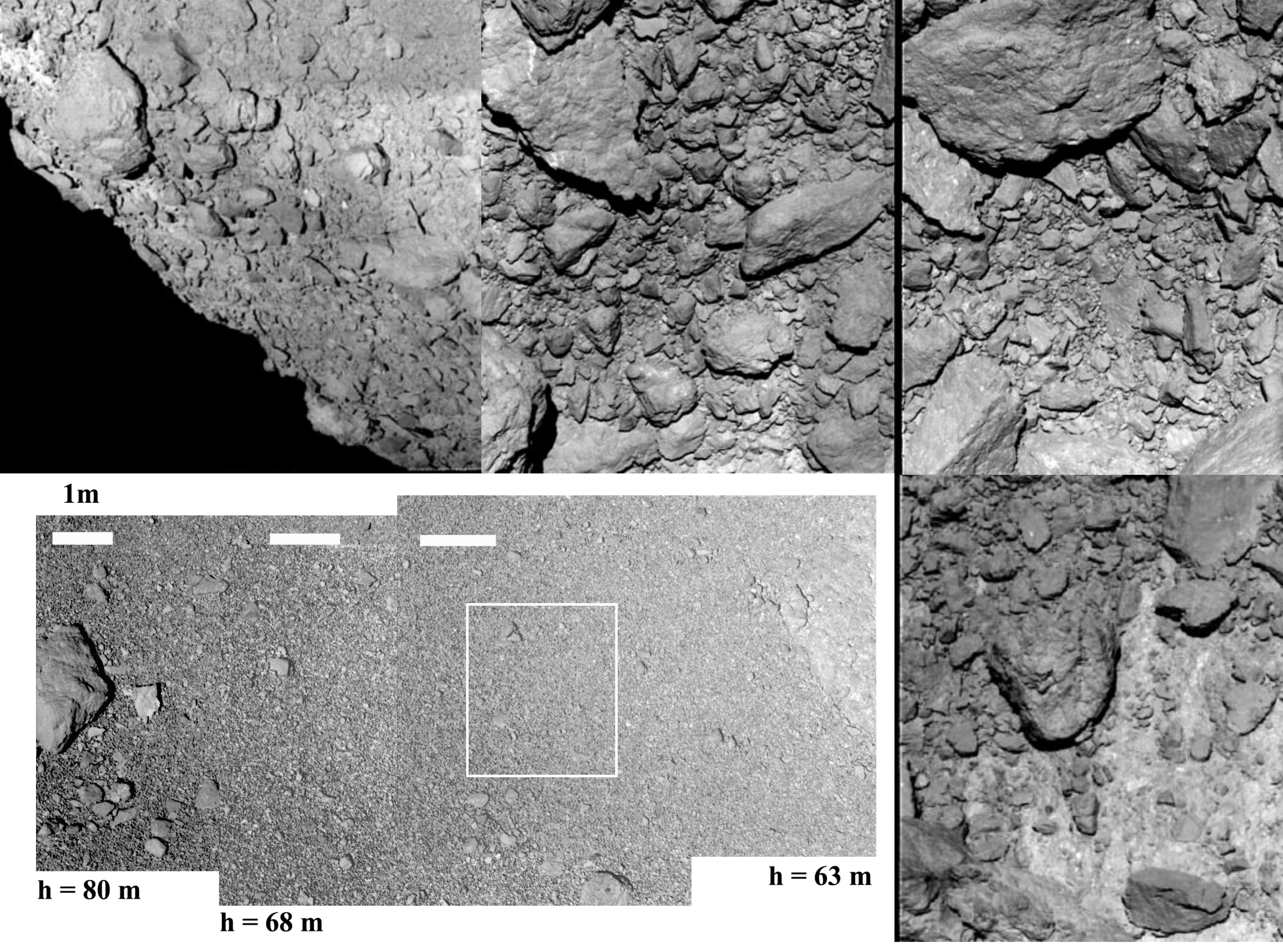
What Do Rubble Piles Look Like?

- Itokawa remains the "poster child" of rubble pile asteroids.
- Clearly comprised of a collection of boulders and grains:
  - Maximum size on the order of 10's of meters
  - Minimum size on the order of of microns (from the Hayabusa Sample)
  - Measured boulder size distribution ~ *1/d³* (Michikami et al. 2008), and confirmed by more recent analysis






Science
HAYABUSA
Dust from Itokawa
AAAS

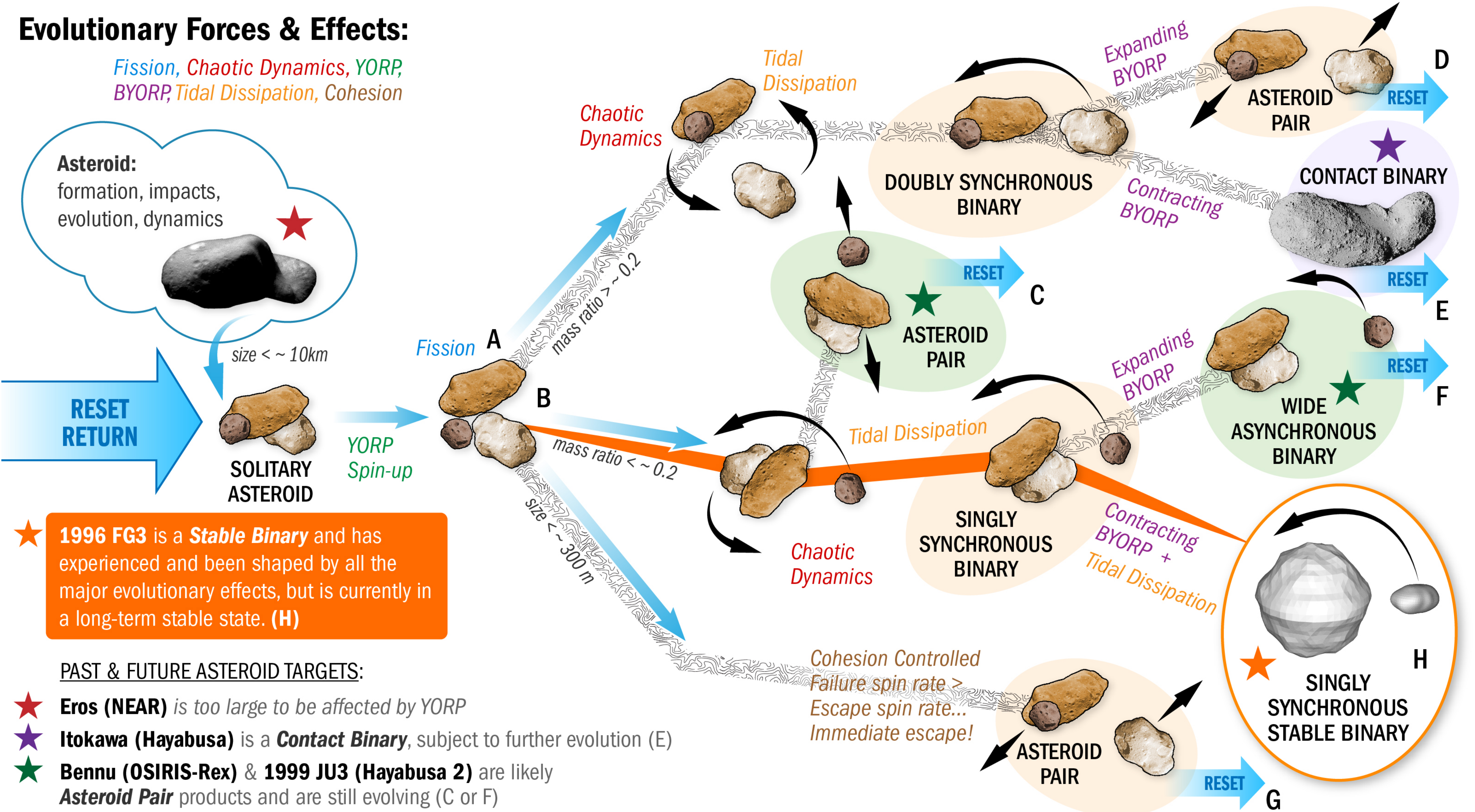

1m
h = 80 m
h = 68 m
h = 63 m

Implications for Evolution of Small Asteroids

Evolutionary Pathways of Asteroids

Evolutionary Forces & Effects:

Fission, Chaotic Dynamics, YORP, BYORP, Tidal Dissipation, Cohesion



★ **1996 FG3** is a ***Stable Binary*** and has experienced and been shaped by all the major evolutionary effects, but is currently in a long-term stable state. **(H)**

PAST & FUTURE ASTEROID TARGETS:

- ★ **Eros (NEAR)** *is too large to be affected by YORP*
- ★ **Itokawa (Hayabusa)** is a ***Contact Binary***, subject to further evolution (E)
- ★ **Bennu (OSIRIS-Rex)** & **1999 JU3 (Hayabusa 2)** are likely ***Asteroid Pair*** products and are still evolving (C or F)

A: Scheeres (2007); B: Jacobson & Scheeres (2011a); C: Pravec et al. (2010); D: Fang & Margot (2012); E: Scheeres et al. (2007); F: Jacobson et al. (2014); G: Scheeres et al. (2010); H: Jacobson & Scheeres (2011b)

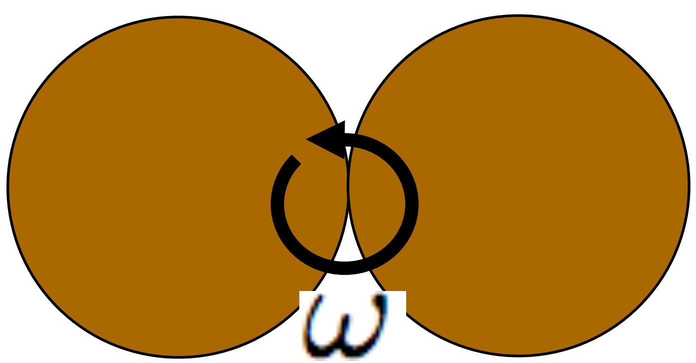
Fission Mechanics

What happens to a rubble pile subject to YORP spin-up?

- Angular Momentum of these bodies change over time
  - Best modeled as a collection of rigid bodies resting on each other
  - The celestial mechanics and geophysics of such collections are poorly understood, and can exhibit complex behavior
- Reorientations can occur as spin rate increases
  - Transitions occur at discrete energy levels
  - Can cause global "landslides" as material seeks out its minimum energy state
- Continued spin-up can lead to *rotational fission* of the asteroid
  - The stability of these proto-binaries controls subsequent evolution
    - *Failed orbital binaries*
      - *Disrupted -> Asteroid pairs*
      - *Re-impacted -> Contact binaries*
    - *Stable binaries*
      - ... subject to continued evolutionary effects

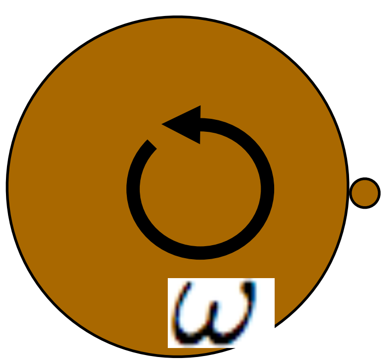
Rotational Fission

- Fission occurs when two portions of a composite body attain orbital rates relative to each other
- The fission spin rate is a strong function of “shape”



$$\omega > \sqrt{\frac{\pi G \rho}{3}}$$

Simple examples of “fission”



$$\omega > 2\sqrt{\frac{\pi G \rho}{3}}$$

For a density of $\rho = 2$ g/cm^3

~ 4.7 hour rotation period

~ 2.3 hour rotation period

Fission occurs at half the spin rate for the equal mass distribution

Rotational Fission

- Fission can be a smooth transition for a rubble pile
- Energy and AM are ideally conserved, but are decomposed:
  - Kinetic Energy

$$\frac{1}{2}\omega \cdot I_0 \cdot \omega = \frac{1}{2}\omega \cdot I_1 \cdot \omega + \frac{1}{2}\omega \cdot I_2 \cdot \omega + \frac{1}{2}\frac{M_1 M_2}{M_1 + M_2}(R\omega)^2$$

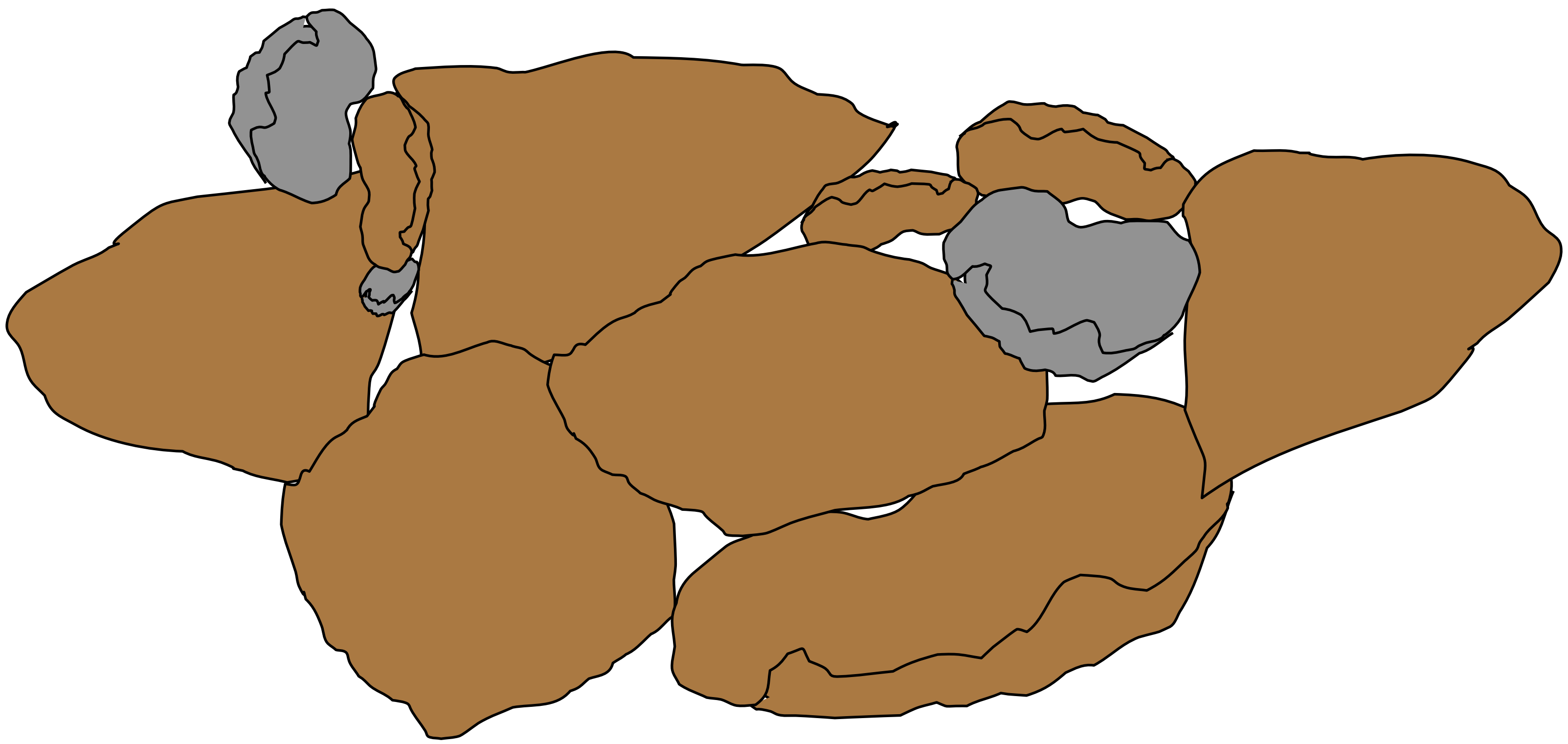
  - Potential Energy

$$\mathcal{U}_{00} = \mathcal{U}_{11} + \mathcal{U}_{22} + \mathcal{U}_{12}$$

  - The mutual potential energy is "liberated" and serves as a conduit to transfer rotational and translational KE

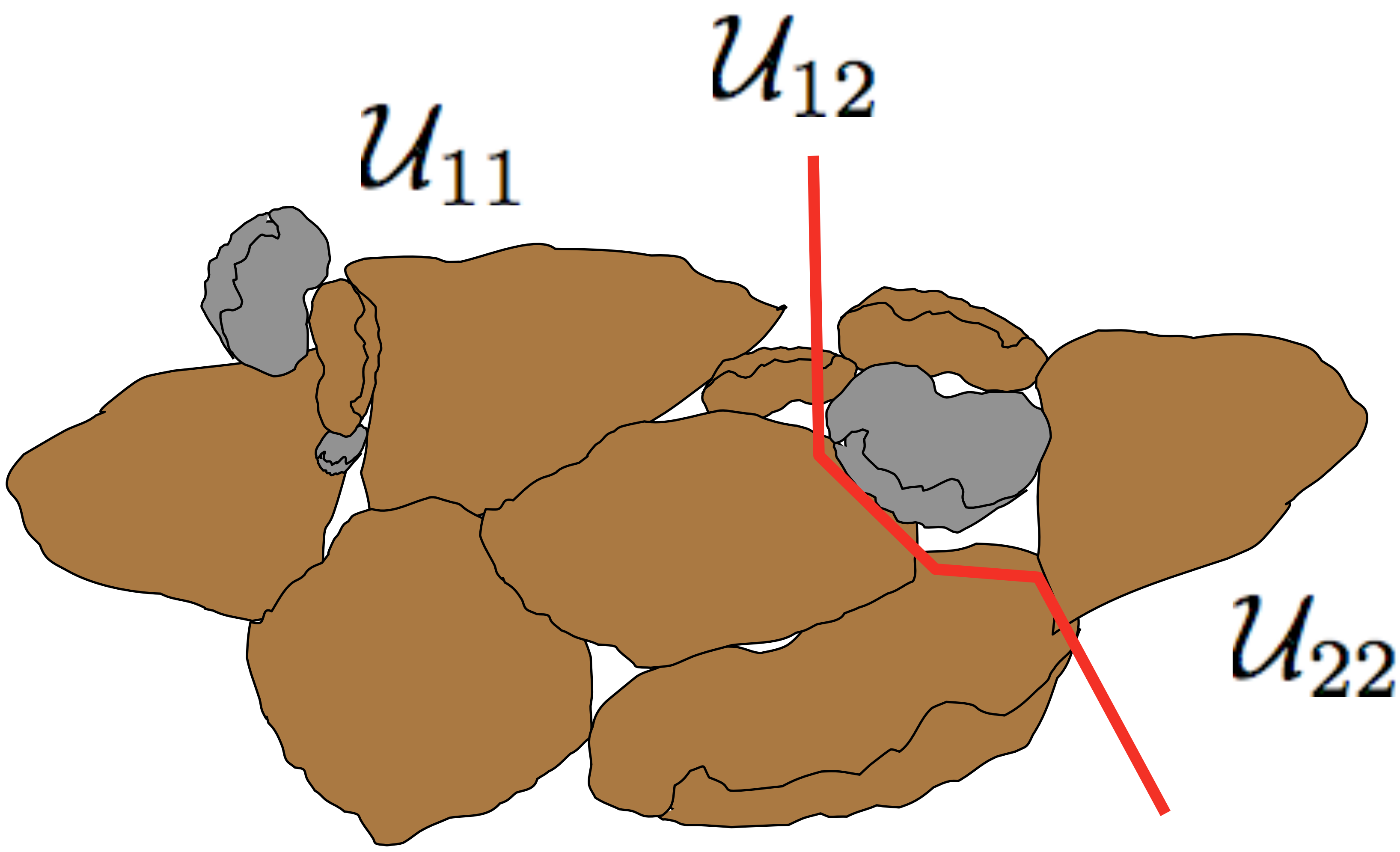
Rotational Fission

$\mathcal{U}_{00}$



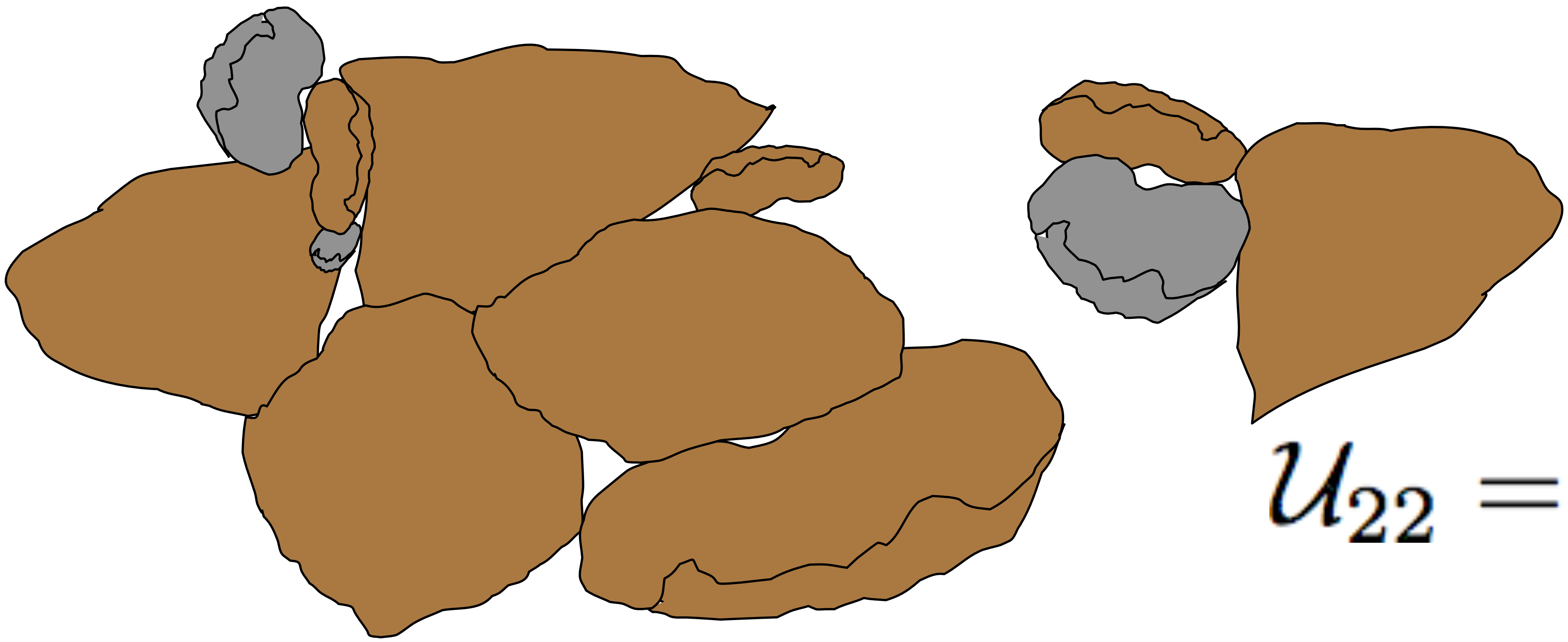
Rotational Fission

$$\mathcal{U}_{00} = \mathcal{U}_{11} + \mathcal{U}_{22} + \mathcal{U}_{12}$$



Orbital Evolution

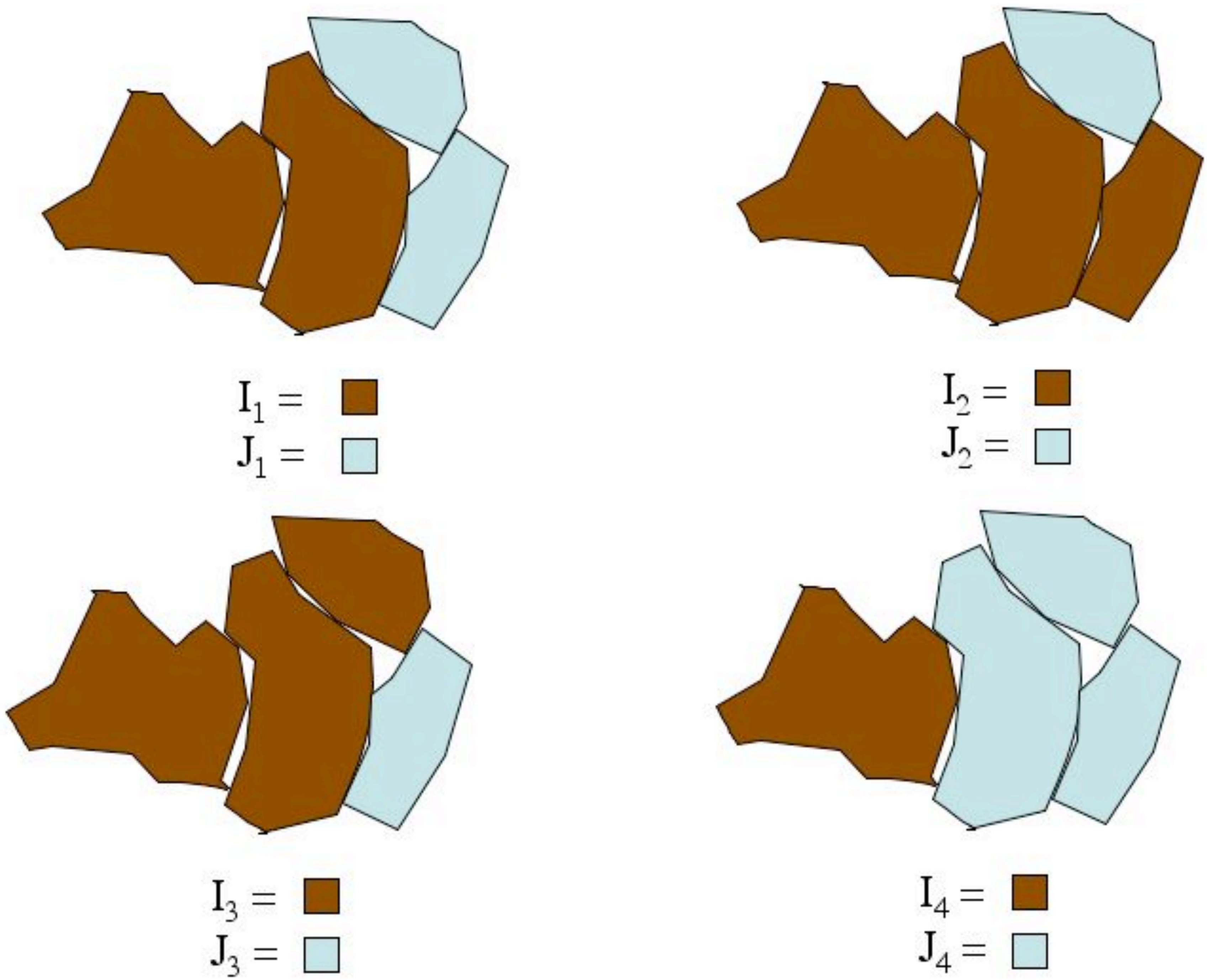
$$\Delta T_{\text{rot}} + \Delta T_{\text{trans}} + \Delta \mathcal{U}_{12} = 0$$



$$\mathcal{U}_{22} = \text{ Constant}$$

$$\mathcal{U}_{11} = \text{ Constant}$$

Possible "Fission Pairs"



Fission Conditions

- For fission of an arbitrary rubble pile split into two collections I and J the general condition becomes:

$$\boldsymbol{R}_{IJ} \cdot \tilde{\boldsymbol{\omega}} \cdot \tilde{\boldsymbol{\omega}} \cdot \boldsymbol{R}_{IJ} \quad \geq \quad -\frac{M_I + M_J}{M_I M_J} \frac{\partial U}{\partial \boldsymbol{R}_{IJ}} \cdot \boldsymbol{R}_{IJ}$$

  – Applying a weak form of Euler's Theorem of Homogenous functions this reduces to

$$T_{IJ} + \alpha U_{IJ} \geq 0$$

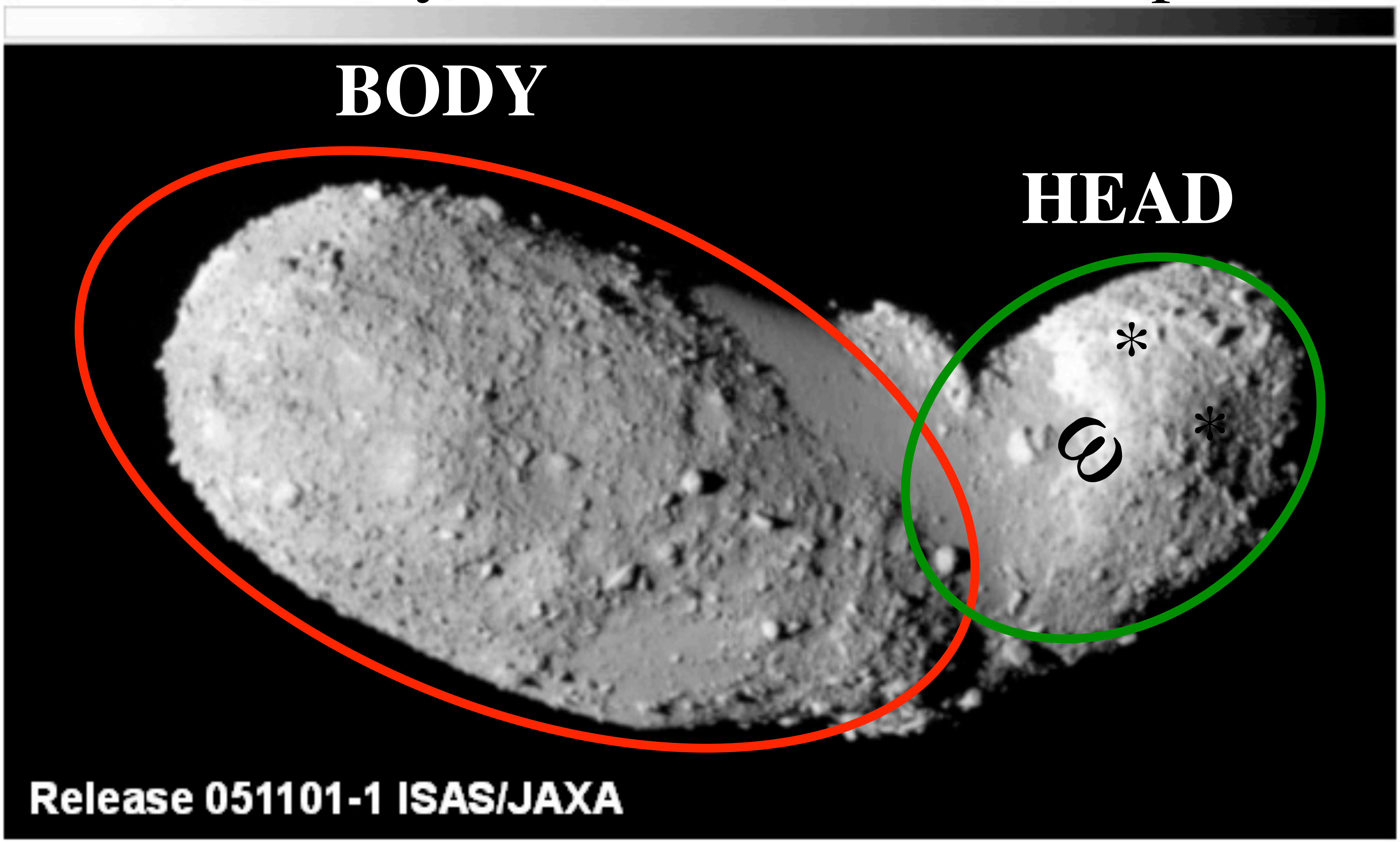
  – which is equivalent to

  "the two components with the largest separation between their centers of mass will fission first at the lowest spin rate"

  D.J. Scheeres. 2009. "Minimum energy asteroid reconfigurations and catastrophic disruptions," Planetary and Space Science 57: 154-164.

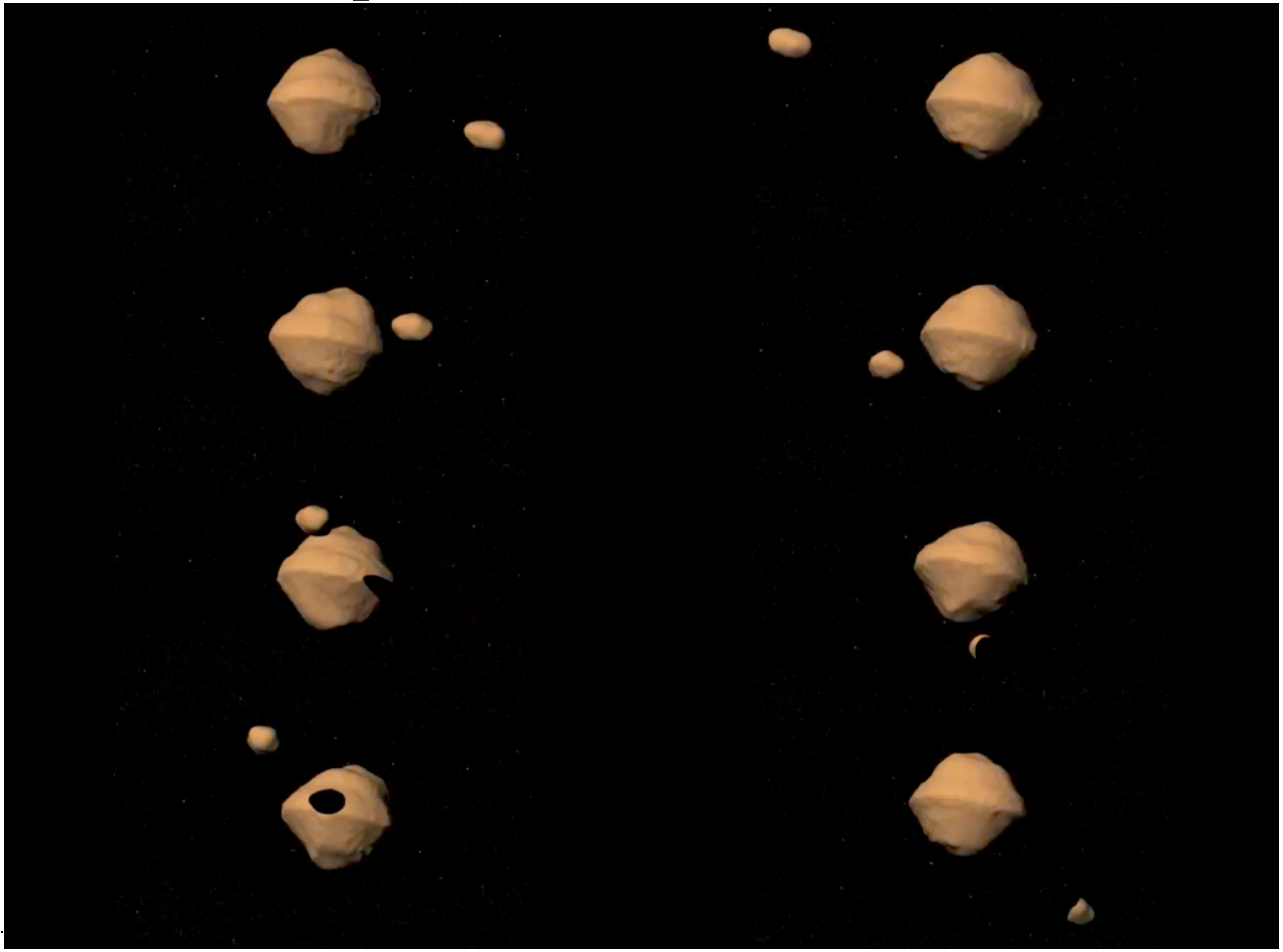
Itokawa

Head and Body will orbit at a ~ 6 hour period

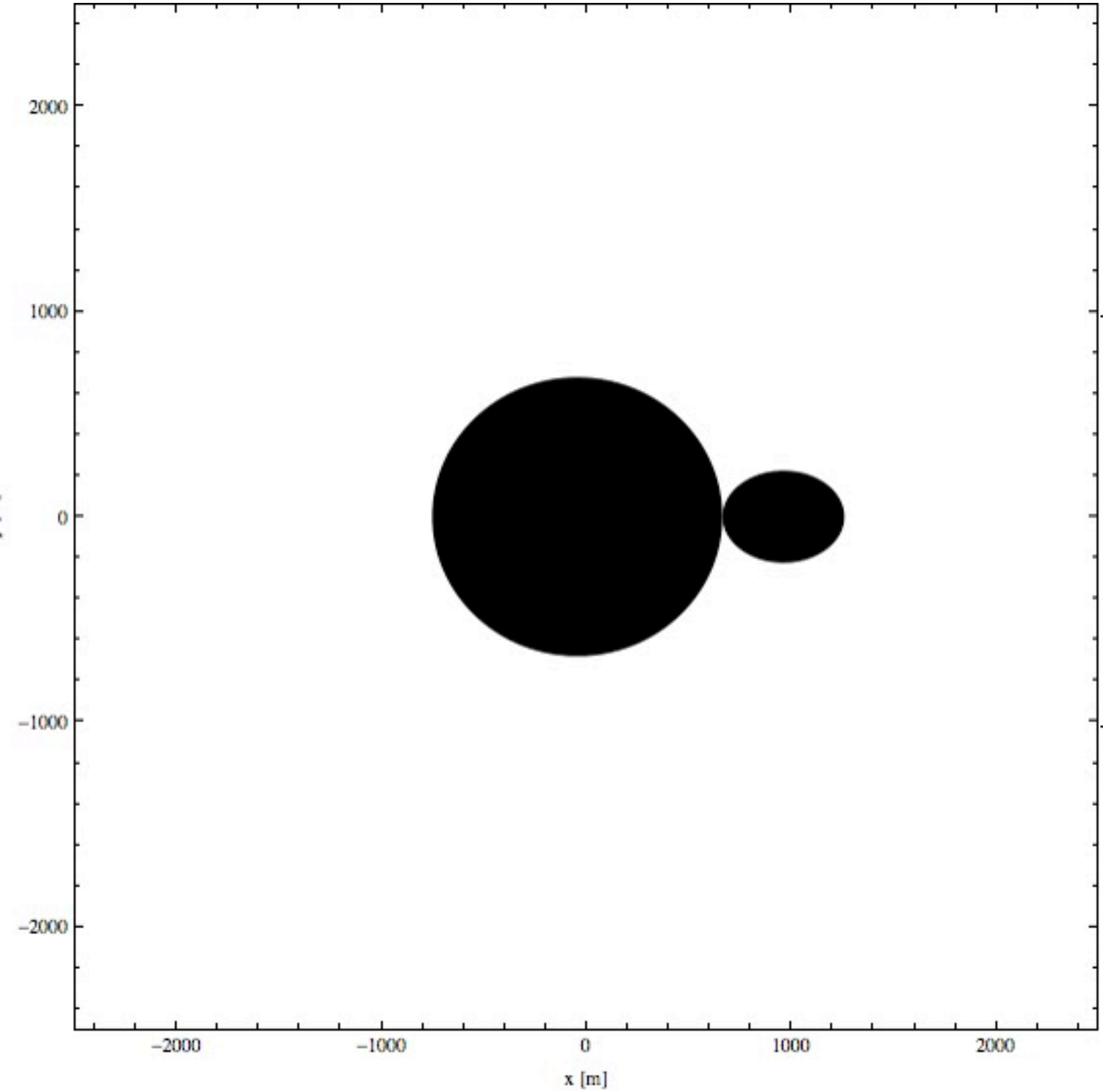


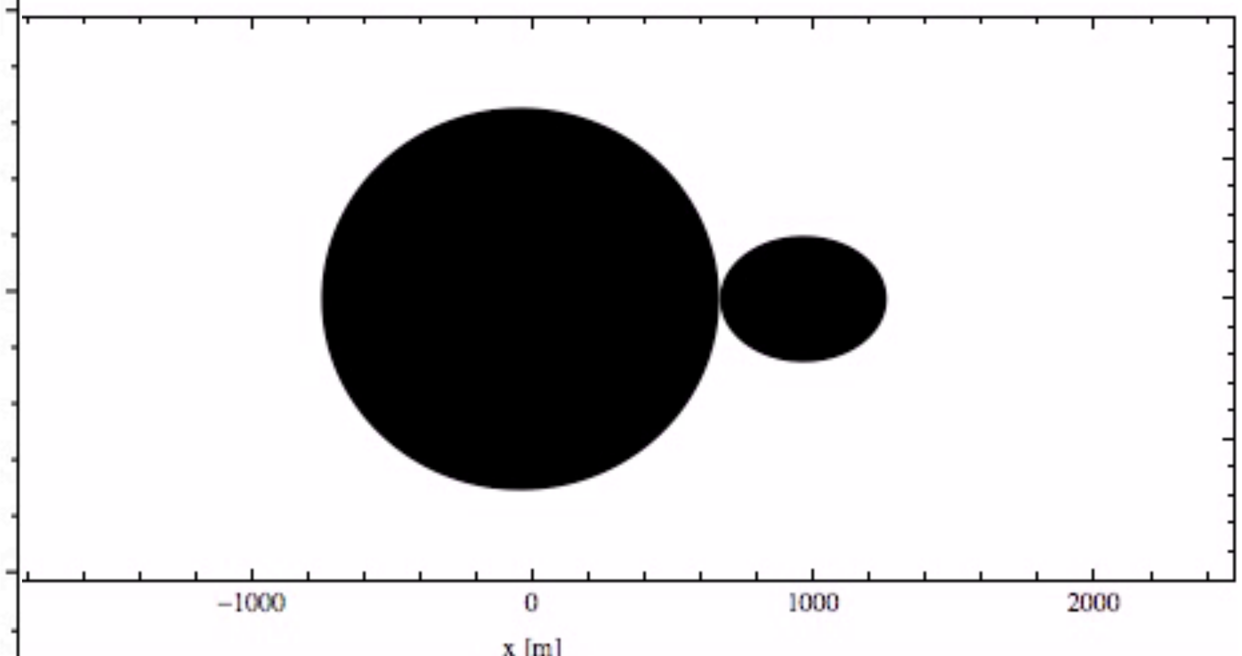
1999 KW4

- Contact binary with Alpha and Beta resting on each other will fission at a spin rate > 4 hours

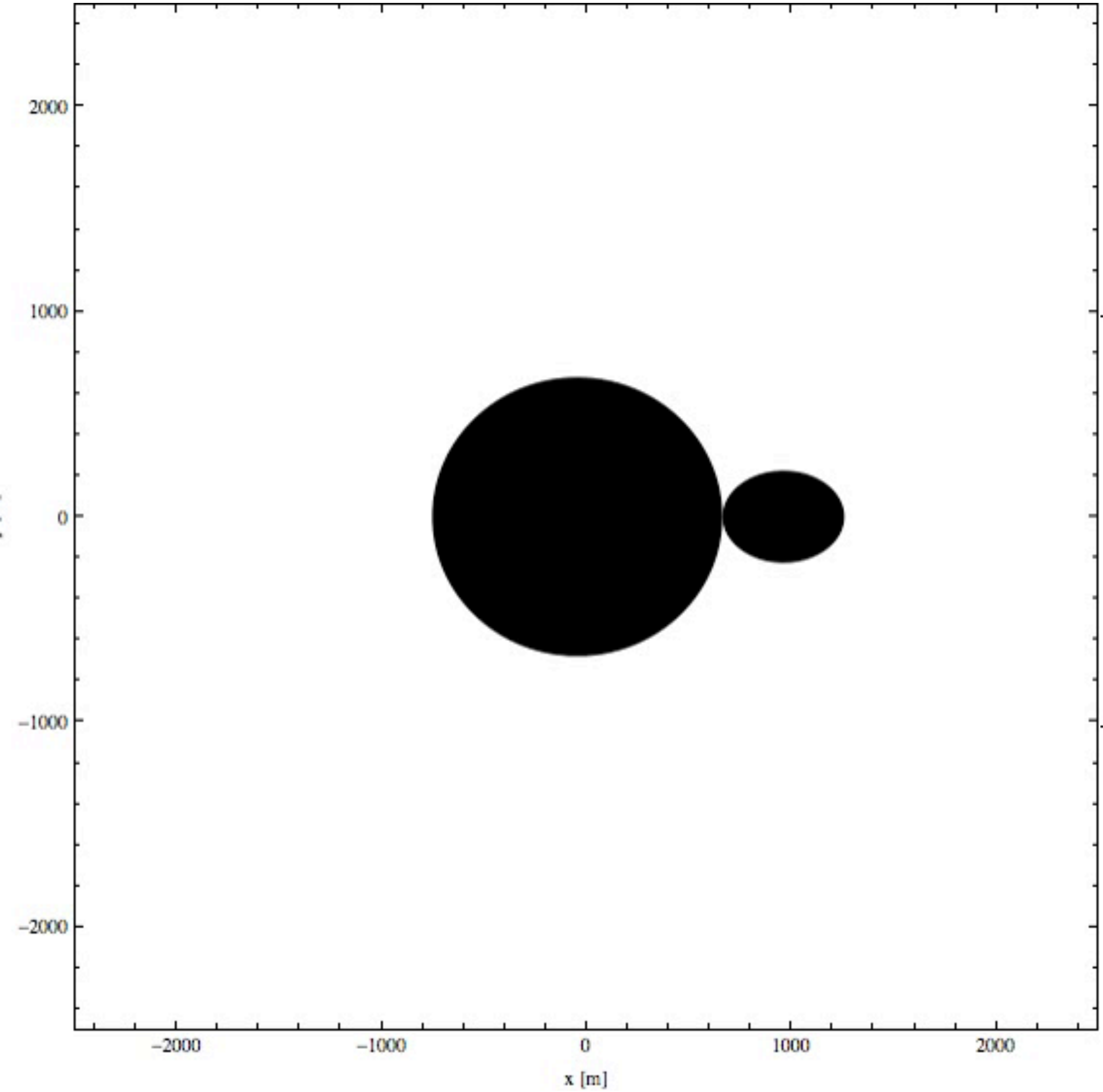


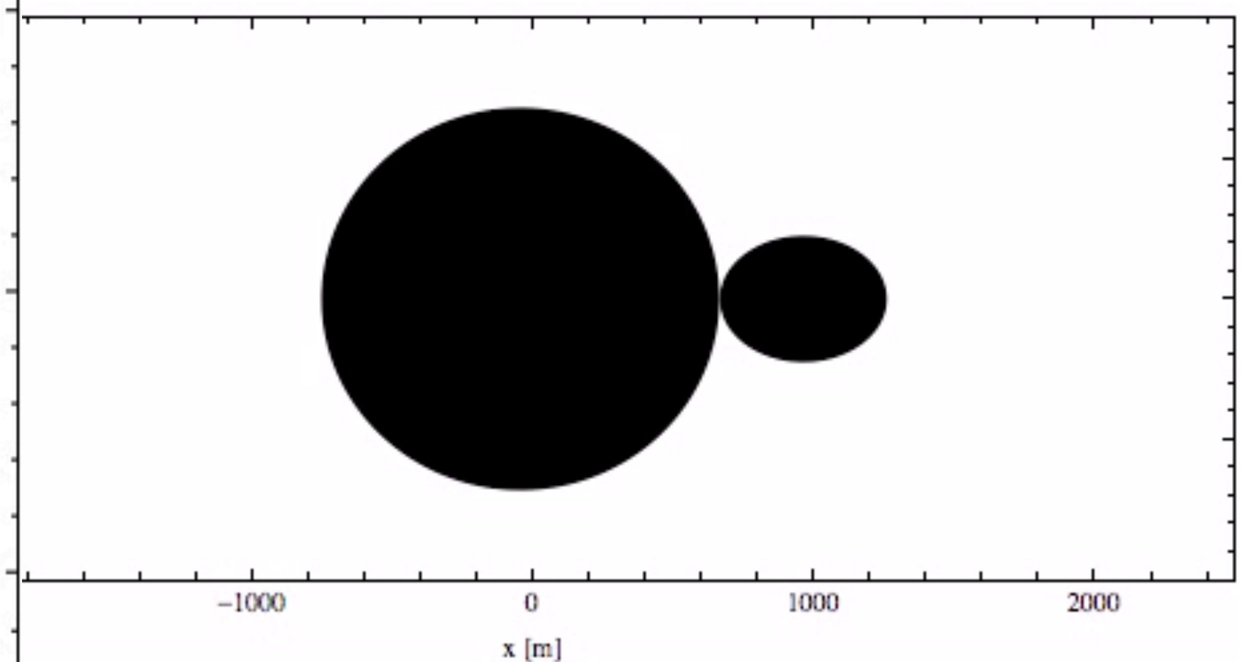
1999 KW4 Fission





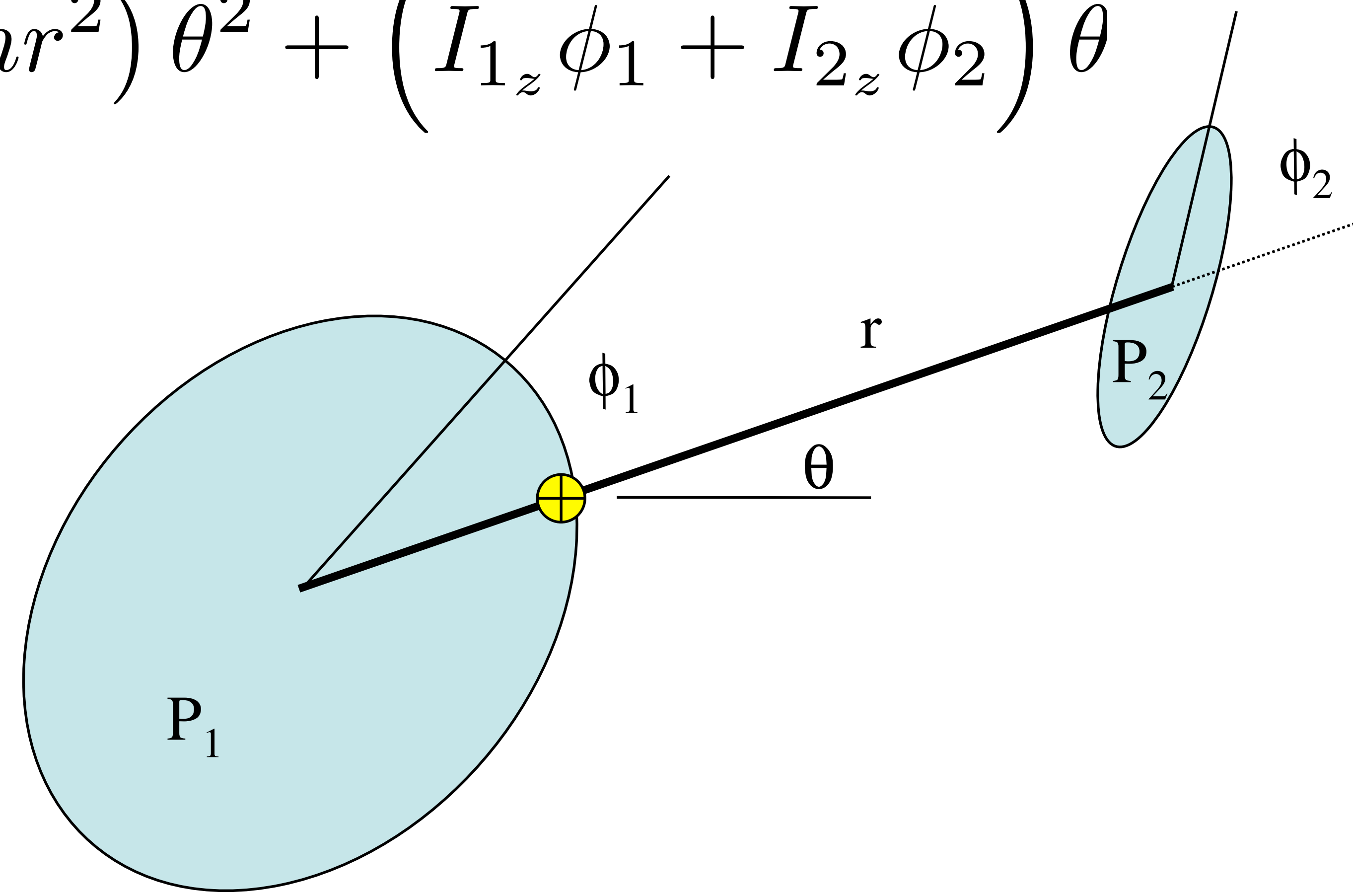
1999 KW4 Fission





Planar Model

$$
\begin{aligned}
L &= \frac{1}{2} I_{1_z} \dot{\phi}_1^2 + \frac{1}{2} I_{2_z} \dot{\phi}_2^2 + \frac{1}{2} m \dot{r}^2 + \\
&\frac{1}{2}\left(I_{1_z} + I_{2_z} + m r^2\right)\dot{\theta}^2 + \left(I_{1_z}\dot{\phi}_1 + I_{2_z}\dot{\phi}_2\right)\dot{\theta} \\
&- V(r, \phi_1, \phi_2)
\end{aligned}
$$

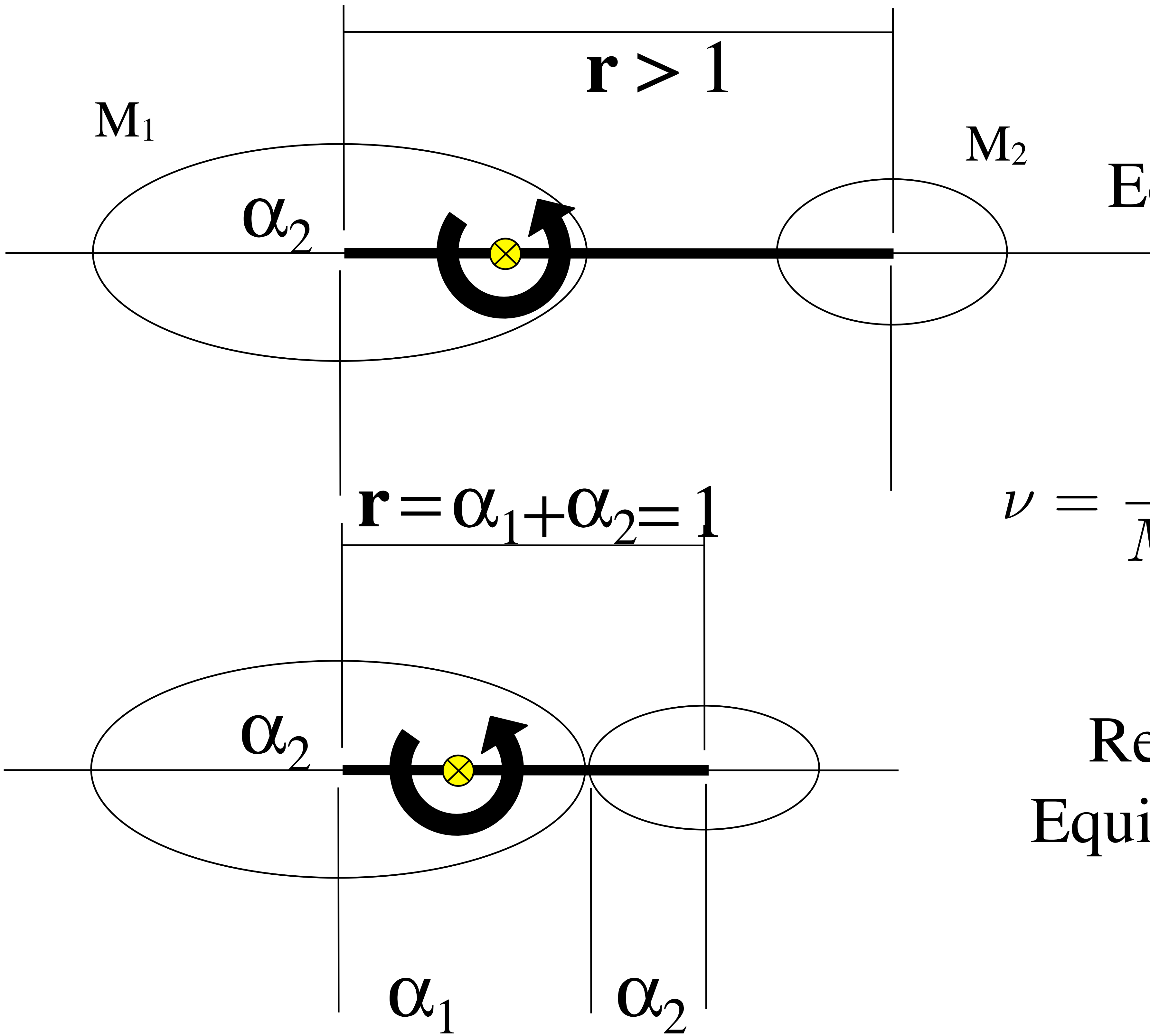


Can develop explicit results for when the system is Energetically stable, Hill stable and Stable Against Impact

$$
\begin{aligned}
V(r, \phi_1, \phi_2) &= -\frac{\mathcal{G} M_1 M_2}{r} \left\{ 1 + \frac{1}{2r^2} \right. \\
&\left. \left[\mathrm{Tr}(\bar{I}_1) + \mathrm{Tr}(\bar{I}_2) - \frac{3}{2}\left(I_{1_x} + I_{1_y} - \cos 2\phi_1 (I_{1_y} - I_{1_x}) + I_{2_x} + I_{2_y} - \cos 2\phi_2 (I_{2_y} - I_{2_x})\right)\right]\right\}
\end{aligned}
$$

Celestial Mechanics and Dynamical Astronomy, *2009*

Equilibrium Geometry



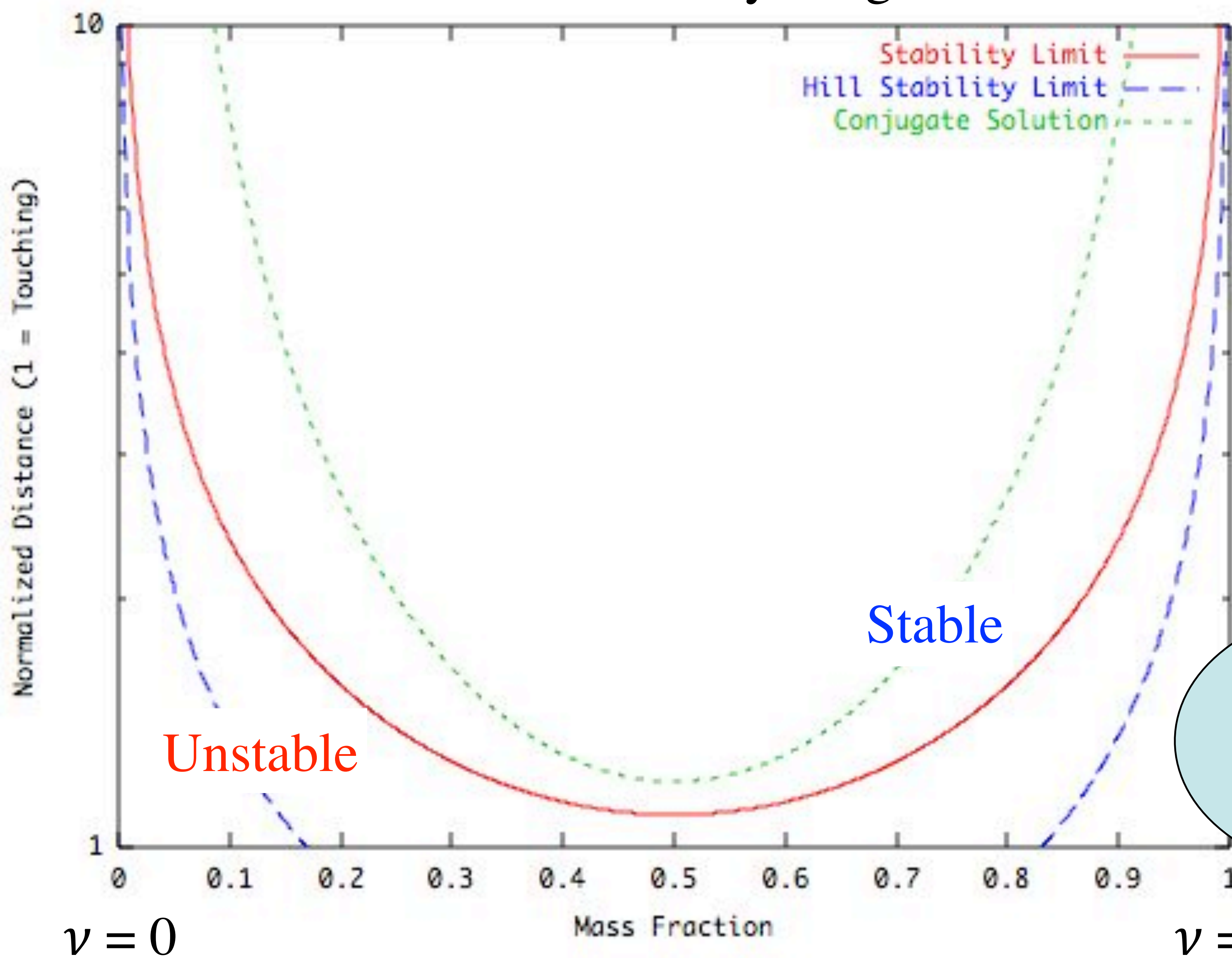
Orbital Equilibrium

$$\nu = \frac{M_2}{M_1 + M_2}$$

Resting Equilibrium

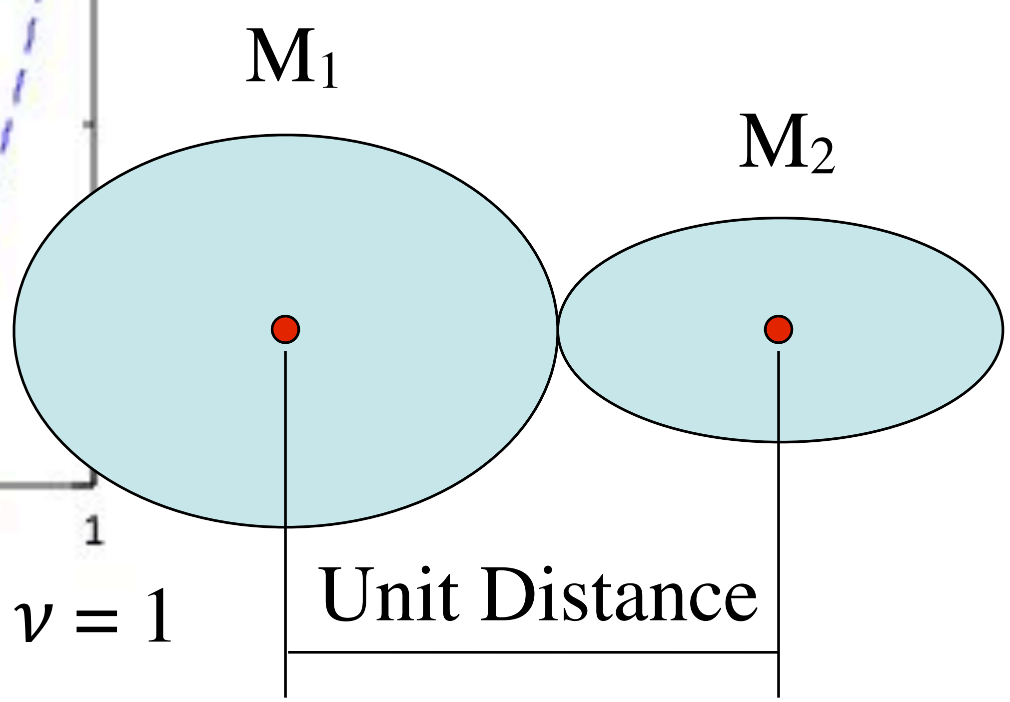
Relative Orbital Equilibrium

Generic Stability Diagram



$$\nu = \frac{M_2}{M_1 + M_2}$$

$$1 - \nu = \frac{M_1}{M_1 + M_2}$$



Relative Orbital Equilibrium

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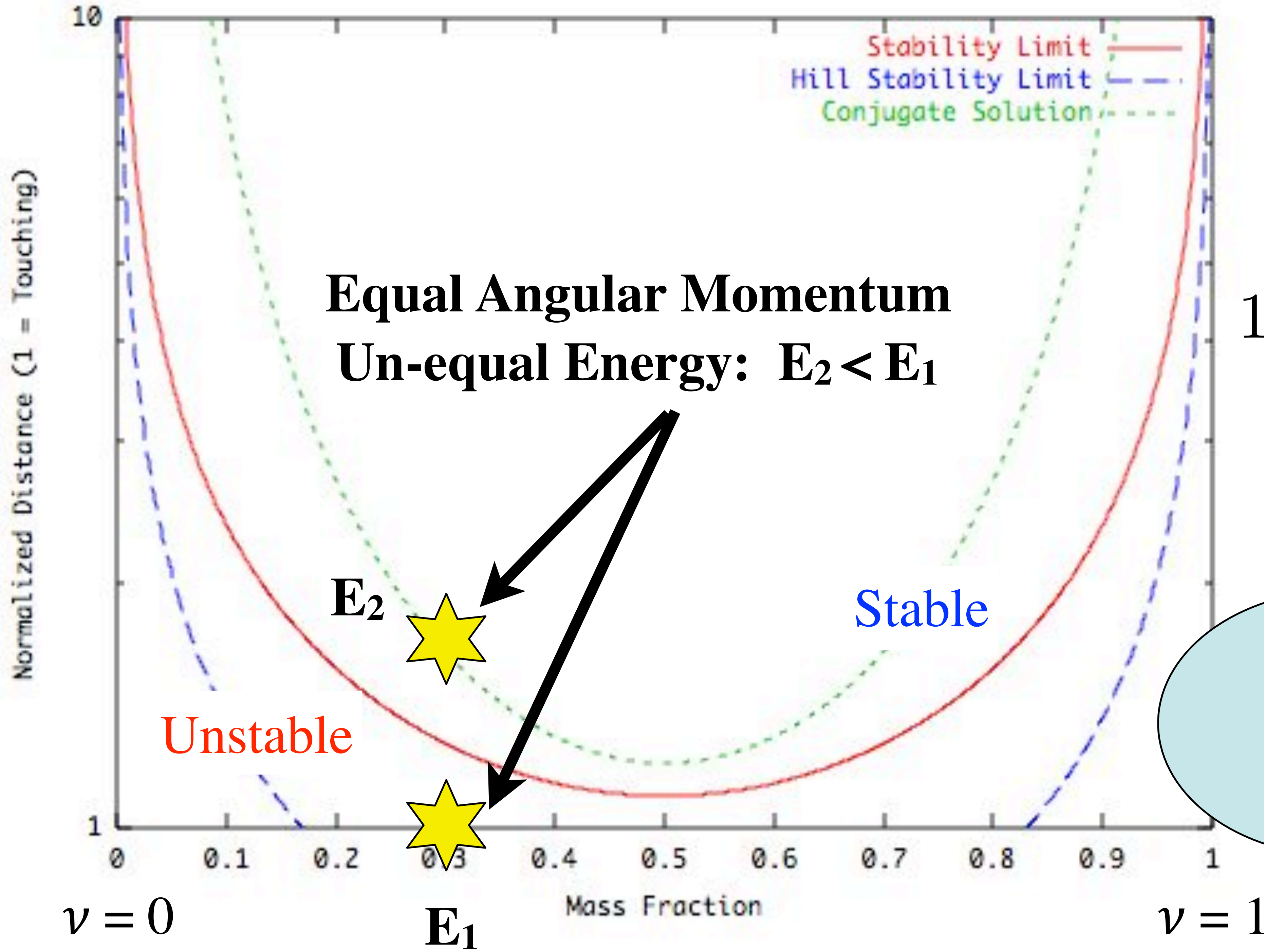

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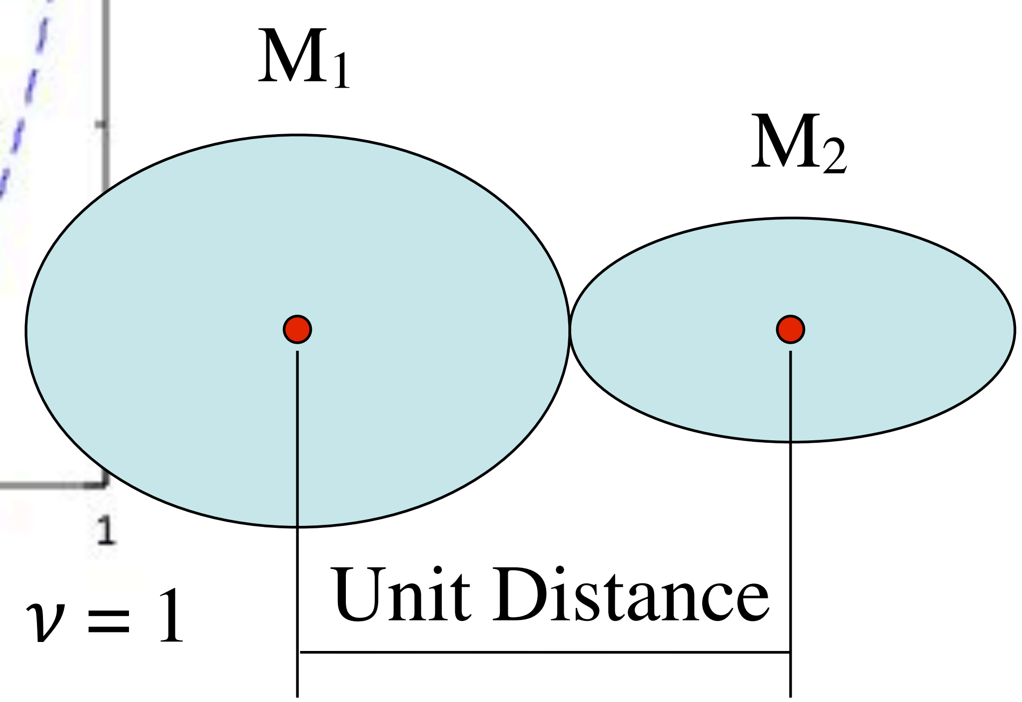
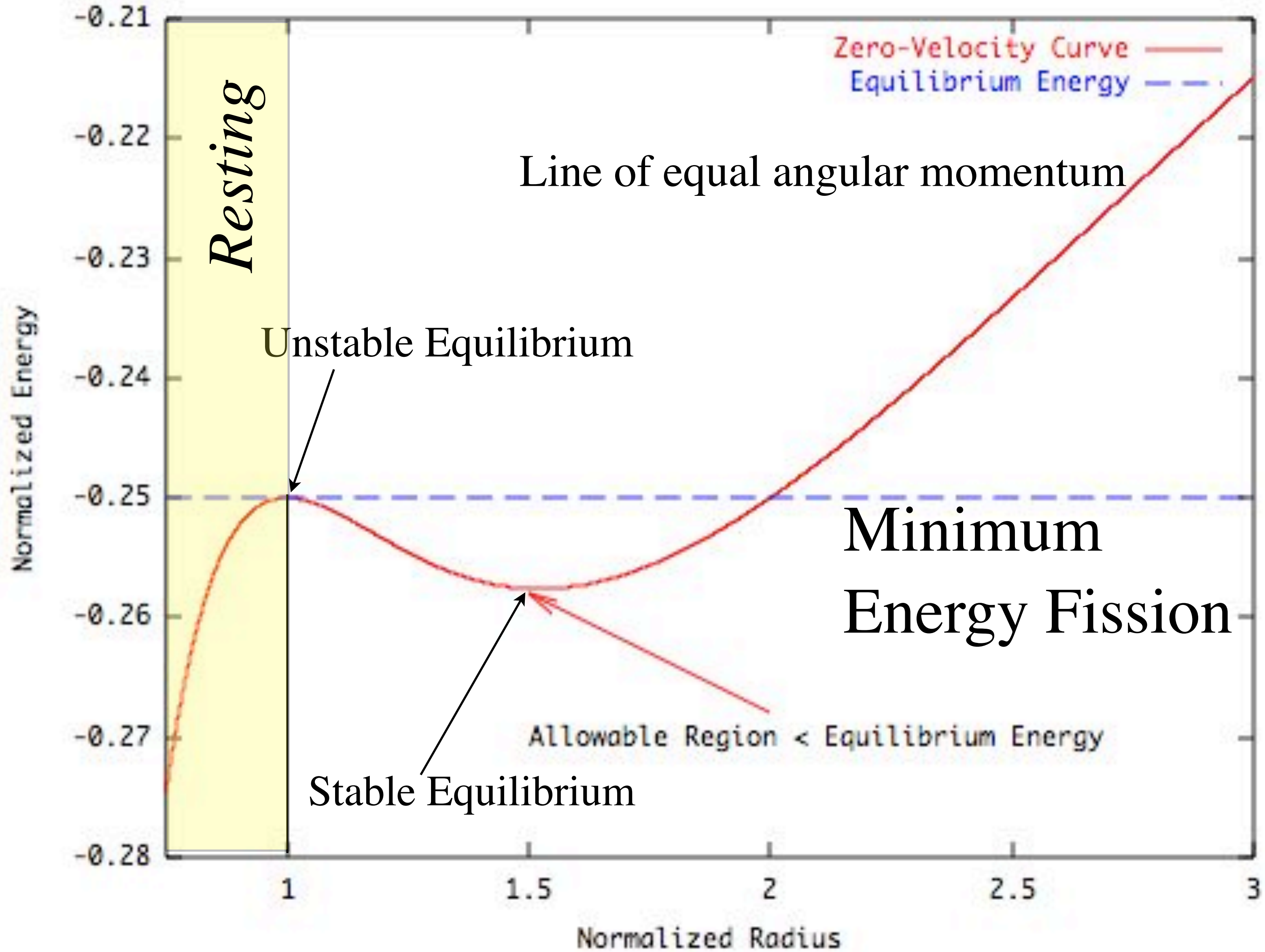


Zero-Velocity Curves and limits on motion $\nu > 0.17$



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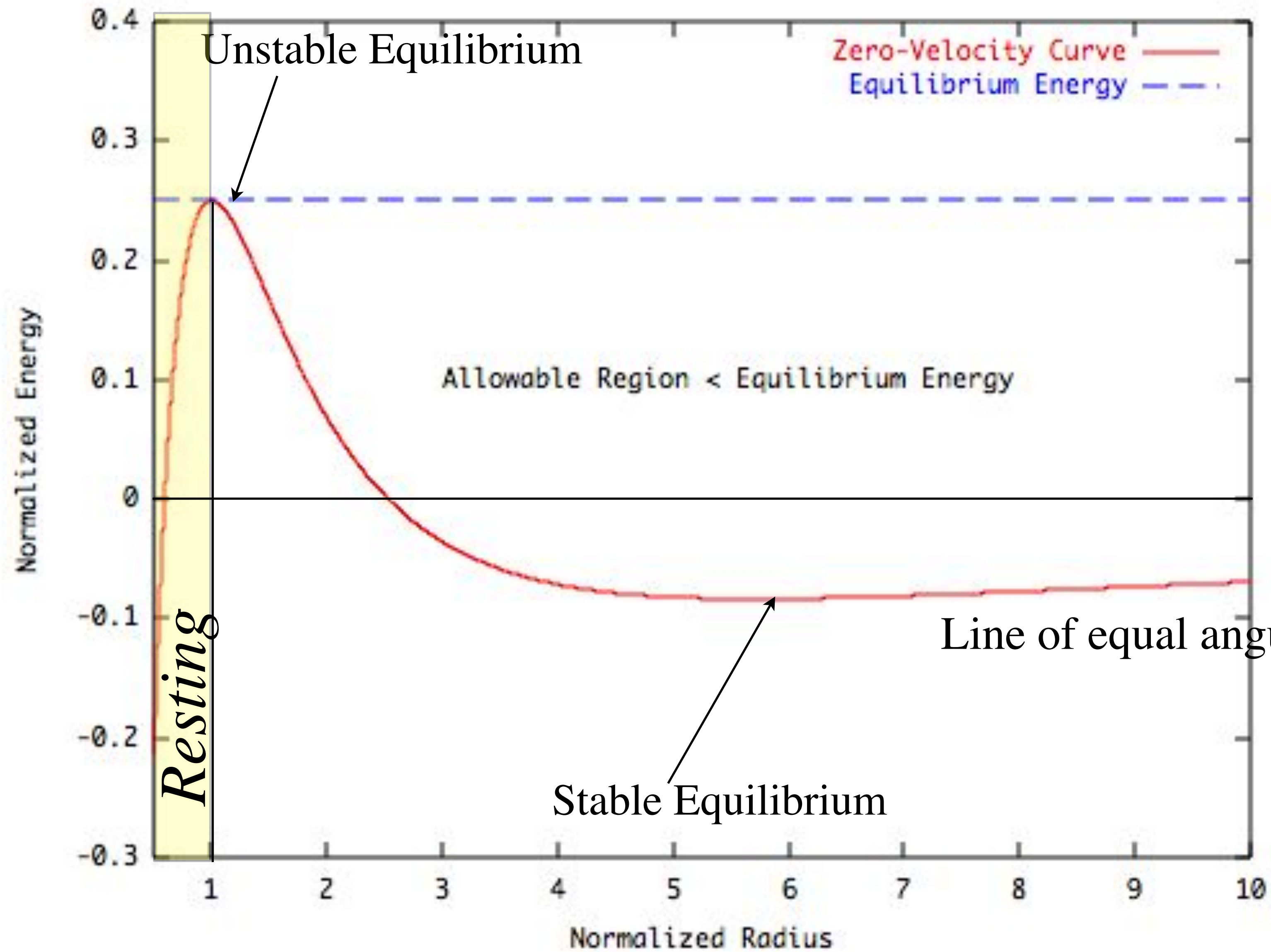


Minimum Energy Fission

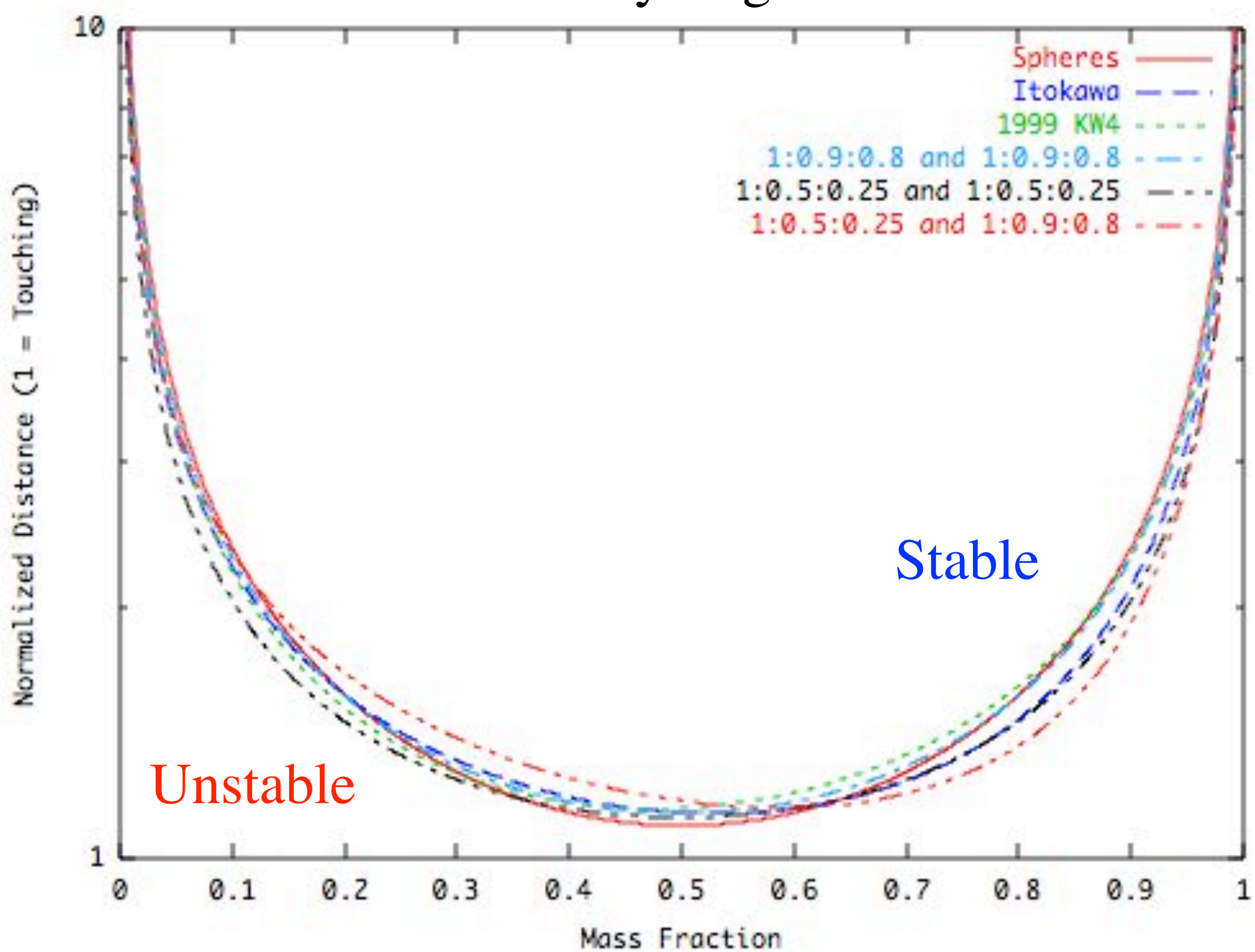
Hill Stability Energy Limit

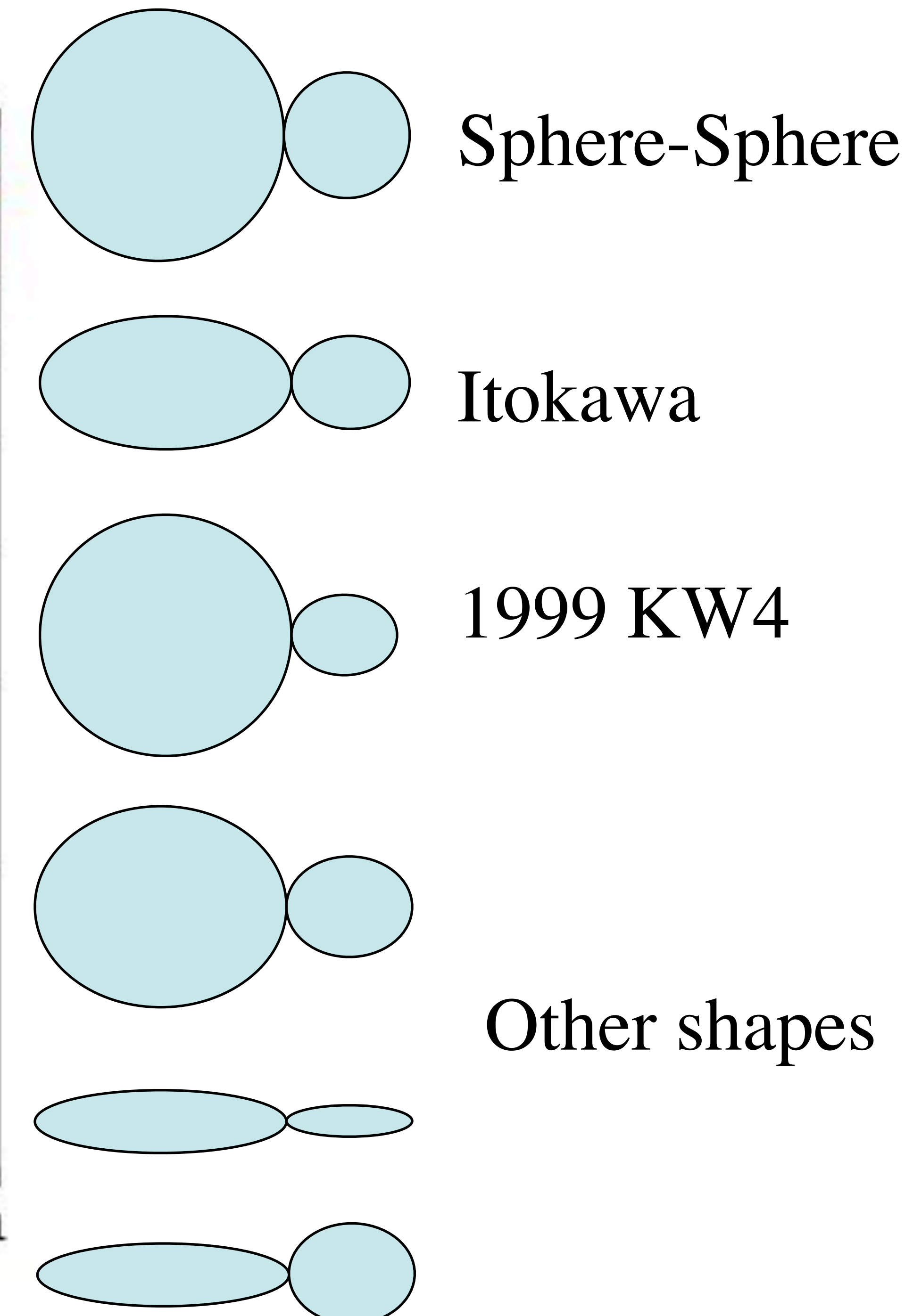
Line of equal angular momentum

Scaled Orbit Stability Results

- Scaling the systems produces generic stability curves

Stability Regions





Scaled Orbit Stability Results

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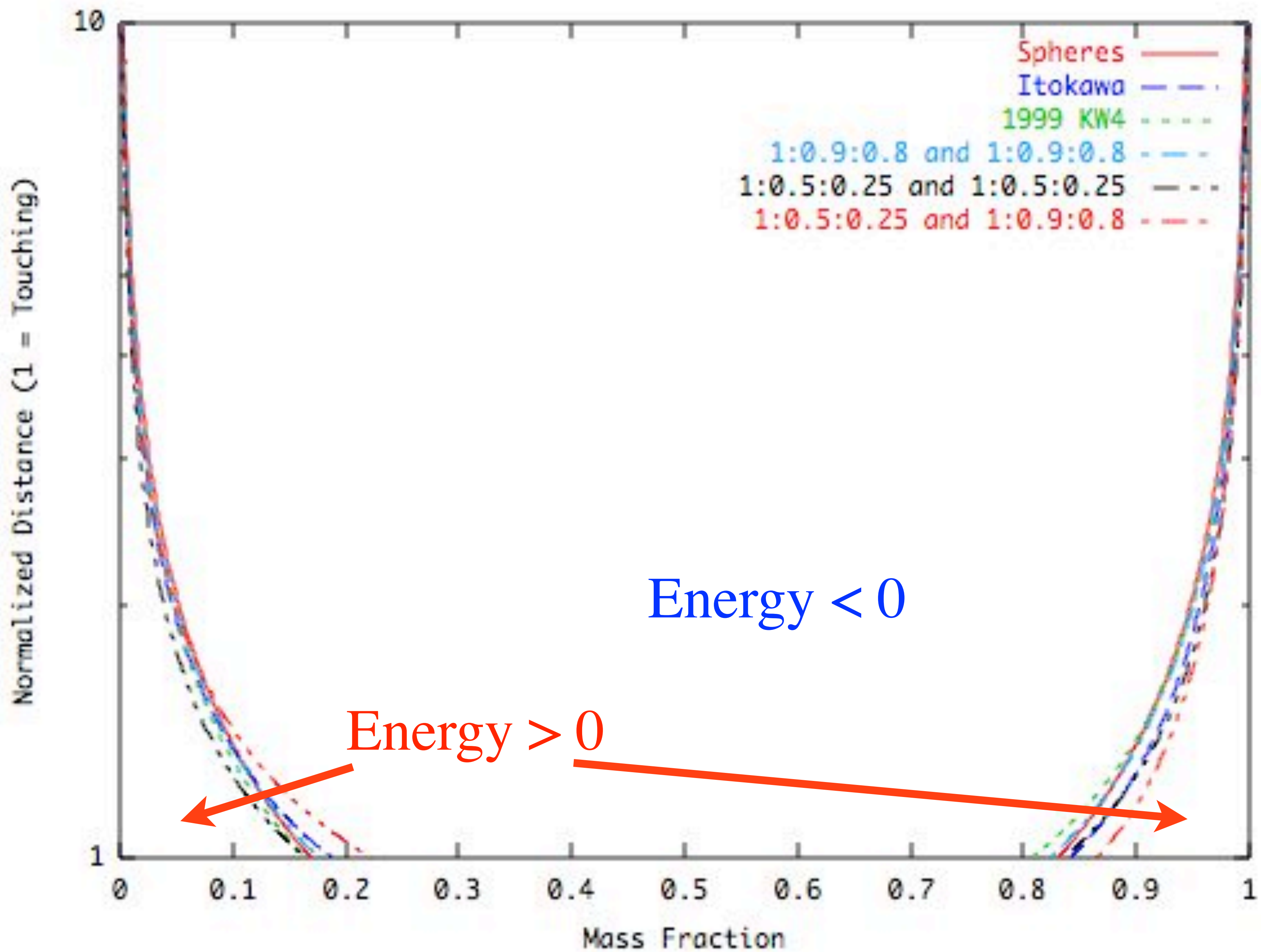
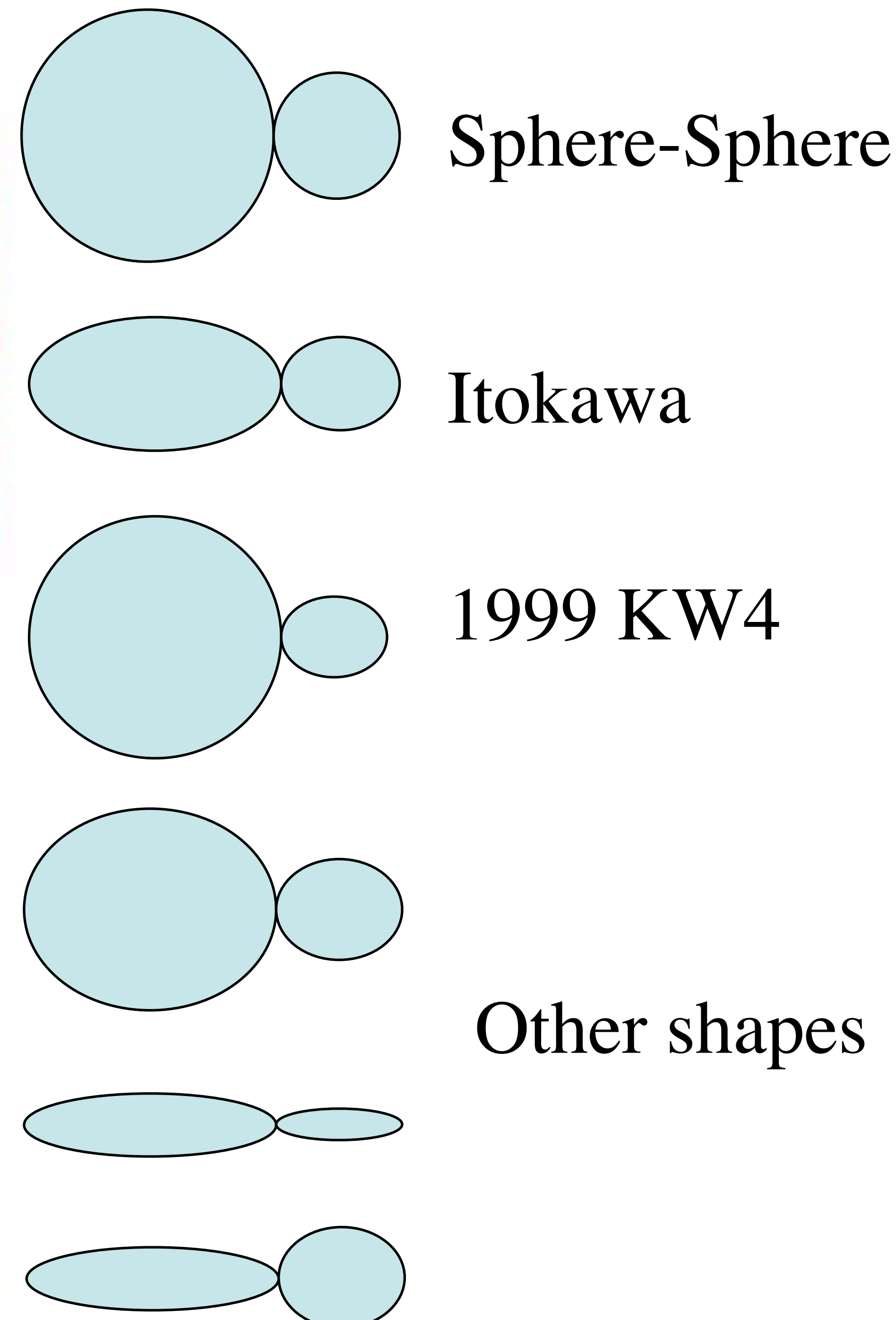
Zero Free Energy Curves





Free Energy

- The "free energy" of the system controls disruption:

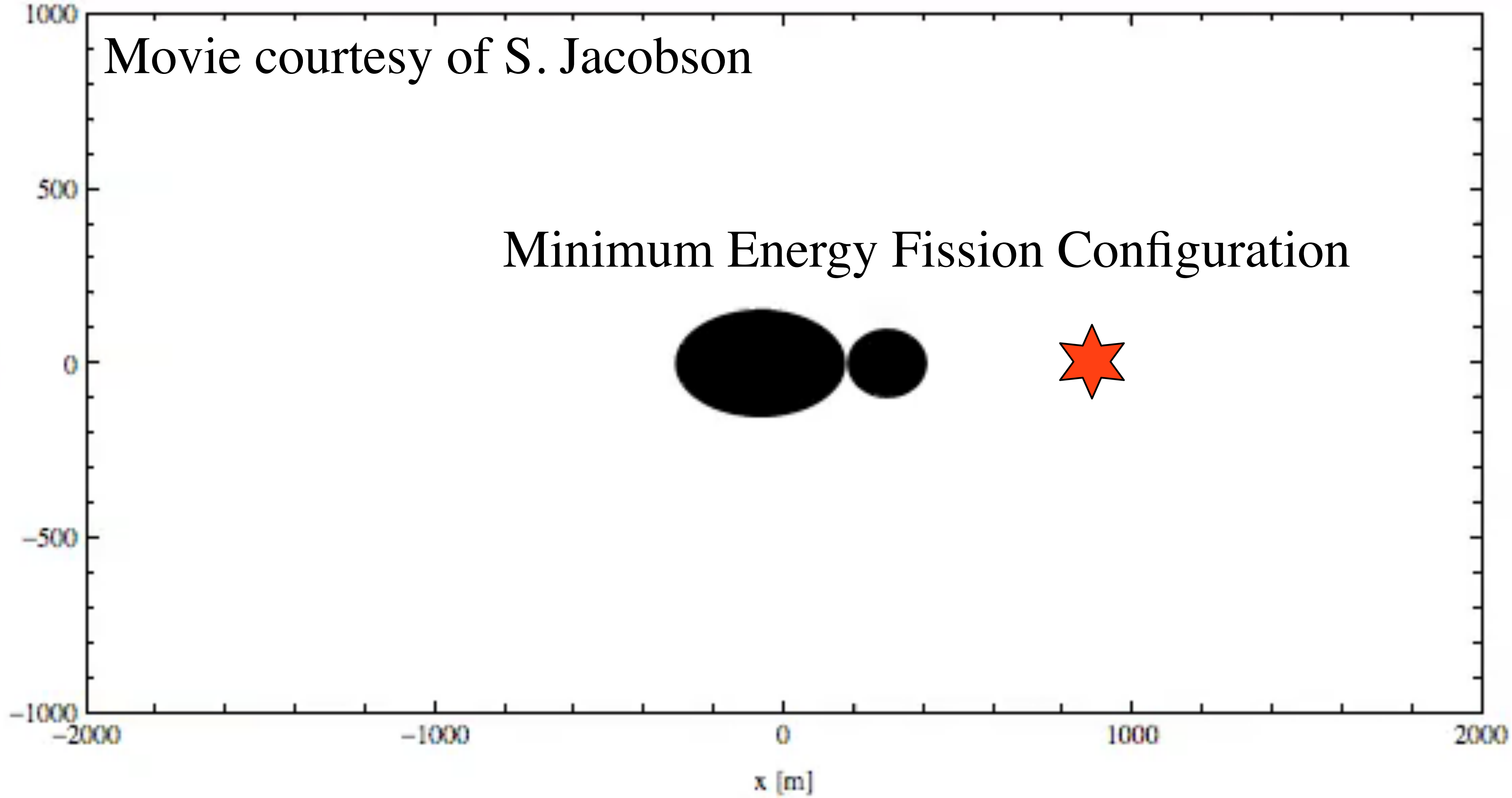
$$E_{\text{Free}} = E - \mathcal{U}_{11} - \mathcal{U}_{22}$$

$$E_{\text{Free}} = \frac{1}{2}\left[\omega_1 \cdot I_1 \cdot \omega_1 + \omega_2 \cdot I_2 \cdot \omega_2 + \frac{M_1 M_2}{M_1 + M_2} V \cdot V\right] + \mathcal{U}_{12}$$

  - If disruption occurs, the mutual potential goes to 0: $\mathcal{U}_{12} \rightarrow 0$
  - If $E_{\text{Free}} > 0$, system can "catastrophically disrupt"
  - If $E_{\text{Free}} < 0$, system cannot "catastrophically disrupt"
  - If $0 < E_{\text{Free}} << 1$ escape leads to a slowly rotating primary

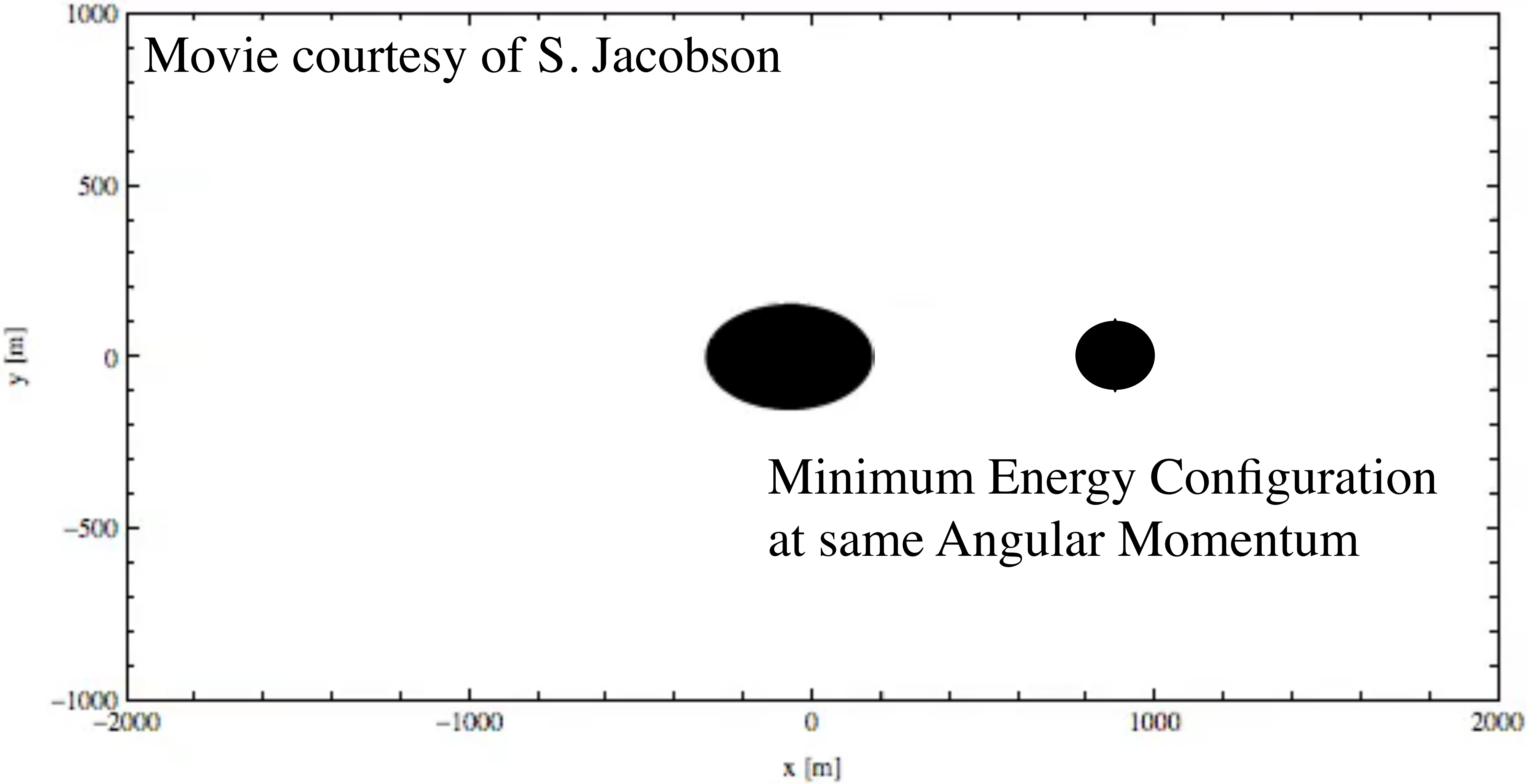
$$\frac{1}{2}\omega_1 \cdot I_1 \cdot \omega_1 \ll 1$$

Itokawa Post-Fission Dynamics



- Total system energy is negative but near zero, disruption impossible
- Re-impact is possible if initial Energy is larger than fission energy
- Relative speeds on the order of cm/s only, allows non-catastrophic re-impacts

Itokawa Post-Fission Dynamics



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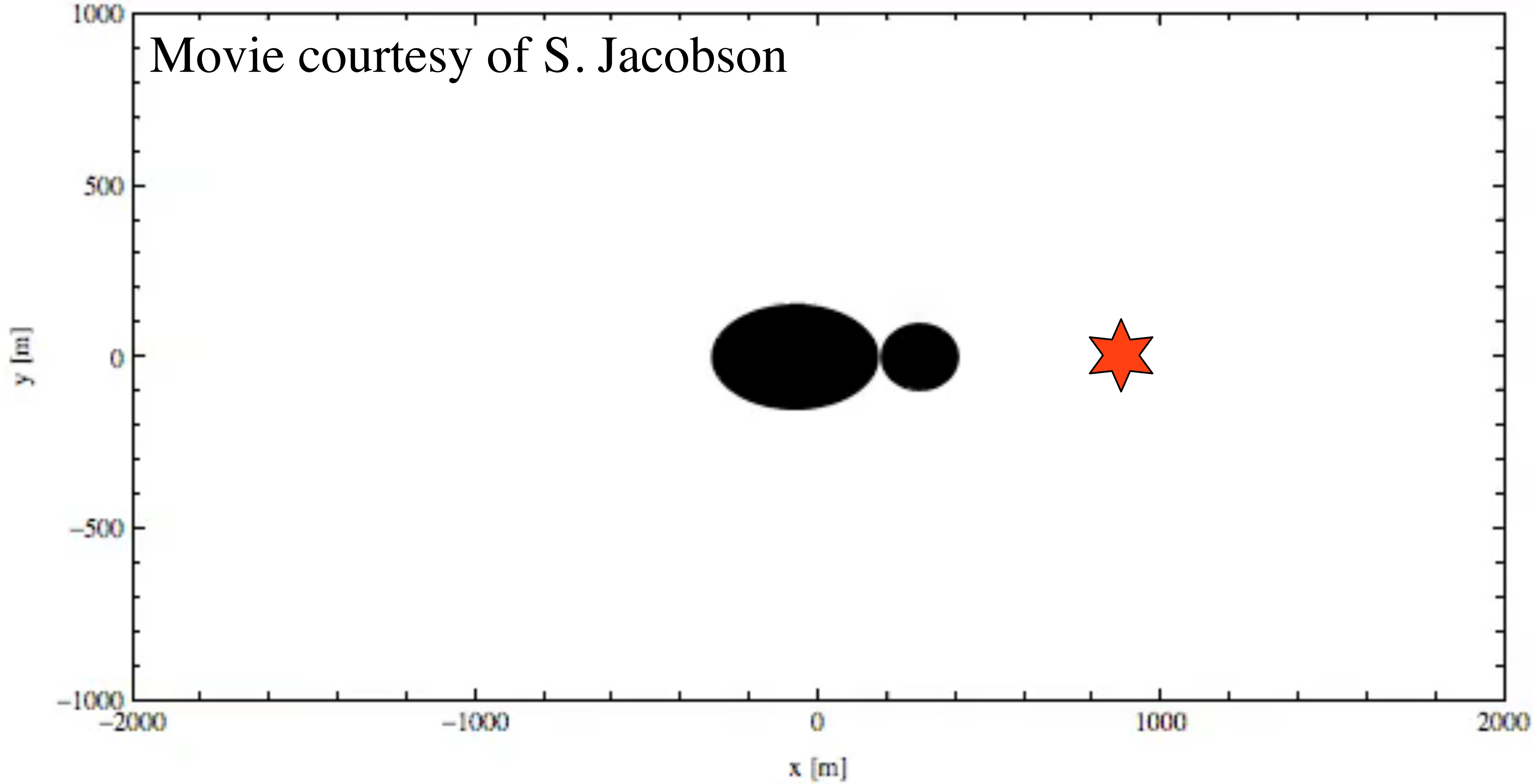

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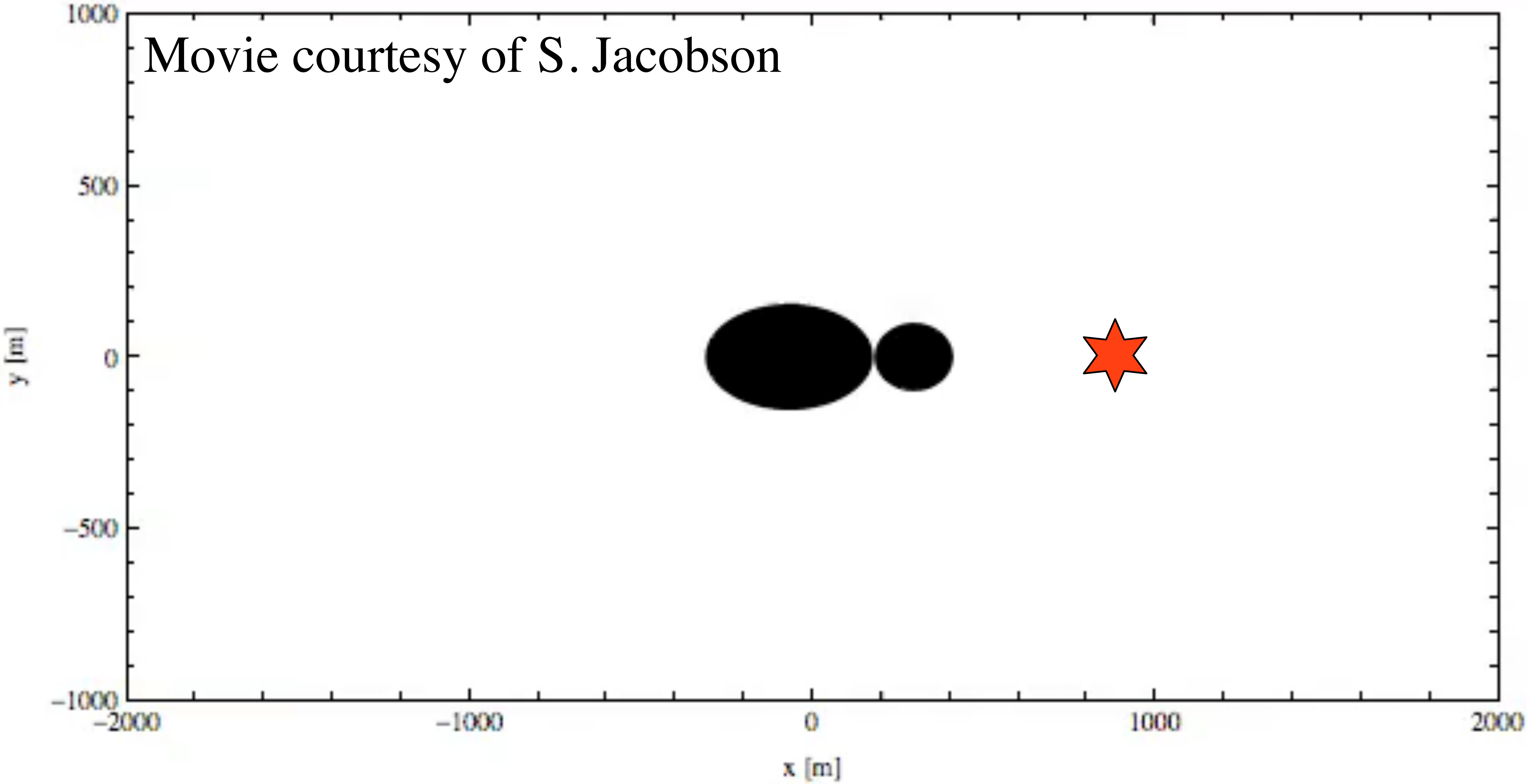

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Implications

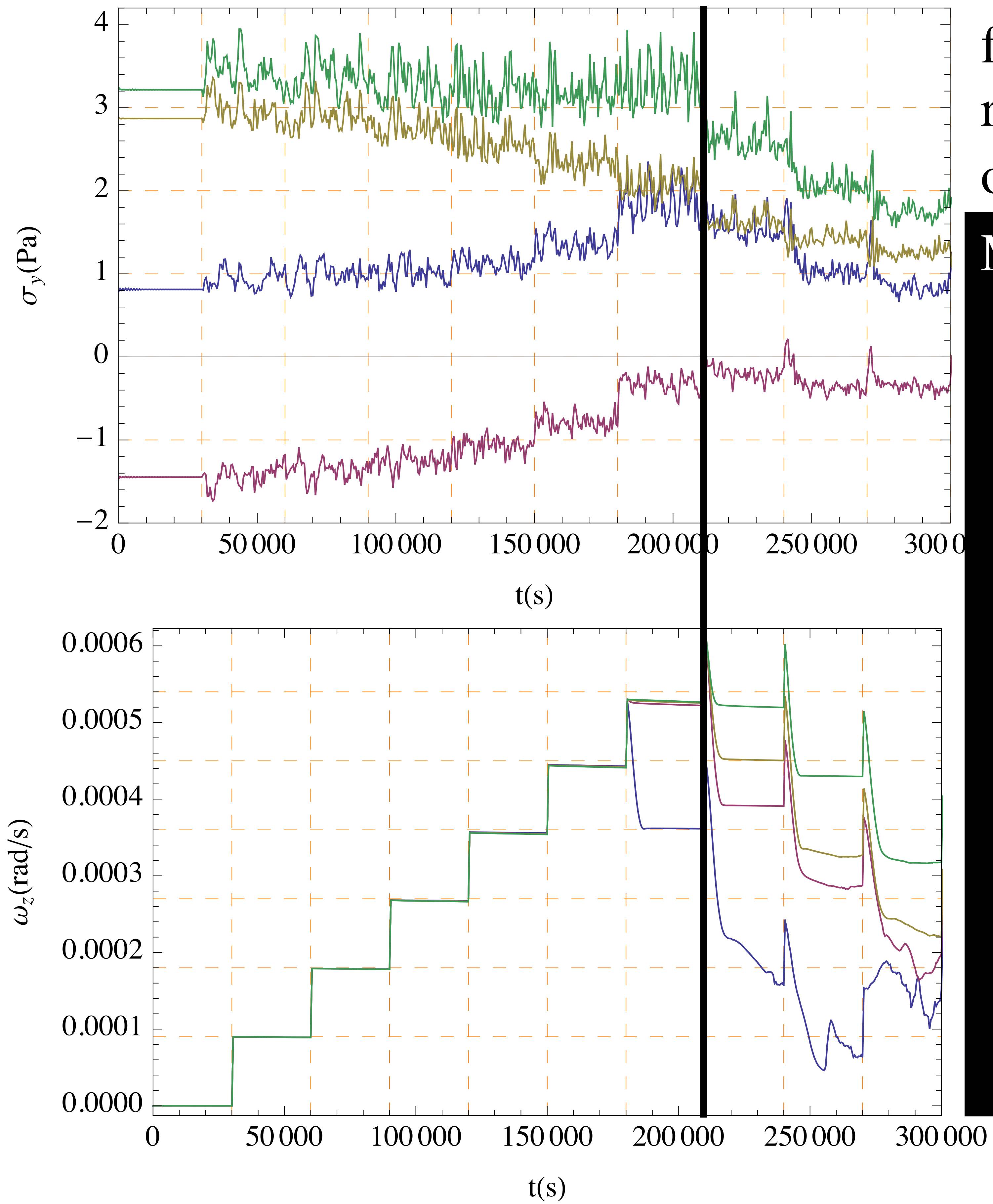
- All contact binaries spun to fission are initially unstable
  - Result holds across all shapes and mass ratios
  - Initial dynamics after fission are strongly chaotic and explore the possible phase space
  - Non-classical tidal dissipation during early evolutionary phases

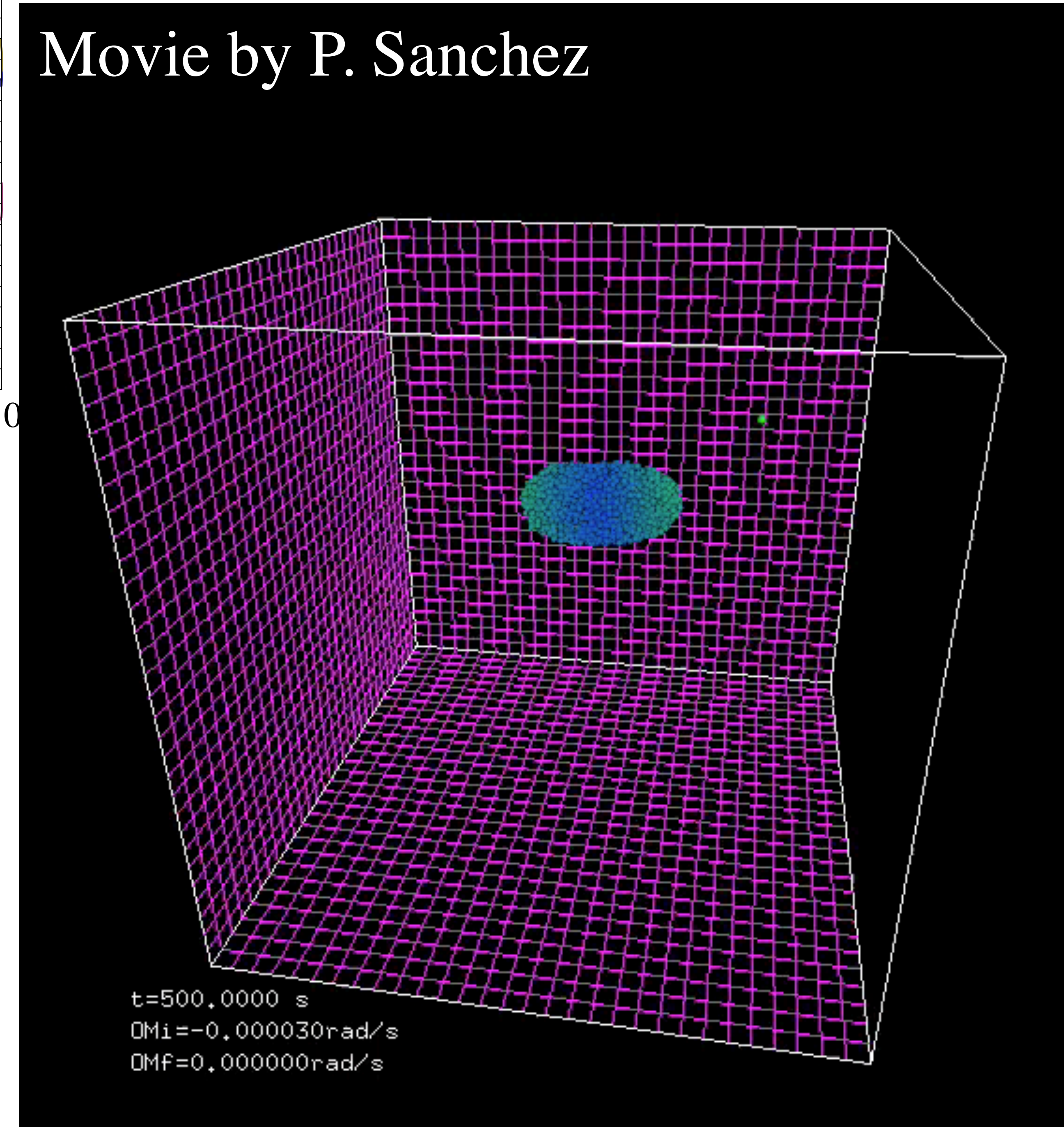
Implications

- All contact binaries spun to fission are initially unstable
  - Result holds across all shapes and mass ratios
  - Initial dynamics after fission are strongly chaotic
- Contact binaries with a small (large) enough mass fraction can mutually escape when spun to fission
  - Mass fraction limits are $\nu < 0.17$ or $\nu > 0.83$
  - Mass ratio limits are < 0.2 or > 0.8
  - Mean radius ratios < 0.59
  - Applies to all "a-synchronous binaries", creating difficulties for simple spin-fission "birth" of binary asteroids

Formation of Asteroid Binaries and Pairs

As spin rate increases, internal stresses increase to the point where failure criteria are violated, leading to reshaping and, in some cases, fission of the body.





Sanchez & Scheeres, *Icarus 2012*

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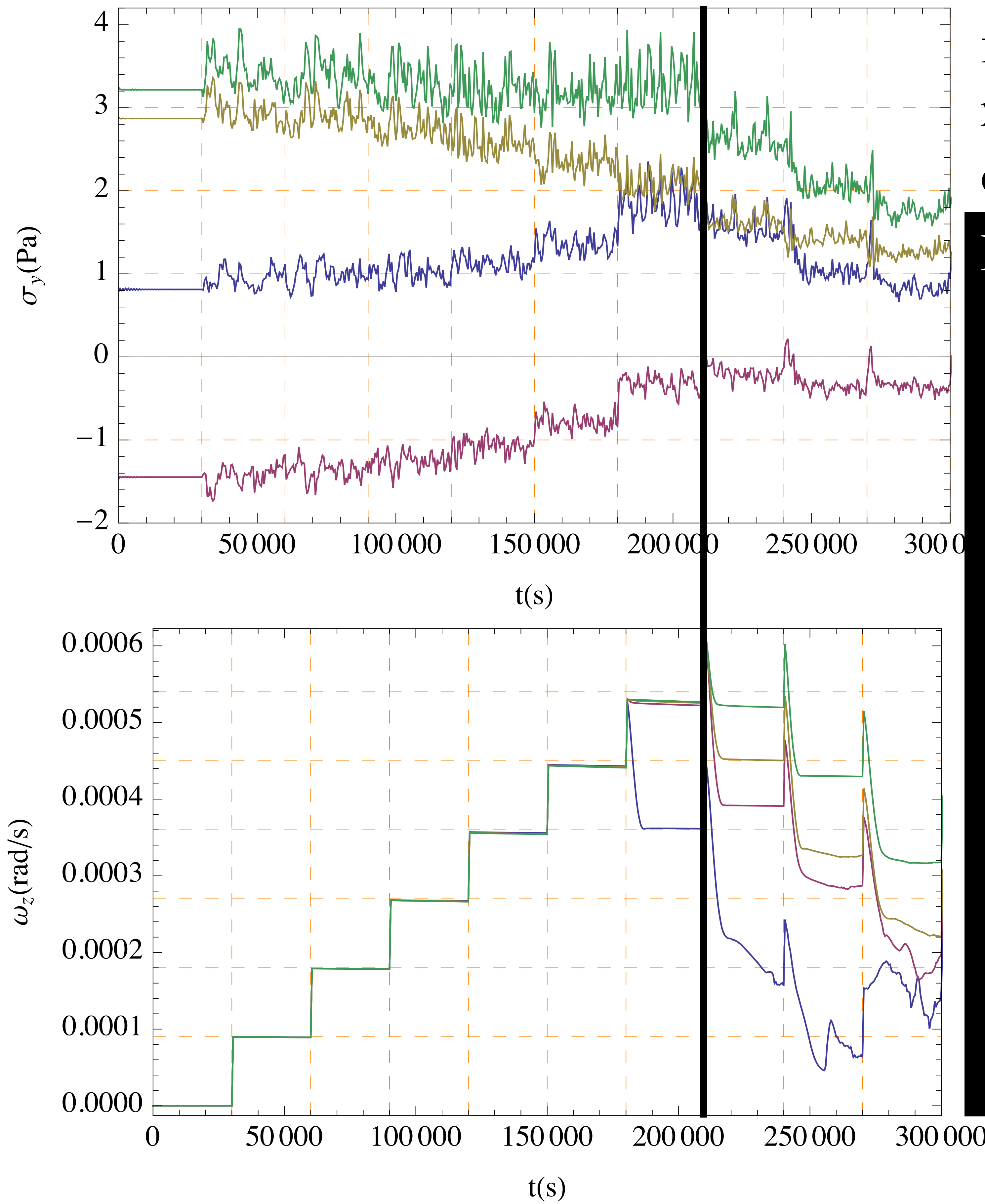

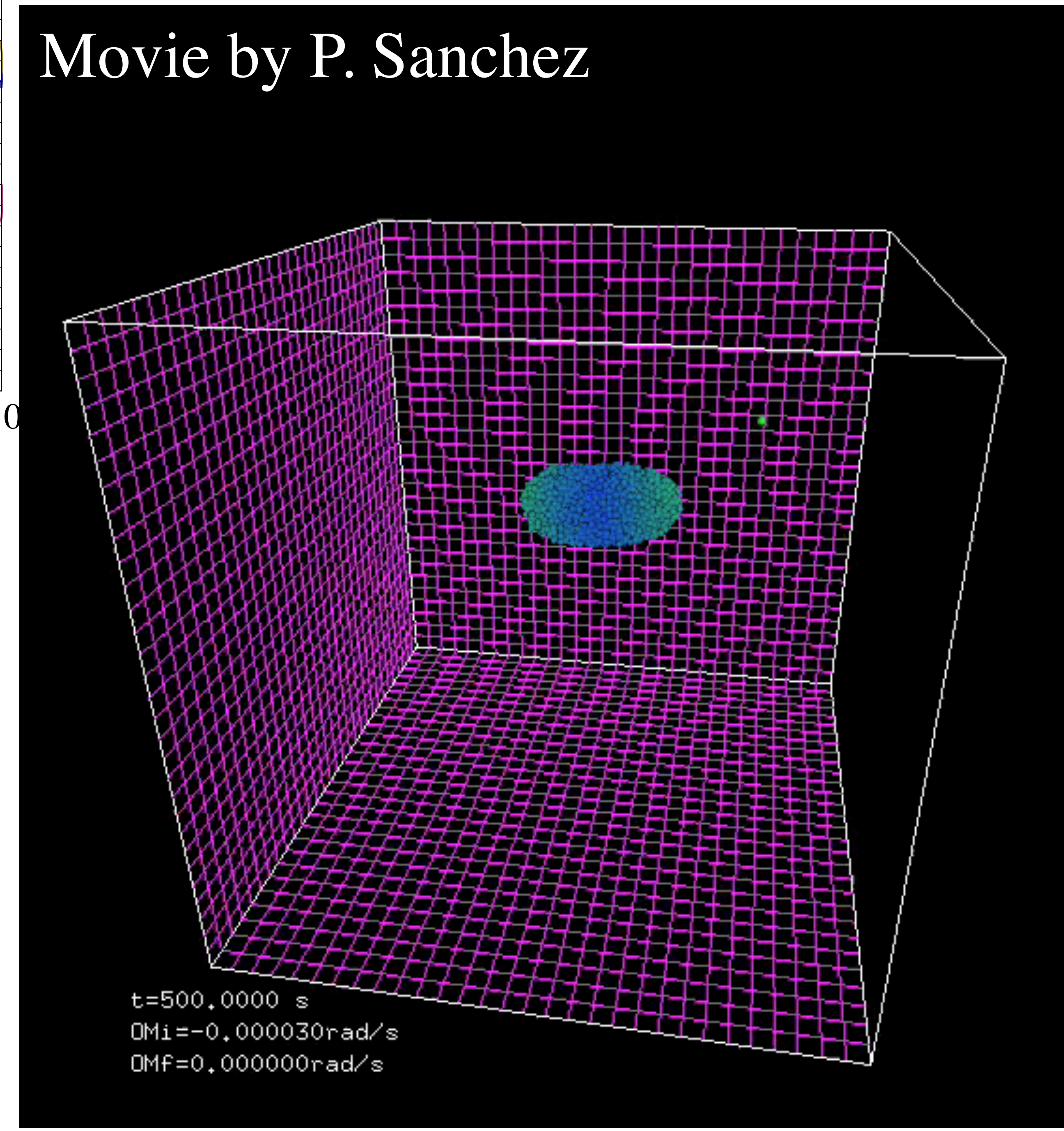

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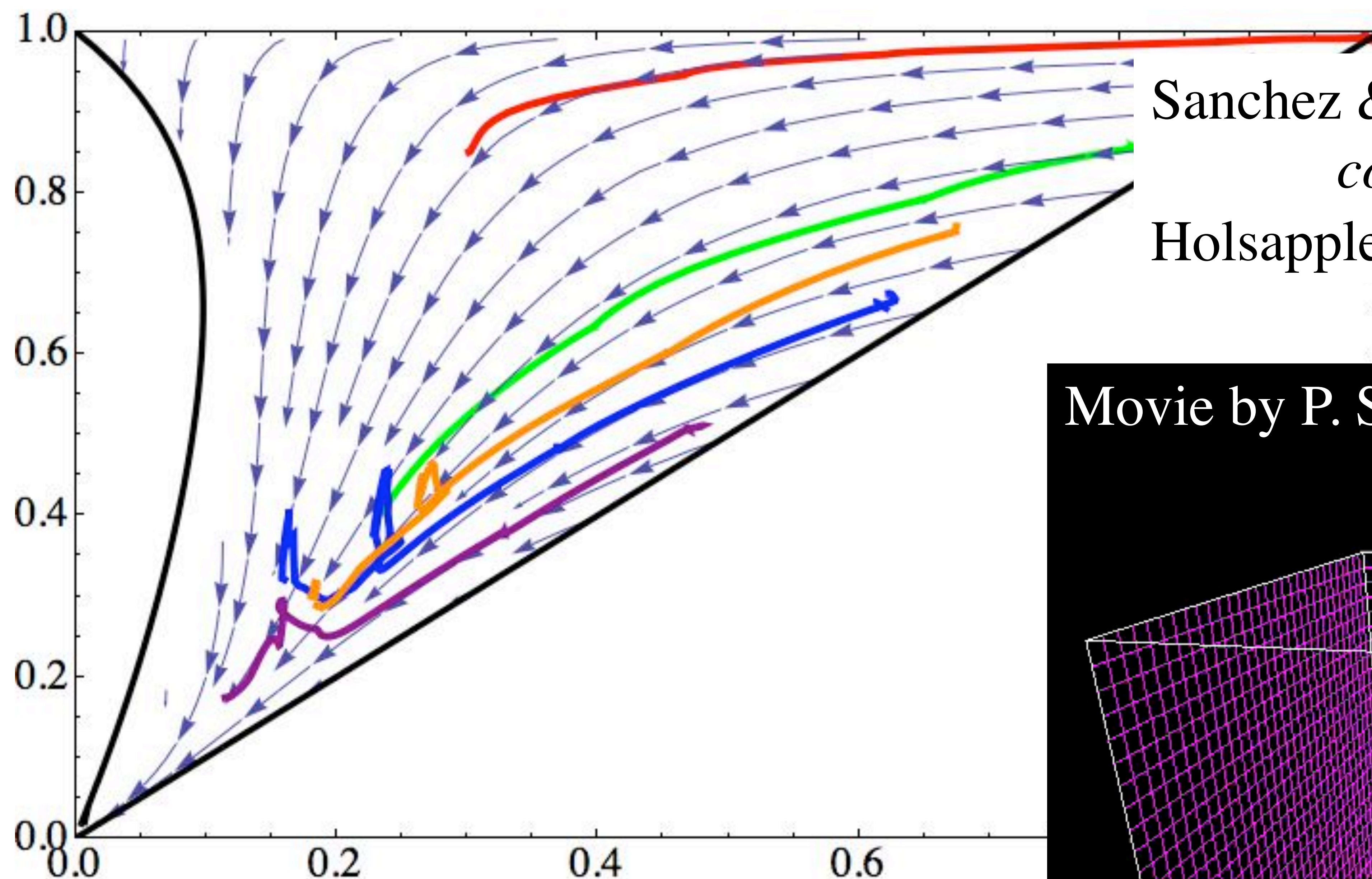
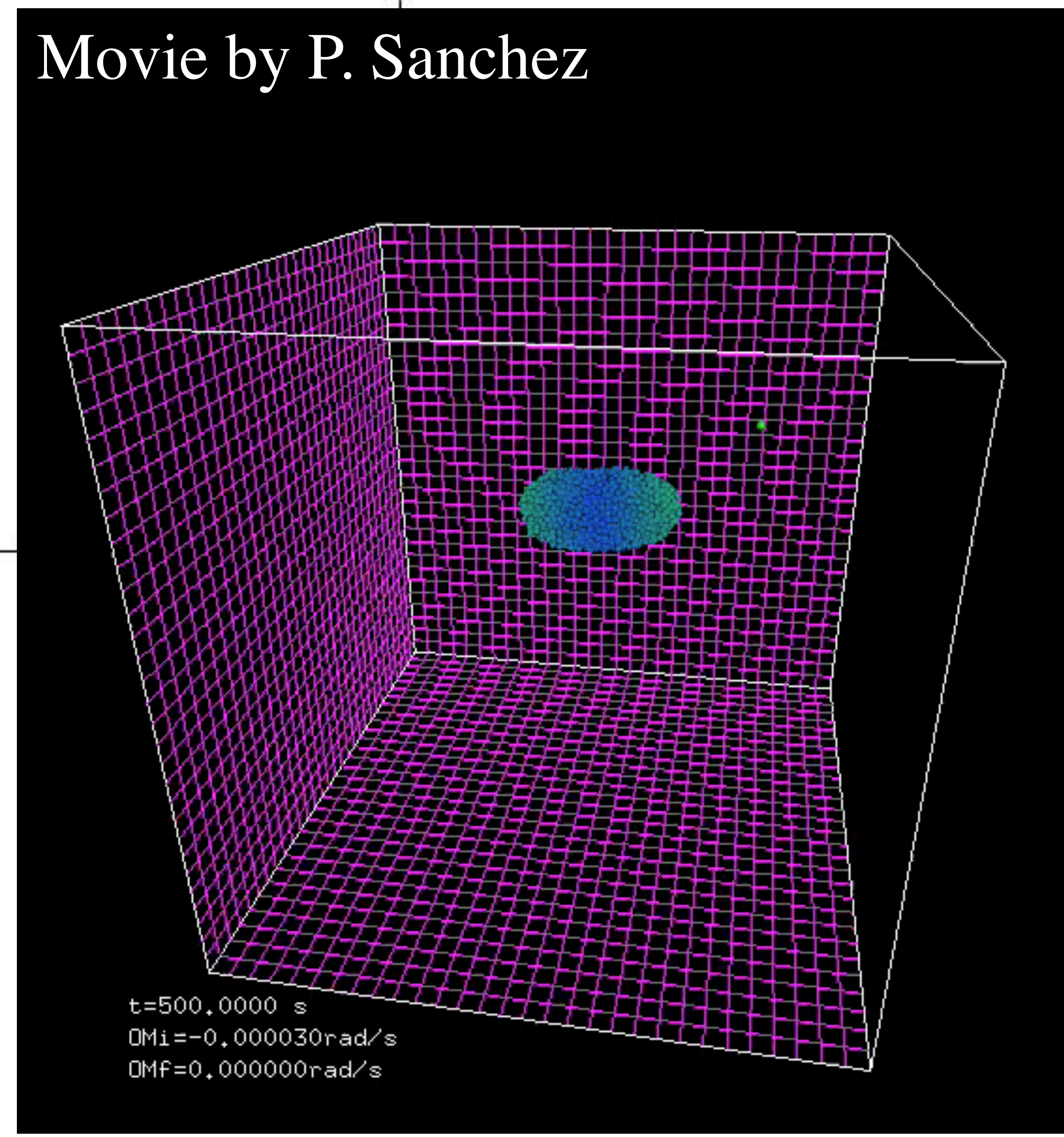


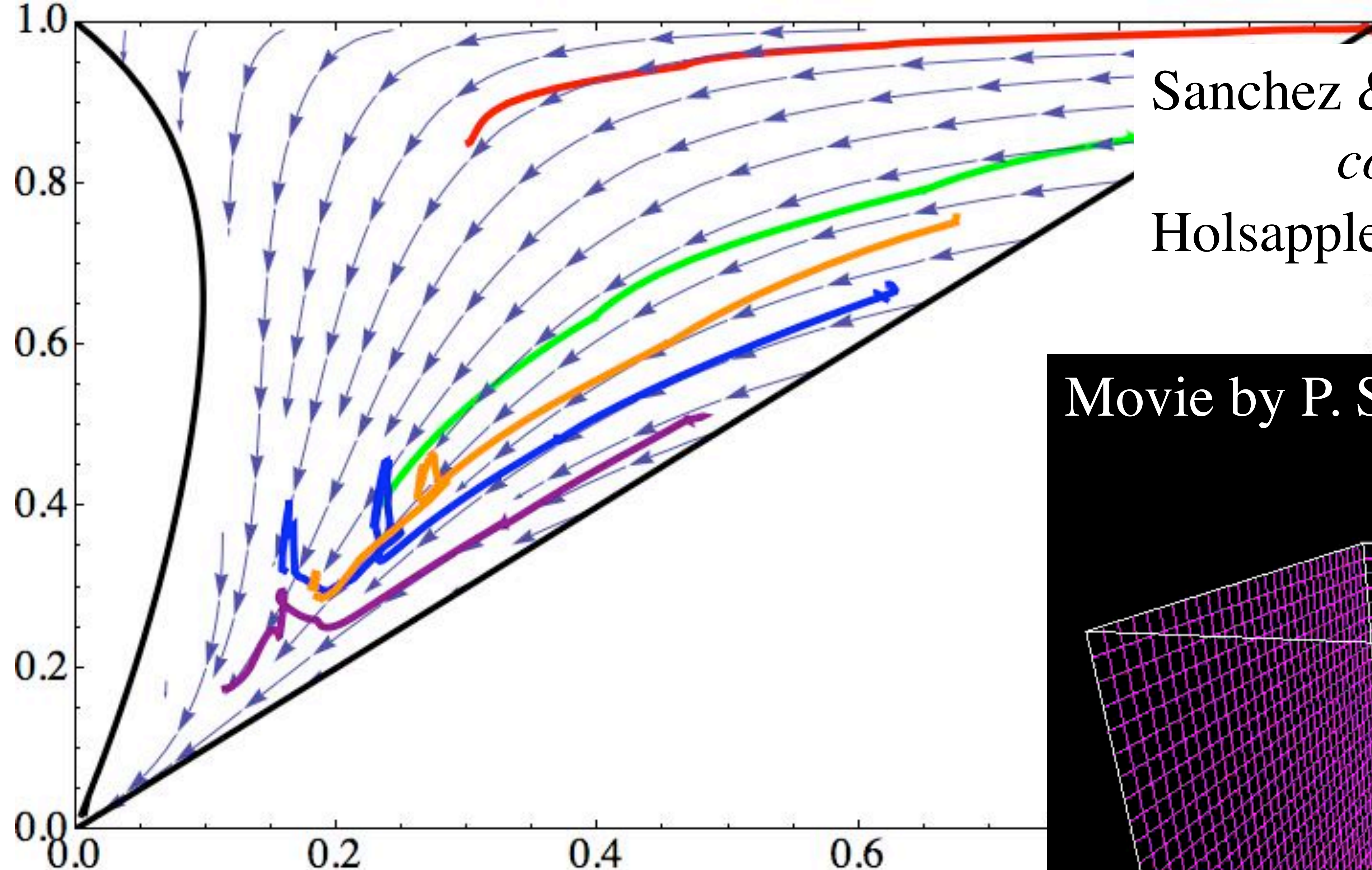
Sanchez & Scheeres, *Icarus 2012*
comparisons with
Holsapple, *Icarus 2007*



Sanchez & Scheeres, *Icarus 2012*

Discrete element method dynamical computations agree with continuum mechanics models.

Can control relative size of components with initial distributions



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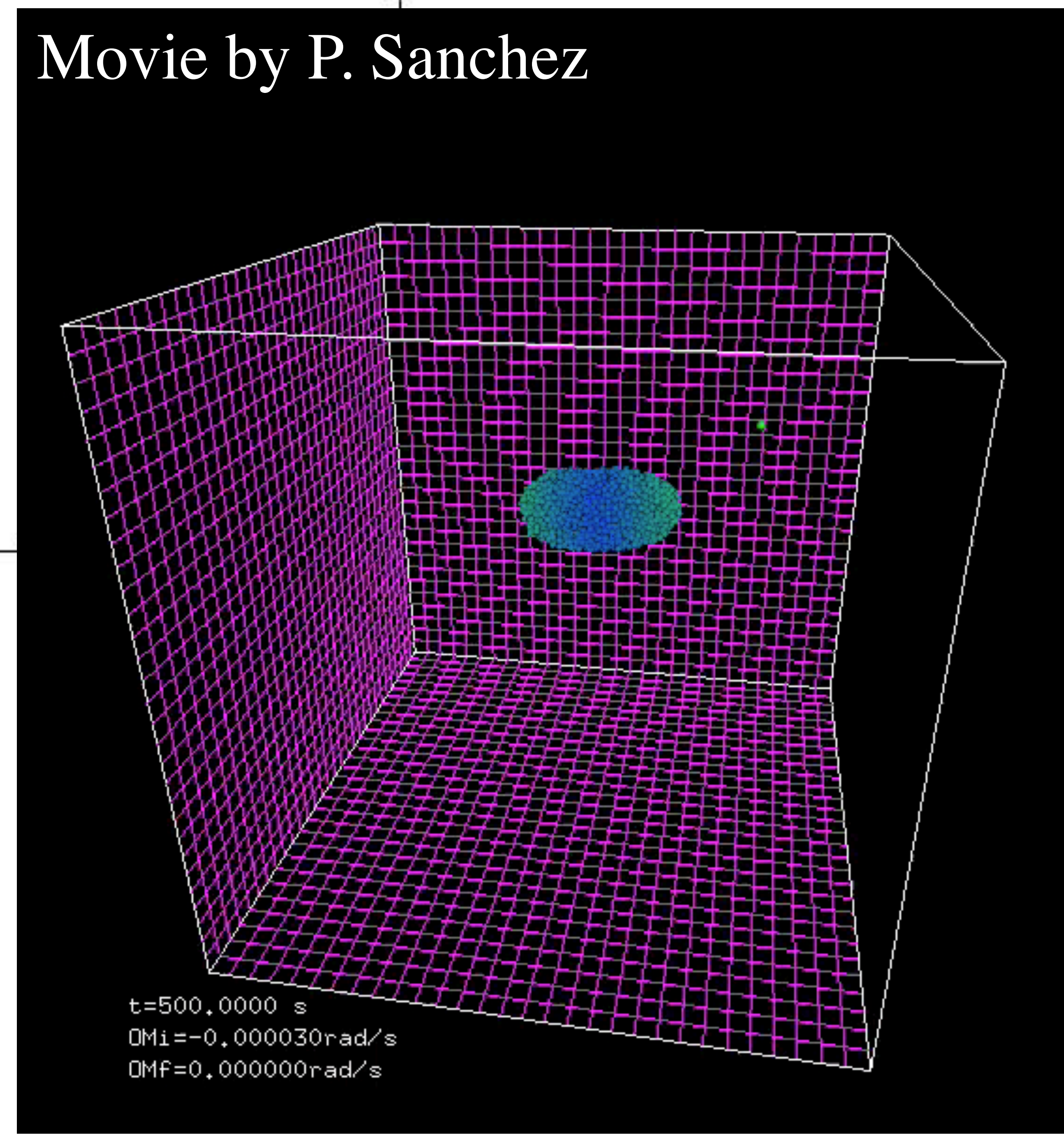

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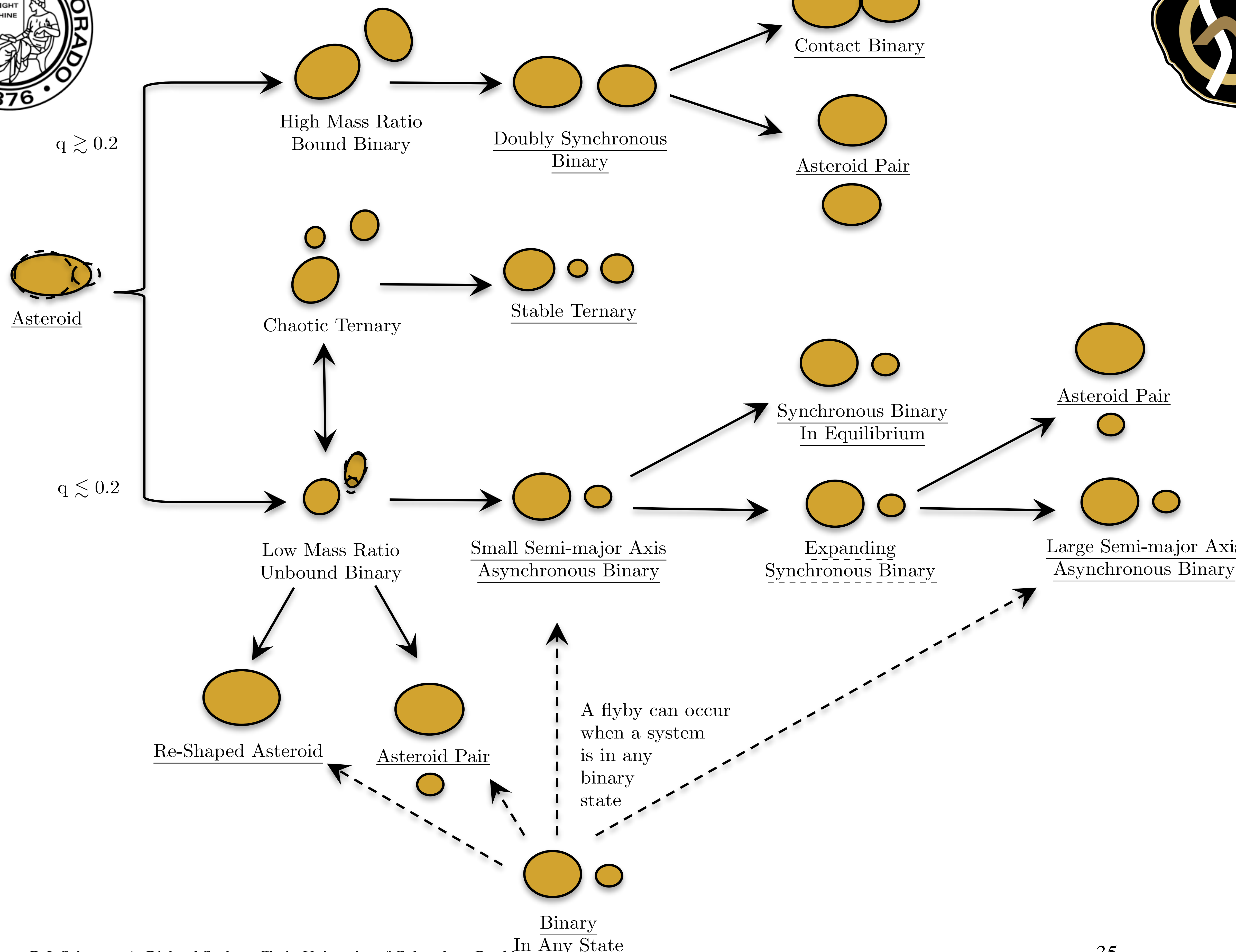
High Mass Ratio
Bound Binary
Doubly Synchronous
Binary
Contact Binary
Asteroid Pair
q ≳ 0.2
Asteroid
Chaotic Ternary
Stable Ternary
Synchronous Binary
In Equilibrium
Asteroid Pair
q ≲ 0.2
Low Mass Ratio
Unbound Binary
Small Semi-major Axis
Asynchronous Binary
Expanding
Synchronous Binary
Large Semi-major Axis
Asynchronous Binary
Re-Shaped Asteroid
Asteroid Pair
A flyby can occur
when a system
is in any
binary
state
Binary
In Any State


Contact Binary
High Mass Ratio
Bound Binary
Doubly Synchronous
Binary
Asteroid Pair
$q \gtrsim 0.2$
Asteroid
Chaotic Ternary
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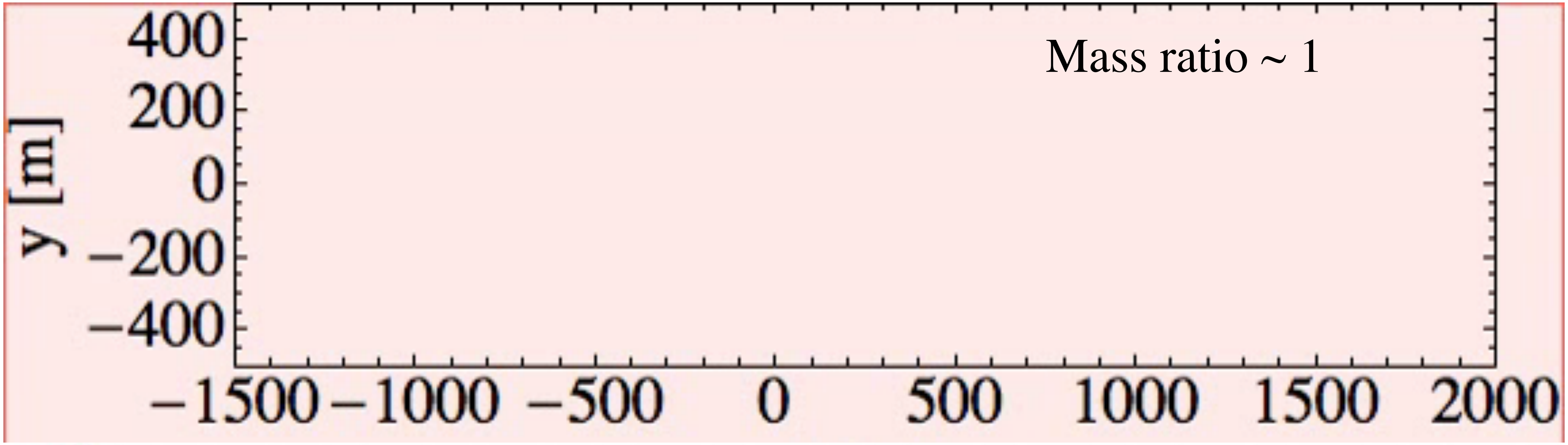
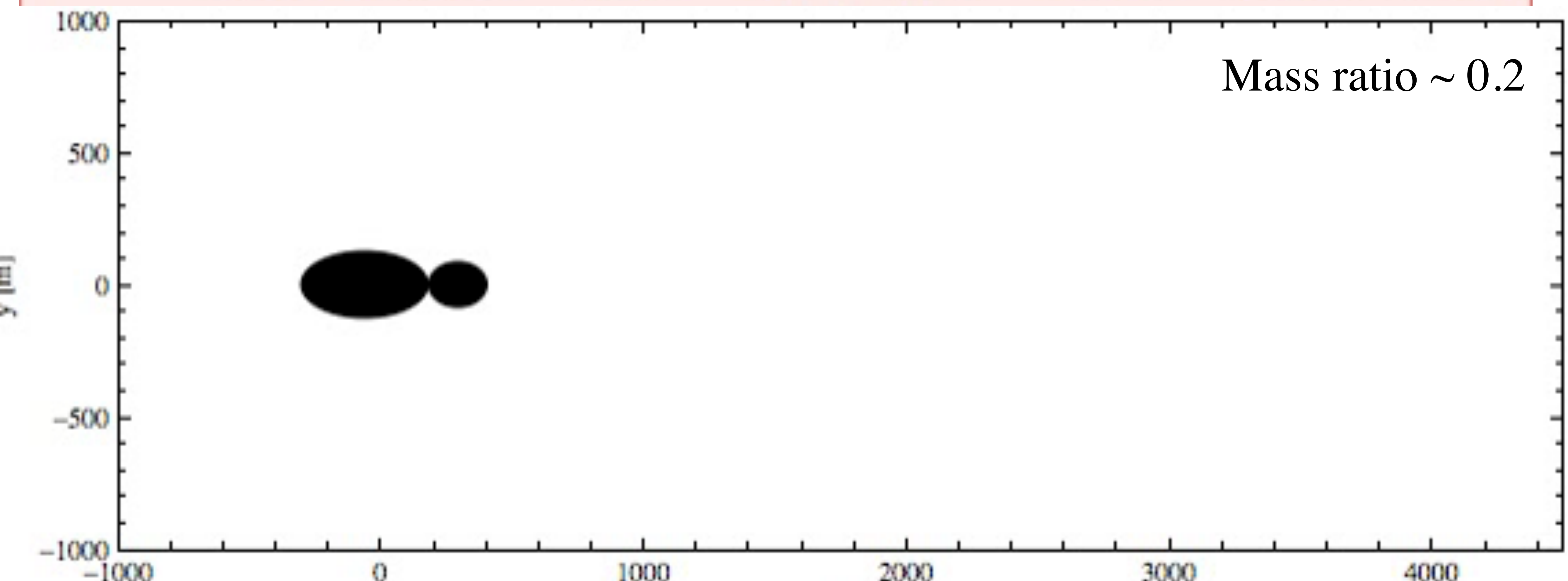
Fission: Mass Ratio > 0.2

- Mass Ratio > 0.2
  - Insufficient energy to mutually escape from each other
  - Insufficient energy to undergo relative circulation / 2nd spin fission
- Tides will cause synchronization of both bodies
  - Following full or partial synchronization will evolve via BYORP
- BYORP effect for each body adds and evolves them…
  - Mutually outwards: More rapid evolution (25%)
    - High Mass Ratio Asteroid Pairs / Susceptible to tidal disruption
  - Mutually inwards: More rapid collapse -> Contact Binaries (25%)
    - Castalia, 1996 HW1, Toutatis, etc…
    - Inability to circulate when spun to fission may lock them into this state
  - Opposite & Competing: Stable relative equilibria possible
    - Hermes?

Hermes/Itokawa Simulations





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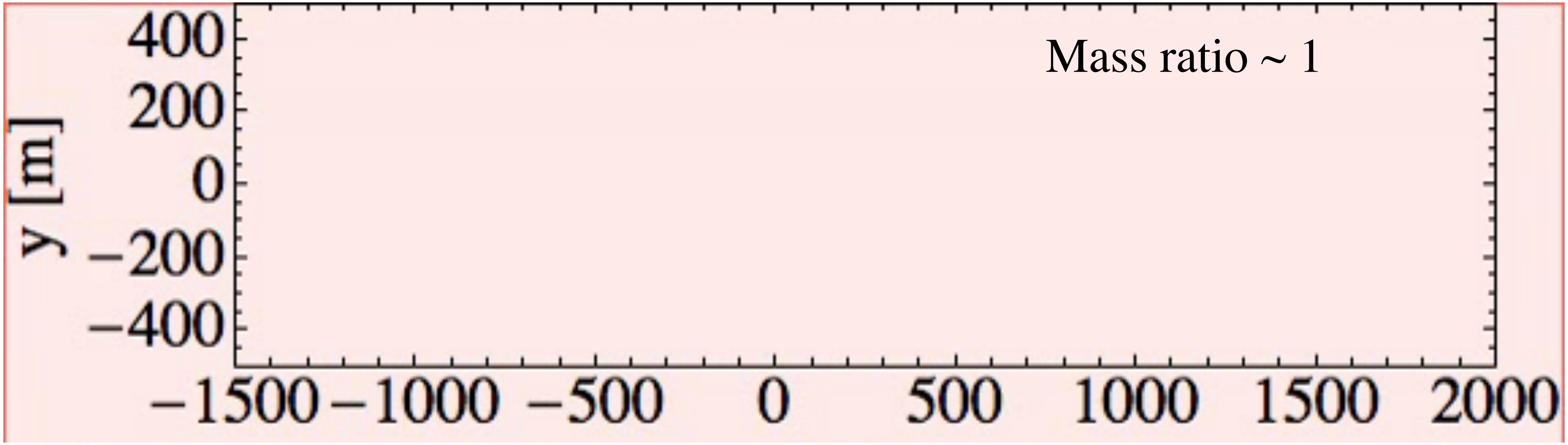

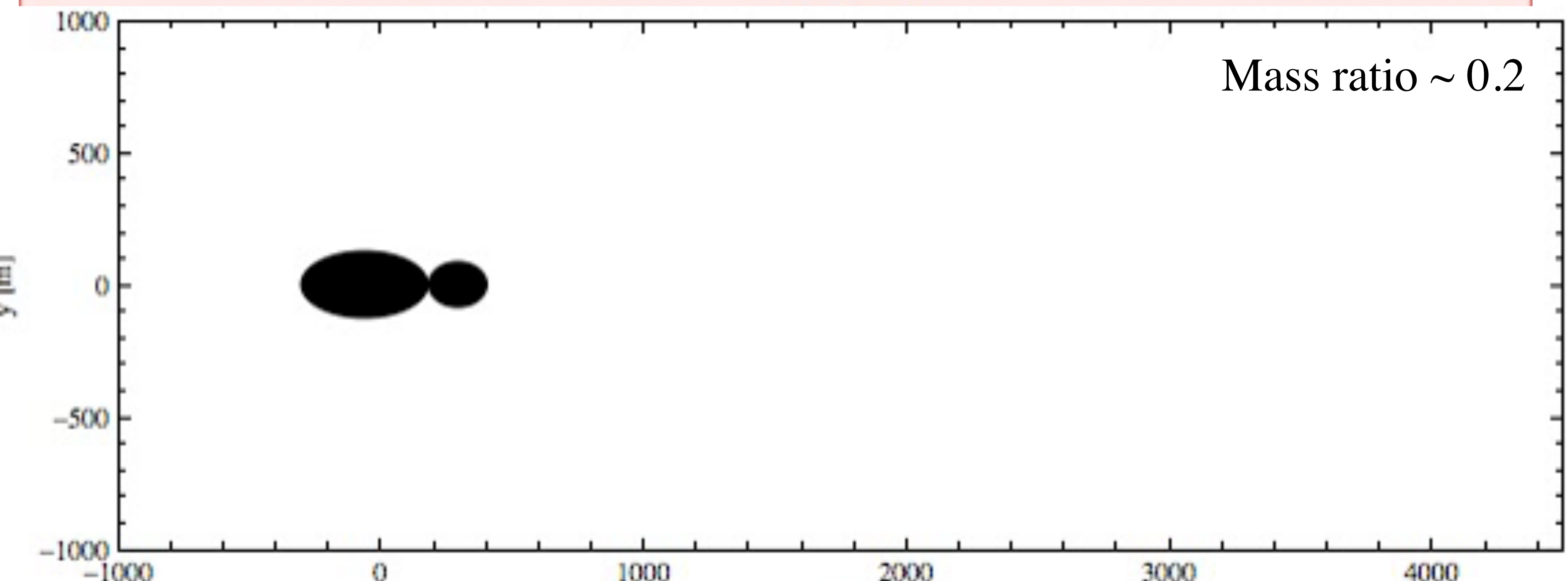

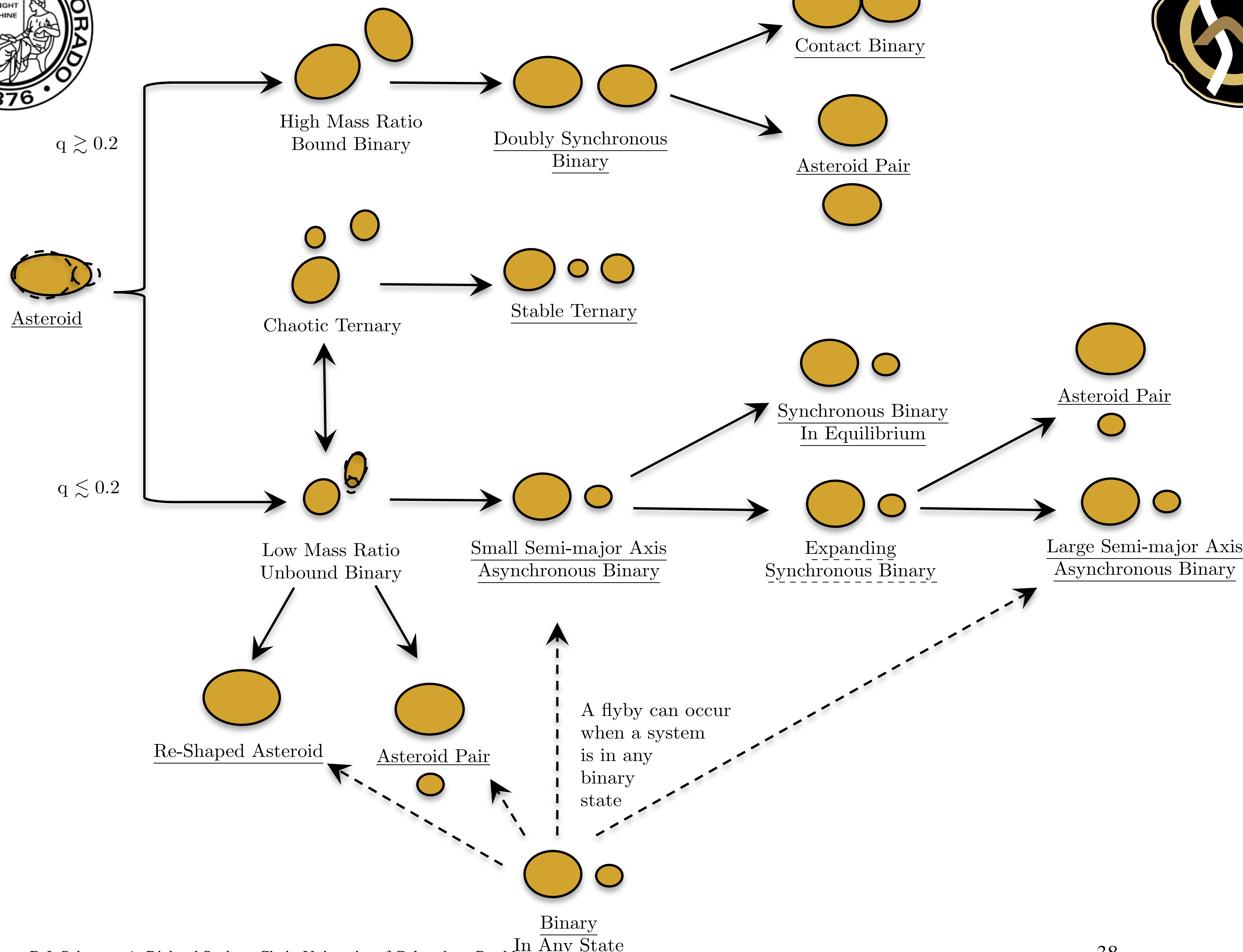
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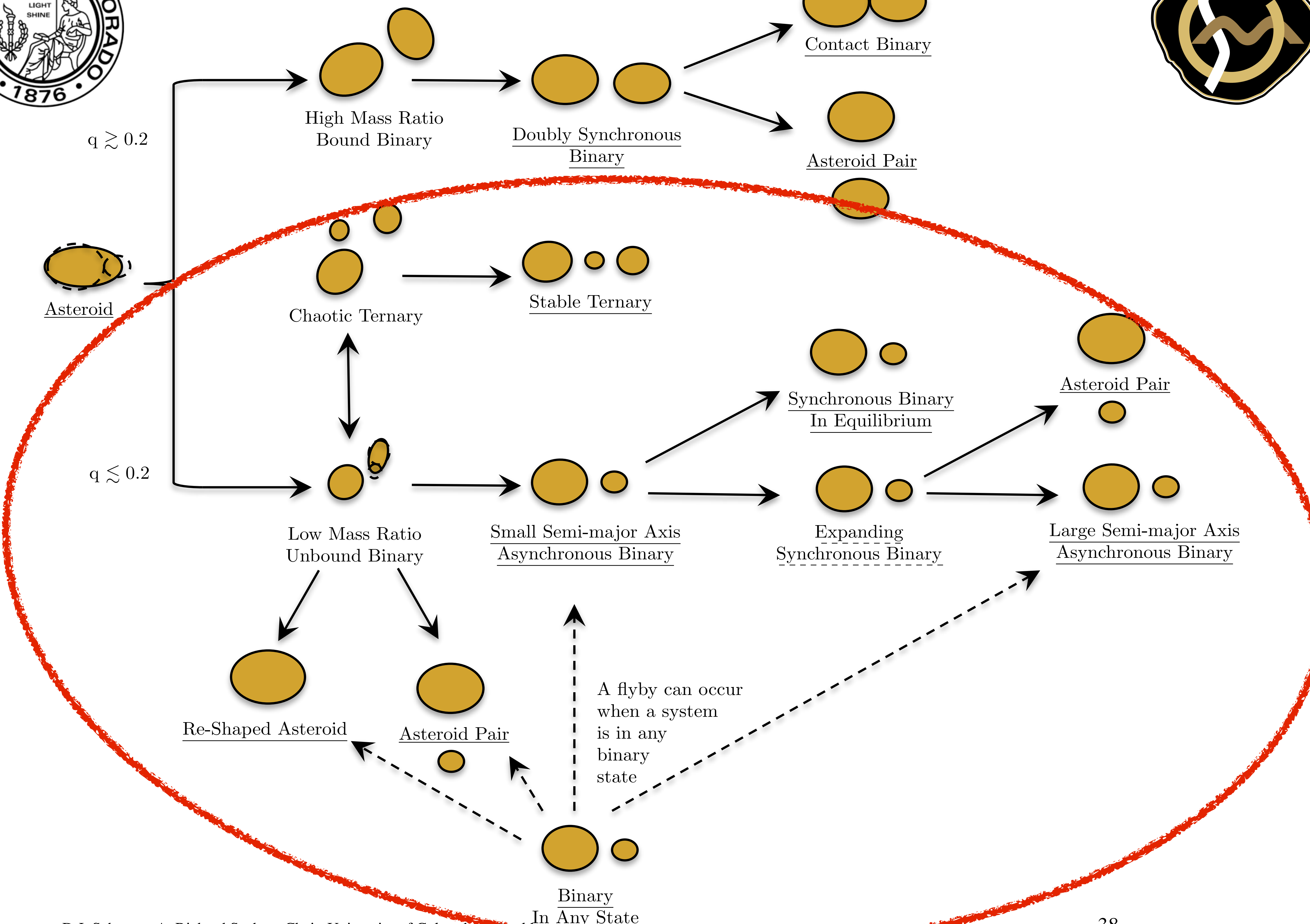
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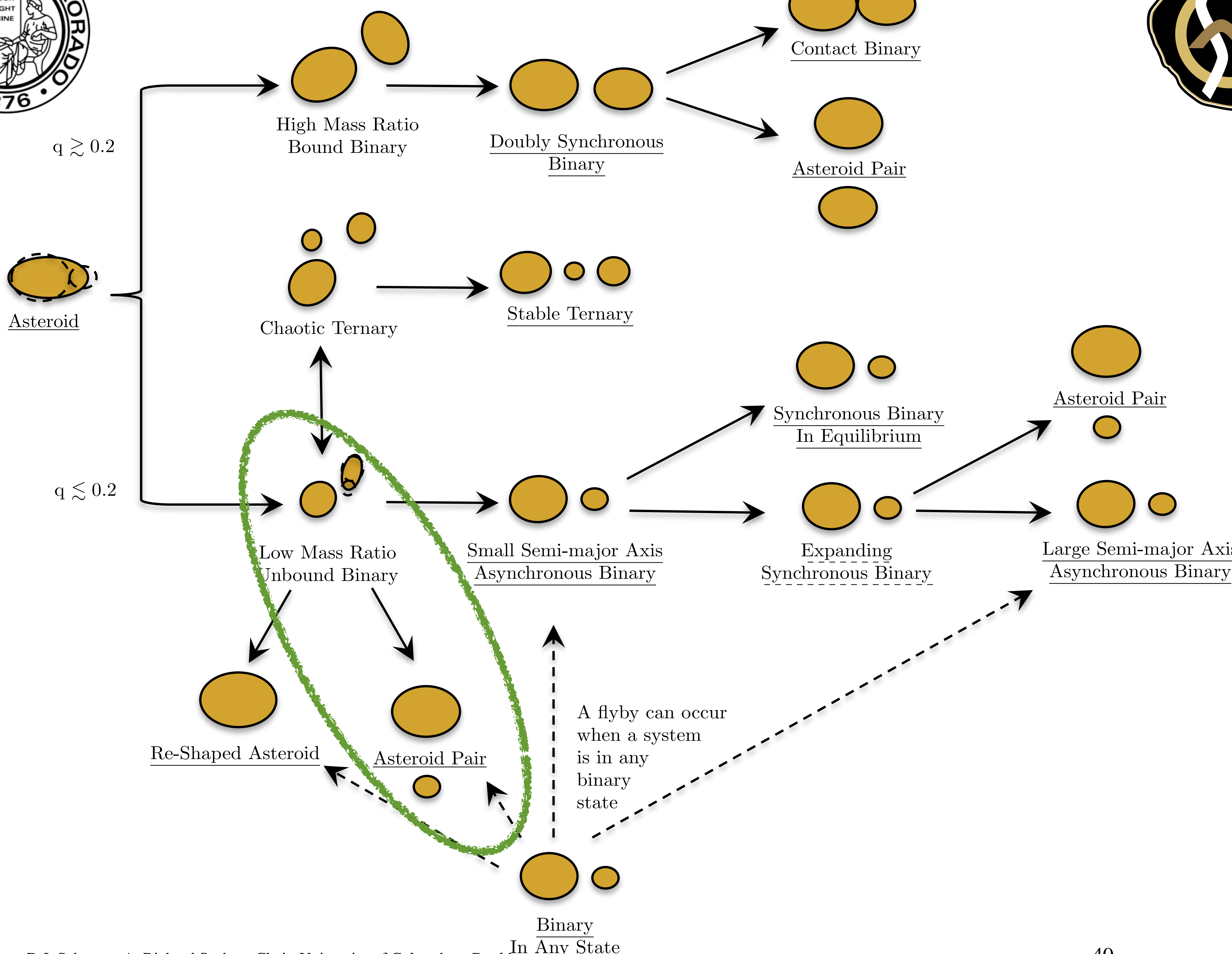
Fission: Mass Ratio < 0.2

- Mass Ratio < 0.2
  – Sufficient energy to escape -> Asteroid Pairs
  – Sufficient energy to undergo secondary spin fission
    - A path to binary stabilization
    - A path to reshaping the primary
- Abrupt / rapid escape:
  – Fissioned bodies with mass ratio < 0.2 have total positive energy
  – Energy to escape is drawn from the primary spin rate – the larger the mass ratio the slower the primary spin after escape
  – Matches very well with observed properties of asteroid pairs



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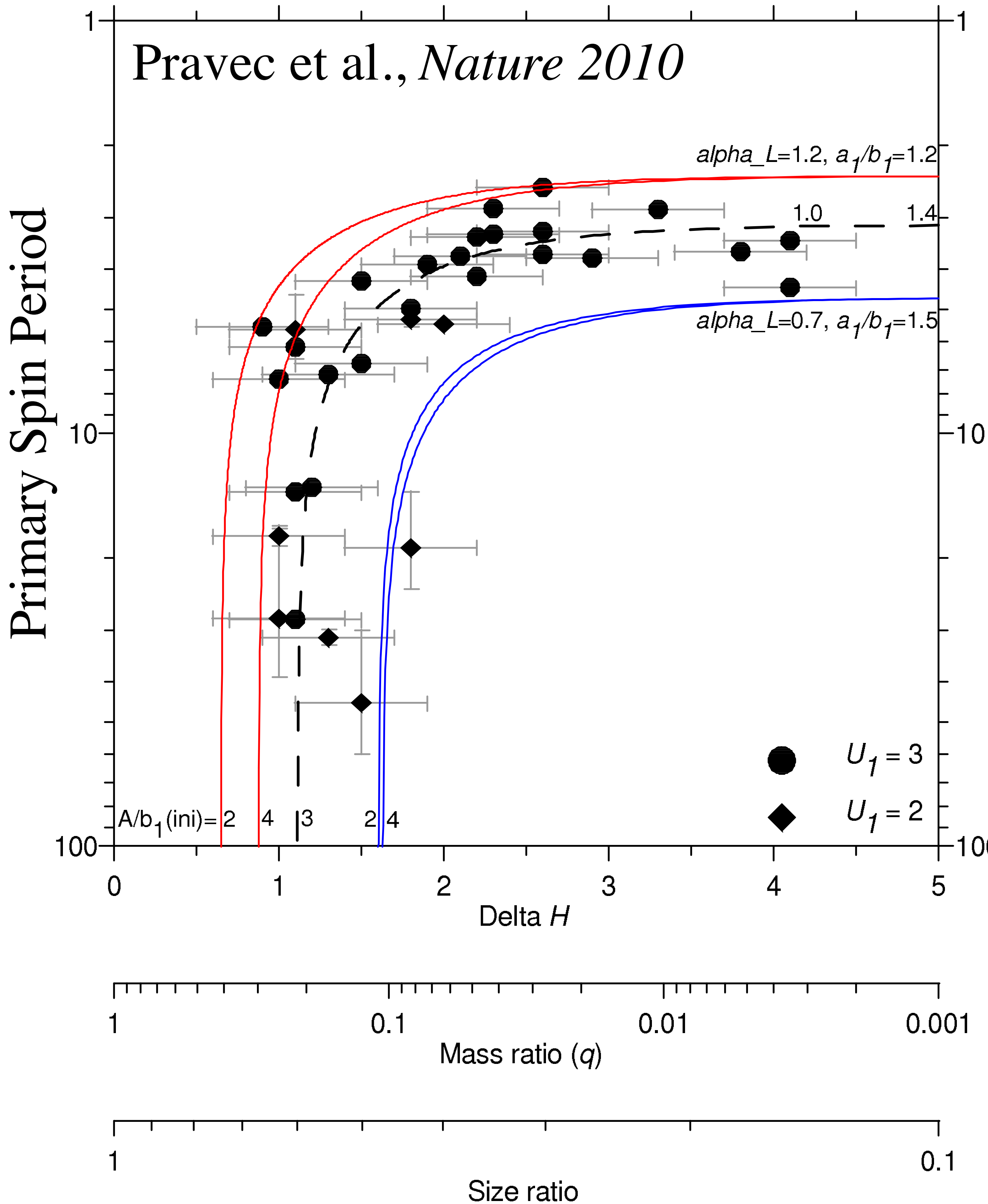
Asteroid Pair Primary Spin vs Mass Ratio



Prediction:

The mass ratio between asteroid pairs formed by direct fission should be < 0.2

The primary spin period should grow long near the cut-off

Observation:

The mass ratios and primary spin periods of Main Belt asteroid pairs match with our Asteroid Fission Theory

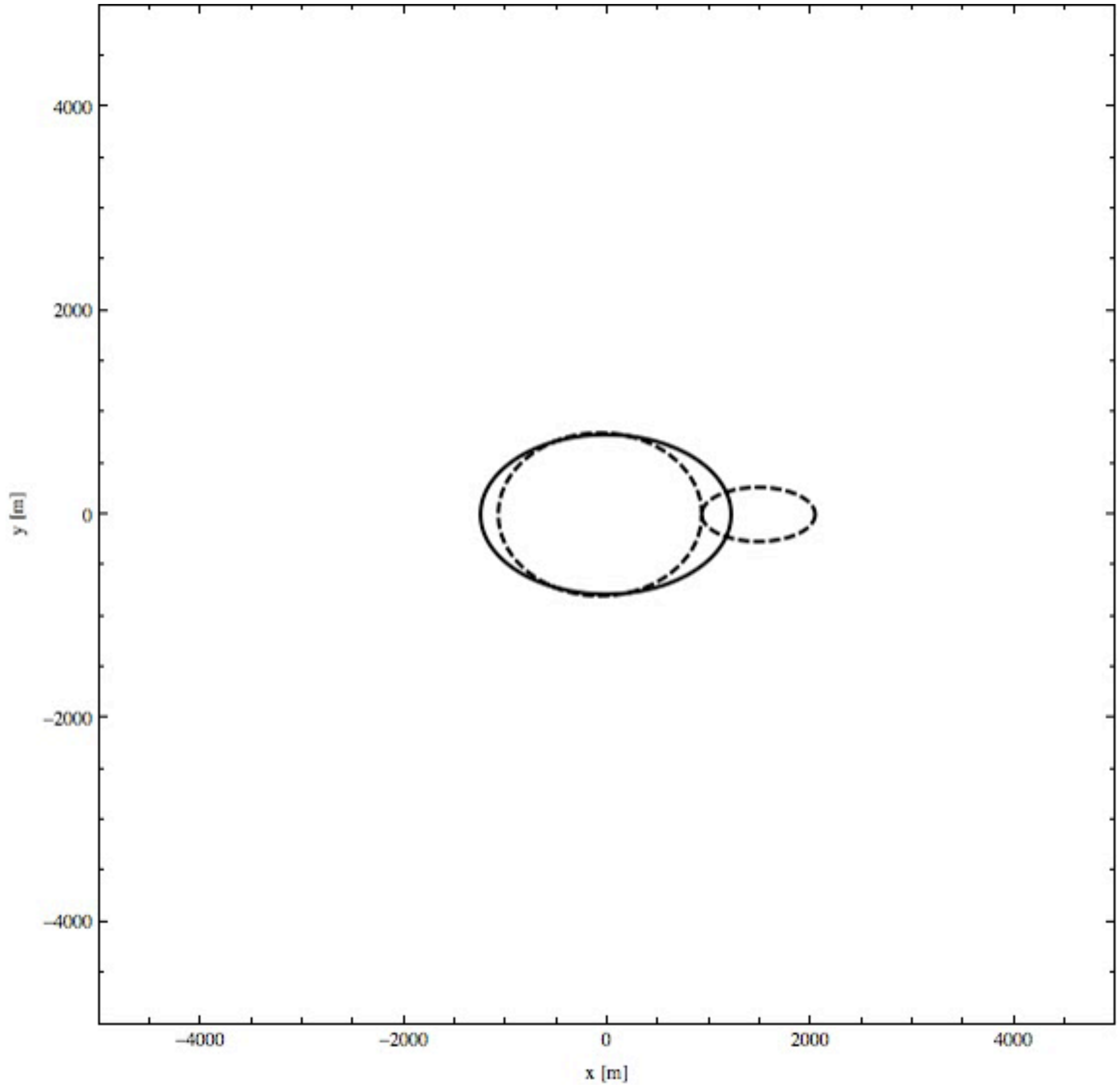
Comment:

The theory matches two independent outcomes, mass ratio cut-off and primary spin period lengthening

Fission: Mass Ratio < 0.2

- Secondary Fission:
  - Prior to escape the secondary is often spun to spin rates beyond the fission rate and can split again
- Can send the inner component towards the primary and stabilize the outer component
  - Impacting components can add angular momentum to the primary and cause reshaping through "relatively slow" impacts
  - Outer components are "impulsively" transferred to an orbit with a higher periapsis = less interaction
  - Repeated fission can cause the system to stabilize, providing the necessary time for tides to synchronize the secondary
- Can eject the outer component creating an asteroid pair
  - Perhaps stabilize the inner component?

Secondary Fission Model: Re-impact



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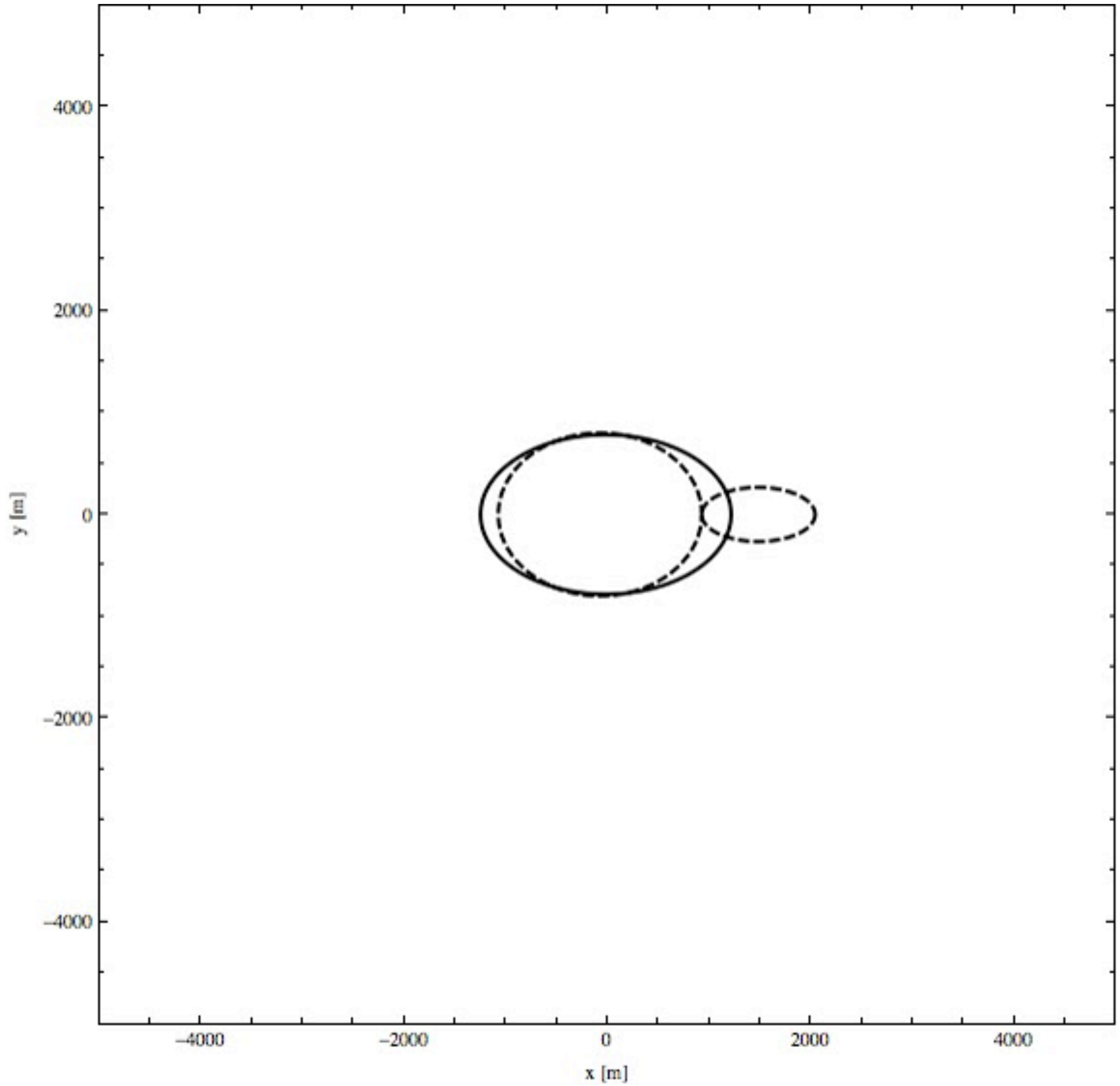
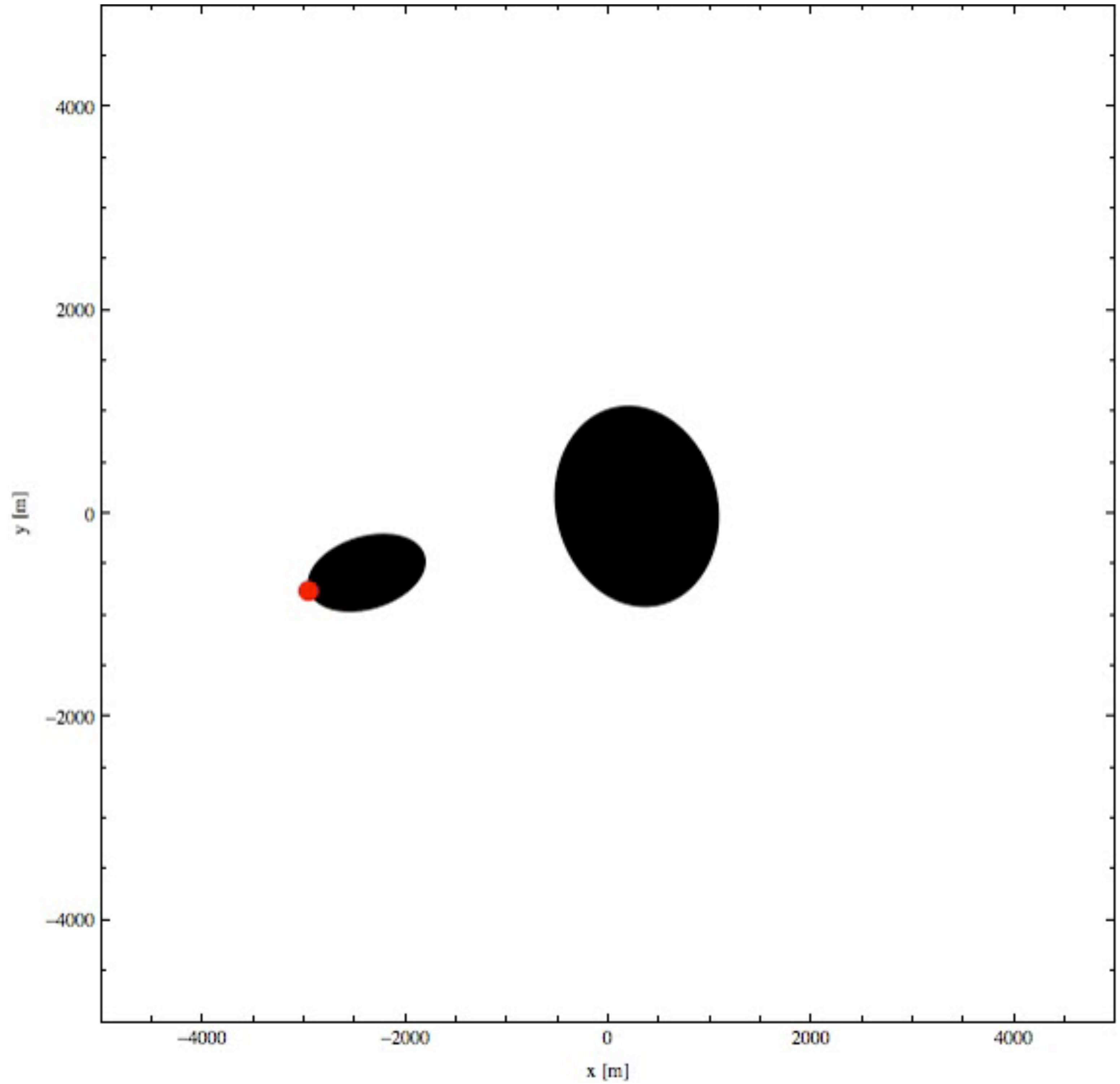


Secondary Fission Model: Escape



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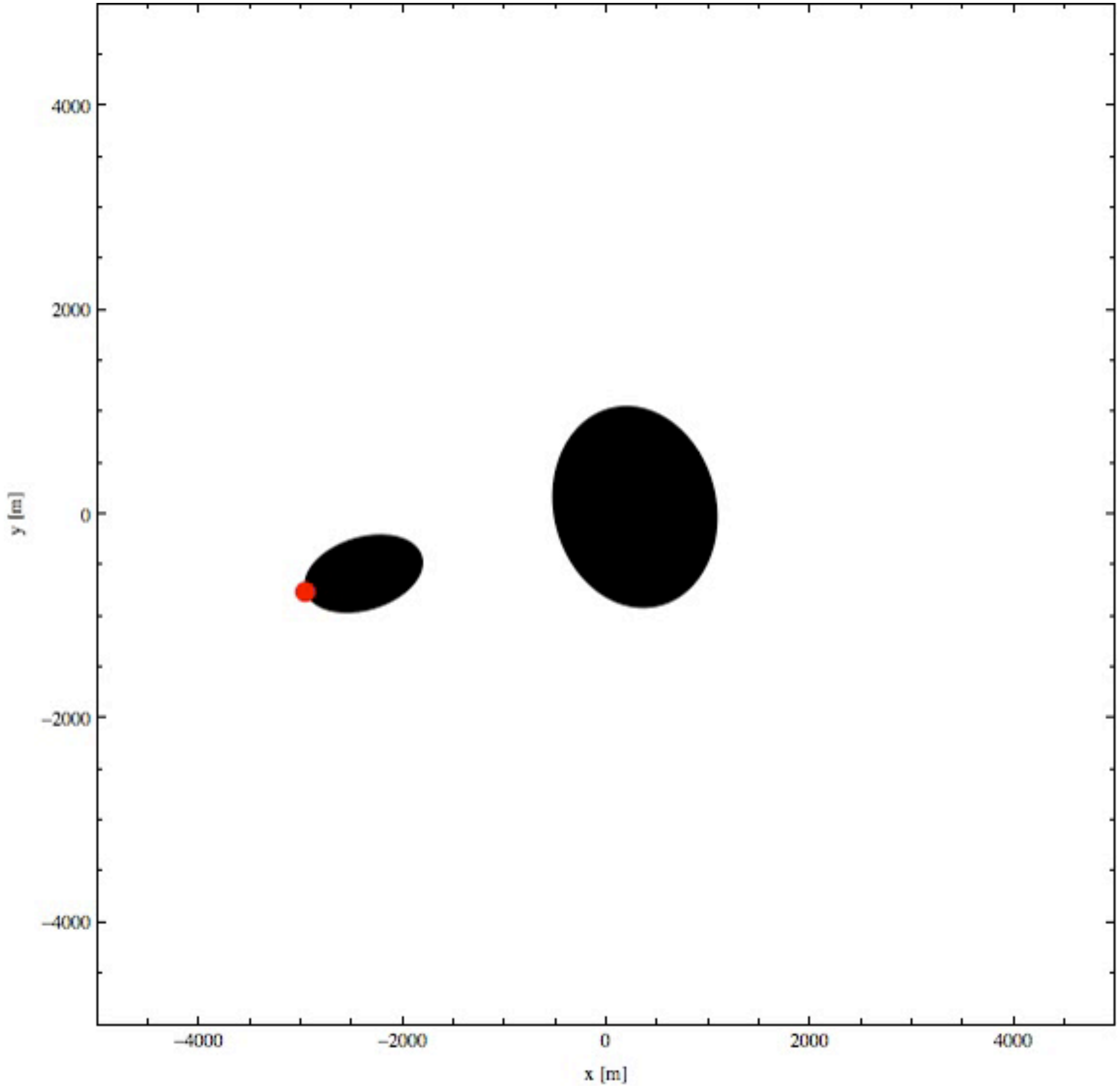

Secondary Fission: Mass Ratio < 0.2

- Singly Synchronous Binaries
  - BYORP becomes active as soon as libration commences – prior to full secondary relaxation
  - BYORP effect either expands or contracts (50/50)
- Expansive BYORP
  - Adiabatic invariance causes librations to grow as orbit expands (*McMahon & Scheeres, 2011*)
  - System can either expand and escape or loose synchronicity and become a wide-asynchronous binary
  - Other resonance effects may occur to break lock (*Cuk and Nesvorny, 2010*)
- Contractive BYORP
  - Stable equilibrium is formed balancing BYORP contraction and tidal expansion (*Jacobson & Scheeres, ApJL 2011*)
  - Resulting systems can persist for arbitrary lengths of time

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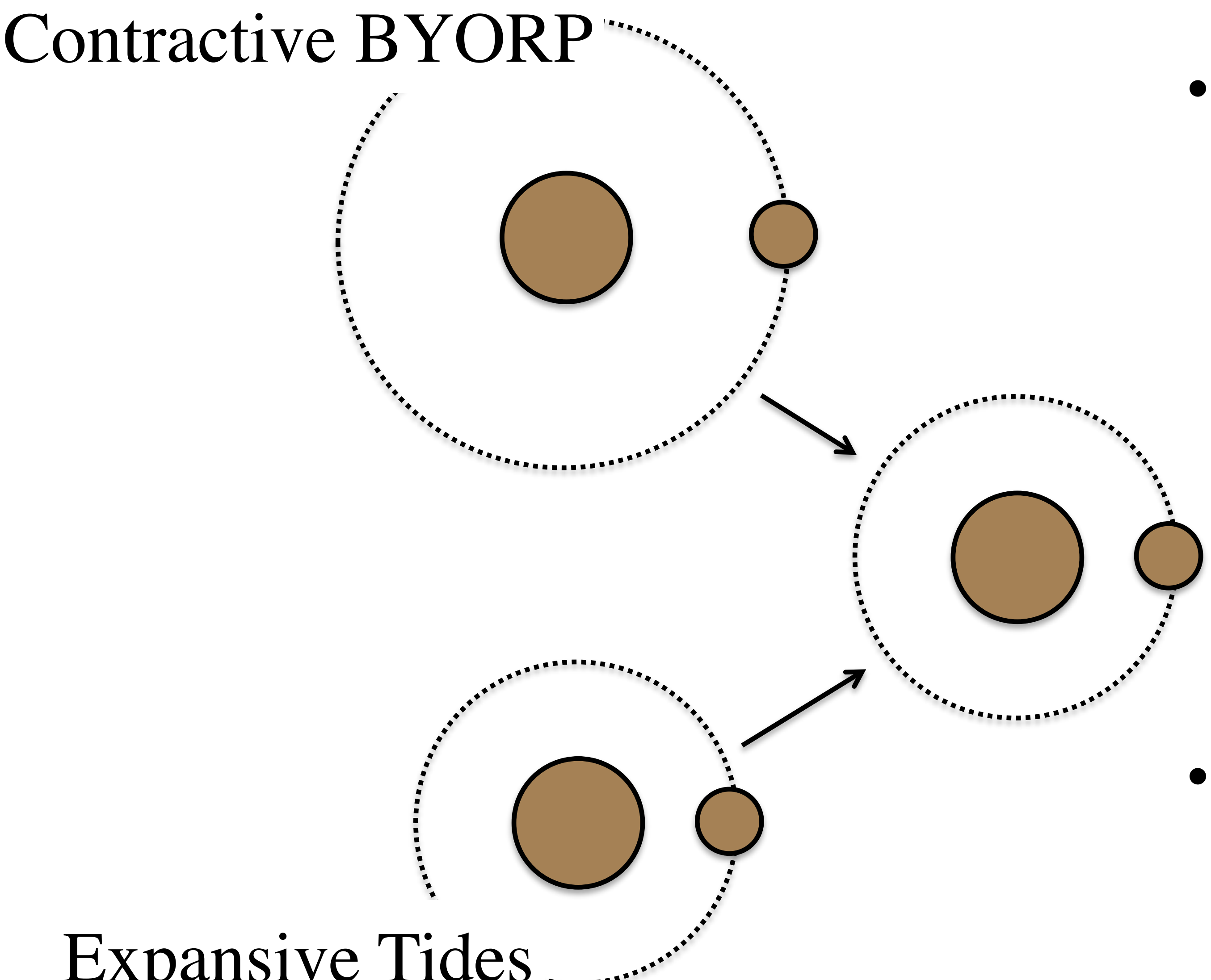
Joint Opposing Evolution



- All such systems evolve in semi-major axis to a stable equilibrium

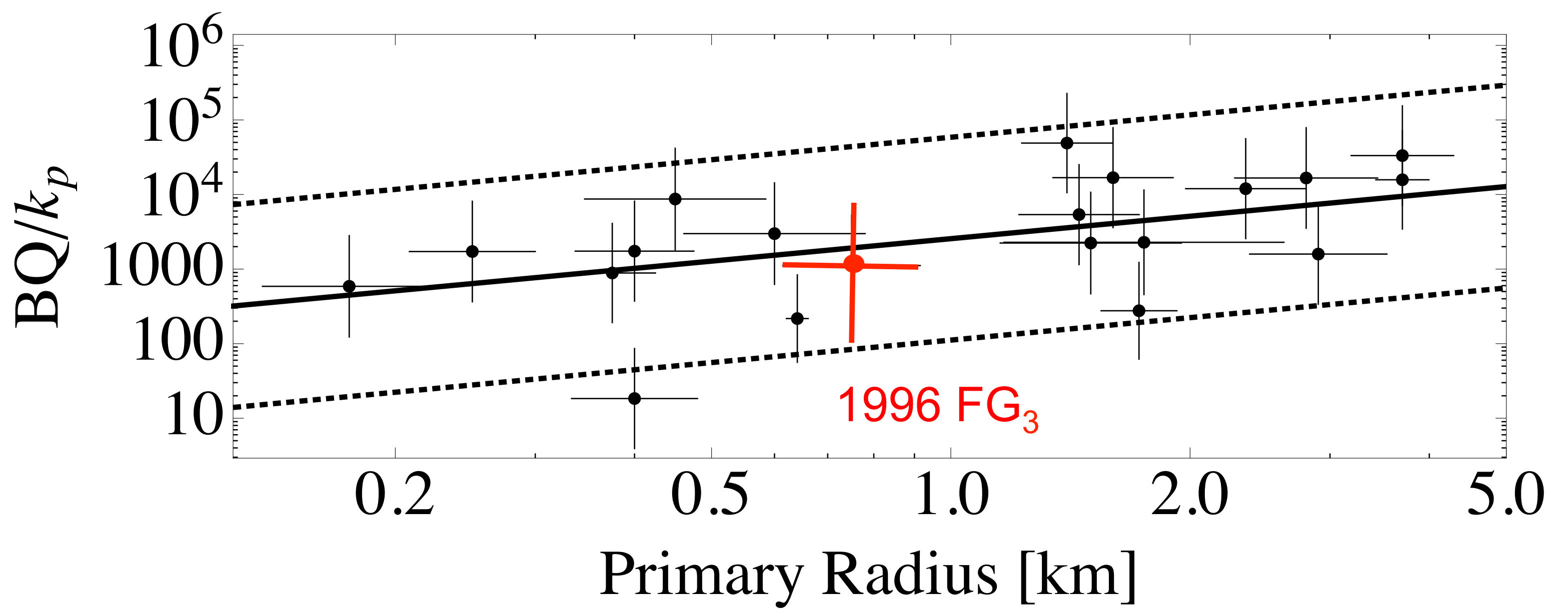
$$\dot{a} = 2\frac{\Gamma_T - \Gamma_B}{mna}$$

$$a^* = \left(\frac{2\pi k_p \omega_d^2 \rho R_p^2 q^{4/3}}{B H_\odot Q}\right)^{1/7}$$

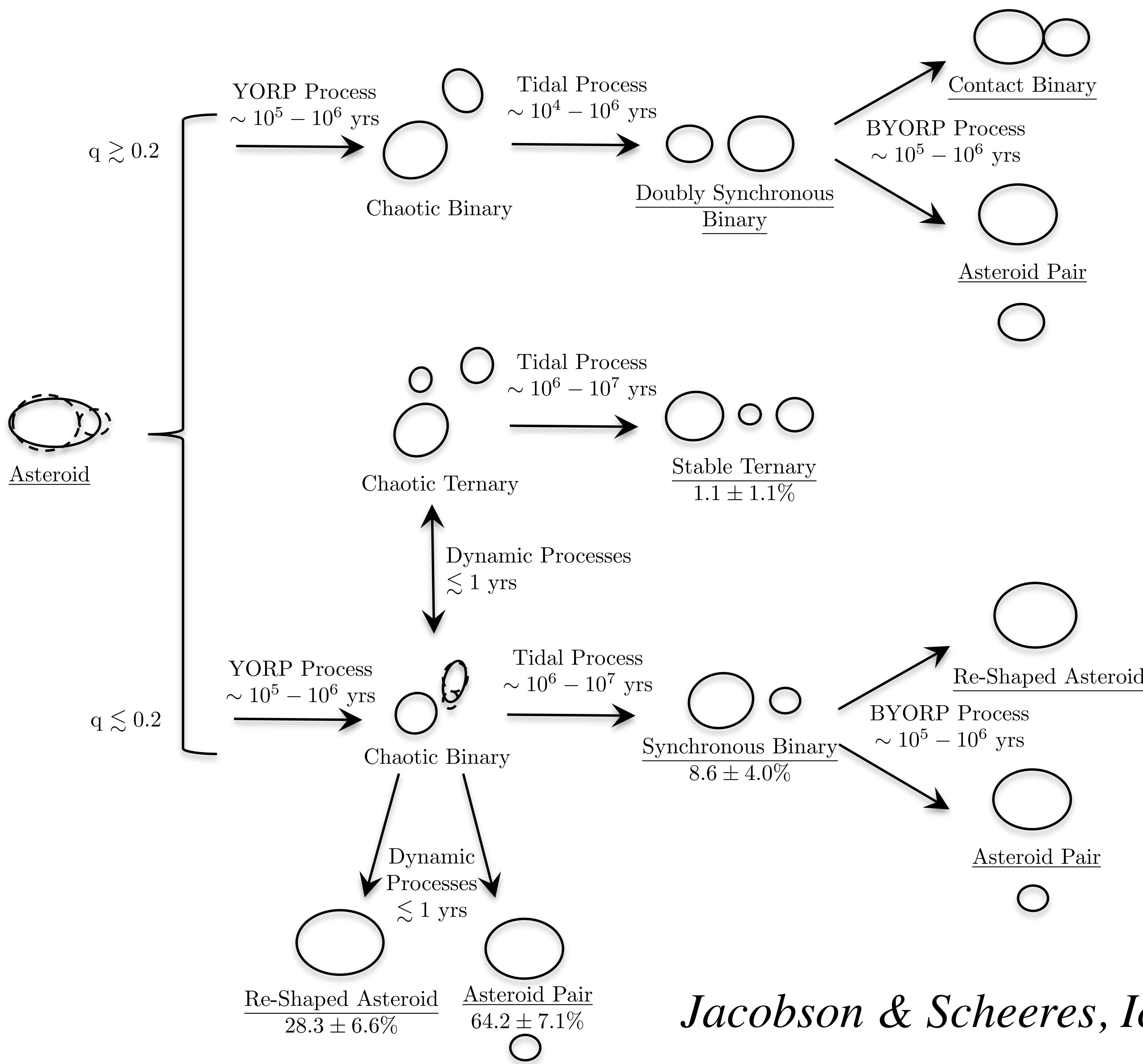
- Tides still dominate eccentricity evolution, so e damps

Stable Singly-Synchronous Binaries

- Semi-major axis of equilibrium a function of BYORP parameter and *Q/k*
  - Means that modeling/measurement of the BYORP parameter provides insight into the internal geophysics of the system
  - Exogenous angular momentum from BYORP acts to slow the primary spin rate – competes with YORP

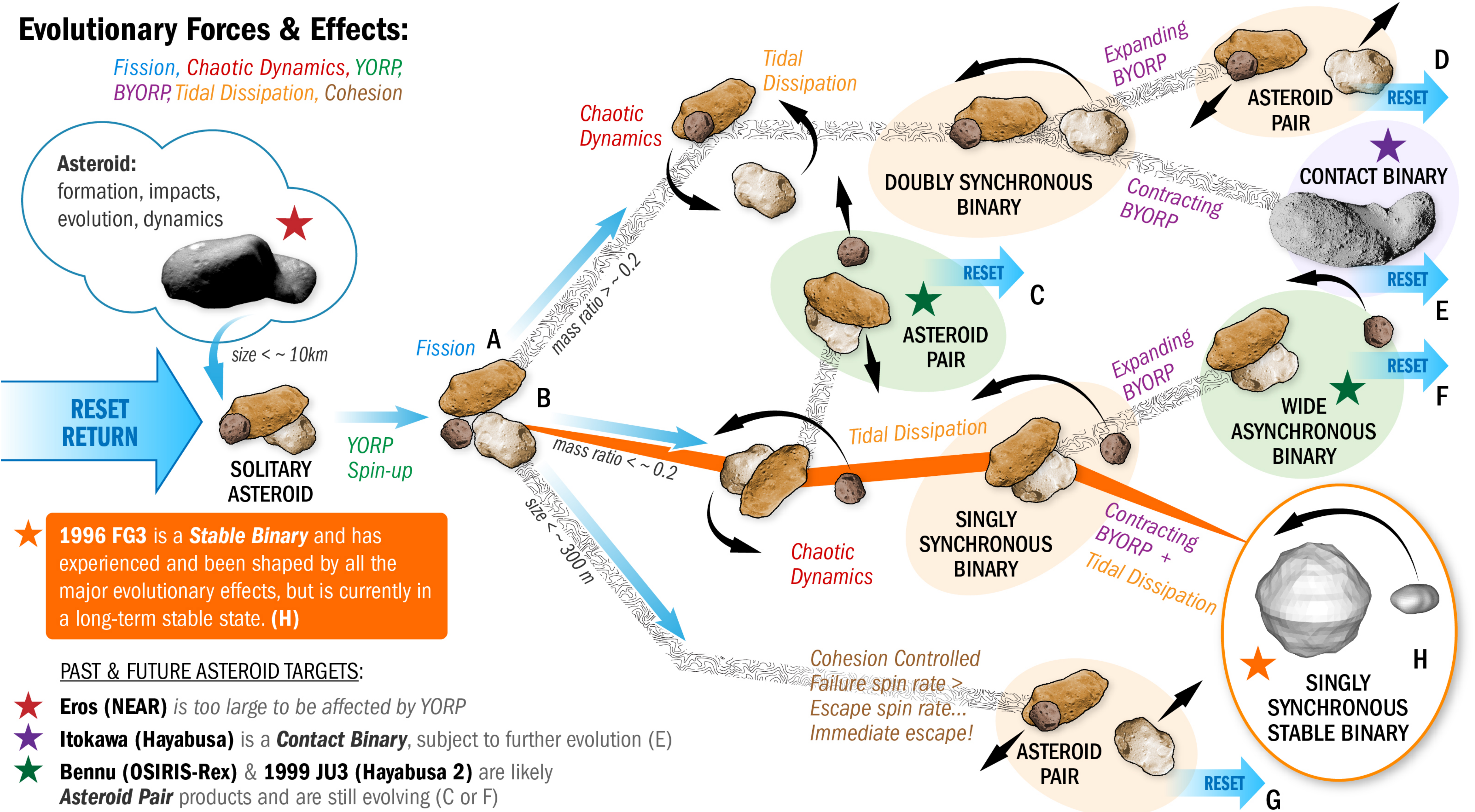


$$\frac{BQ}{k_p} = \frac{2\pi\omega_d^2\rho R_p^2 q^{4/3}}{F_\odot a^7} a_\odot^2\sqrt{1-e_\odot^2} = 2557\left(\frac{R_p}{1\text{ km}}\right)$$



Jacobson & Scheeres, Icarus 2011

Evolutionary Pathways of Asteroids

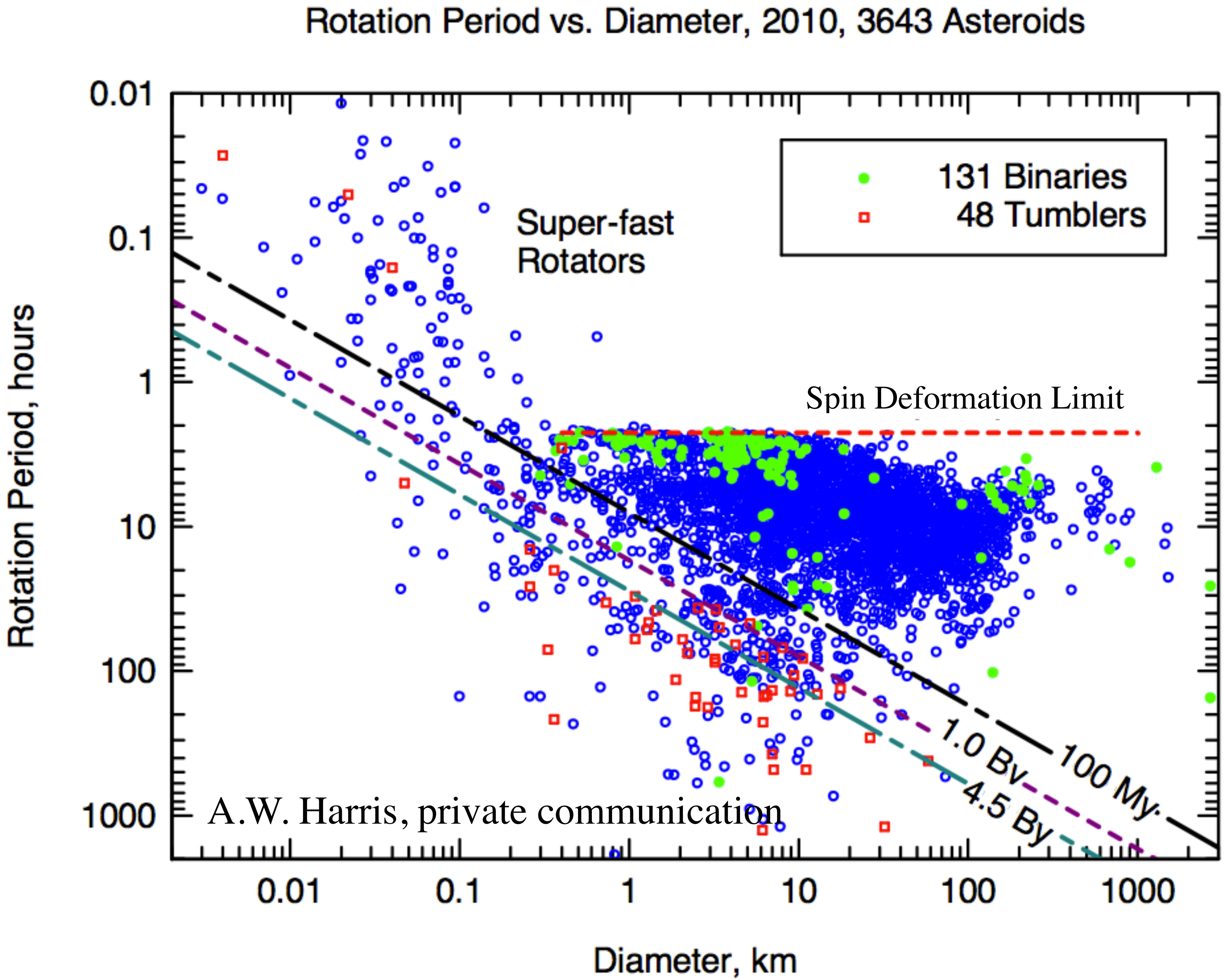


A: Scheeres (2007); B: Jacobson & Scheeres (2011a); C: Pravec et al. (2010); D: Fang & Margot (2012); E: Scheeres et al. (2007); F: Jacobson et al. (2014); G: Scheeres et al. (2010); H: Jacobson & Scheeres (2011b)

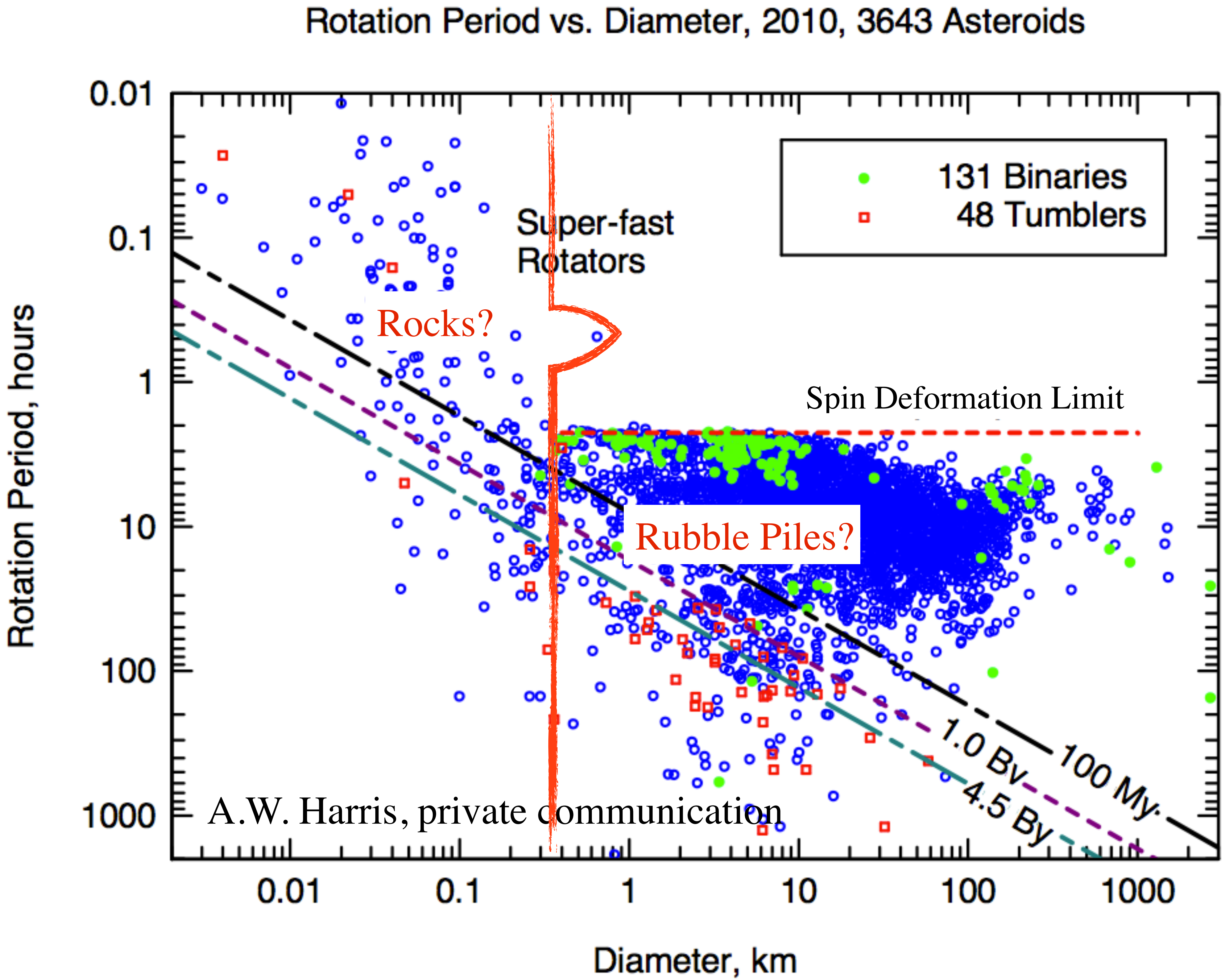
Do Rubble Piles Have Strength?

Spin / Size Relation

- The increase in asteroid spin rates with decreasing size has been well established since Pravec and Harris 2000.
- The spin limit for larger bodies is consistent with the spin deformation limit for spheres of density ~2-3 g/cm^3.
  - A simple interpretation is that the maximum block size from which asteroids are built is ~100+ meters and that asteroids spun beyond this limit “disassemble” into smaller pieces.



Rotation Period vs. Diameter, 2010, 3643 Asteroids
131 Binaries
48 Tumblers
Super-fast
Rotators
Spin Deformation Limit
1.0 By
100 My
4.5 By
A.W. Harris, private communication
Rotation Period, hours
Diameter, km
0.01
0.1
1
10
100
1000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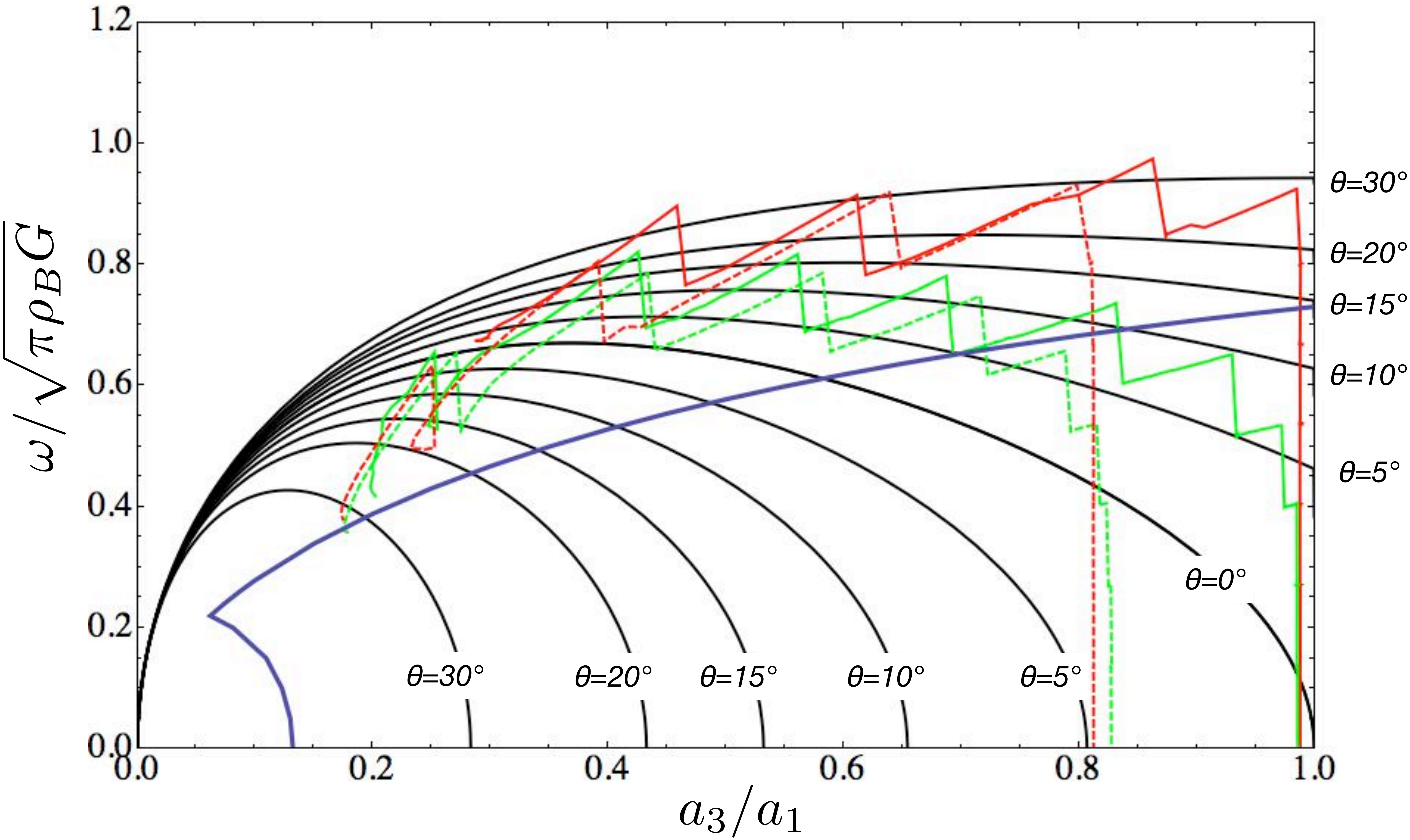
  - A simple interpretation is that the maximum block size from which asteroids are built is ~100+ meters and that asteroids spun beyond this limit “disassemble” into smaller pieces.
- The real picture seems a bit more complicated, however...
  – Direct Observations of asteroid Itokawa and radar shapes
  – The existence of tumbling fast rotators in the small size population
  – The computed mechanics of asteroid fission
  – The predicted physics of rubble pile asteroid cohesive strength...

Under addition of angular momentum, a "deforming" body will not necessarily spin faster



Spin Deformation Limit

- The Drucker-Prager Plastic Failure theory predicts that once a body starts to deform, it will in general change its shape and spin ***at a slower rate***!
  - Theoretical predictions by Holsapple (Icarus, 2007)
  - Simulation verification by Sánchez and Scheeres (Icarus, 2012)
- The deformation spin limit is:

$$\omega = \sqrt{\frac{4\pi}{3}G\rho}\sqrt{\frac{2\sin\phi}{1+\sin\phi}}$$

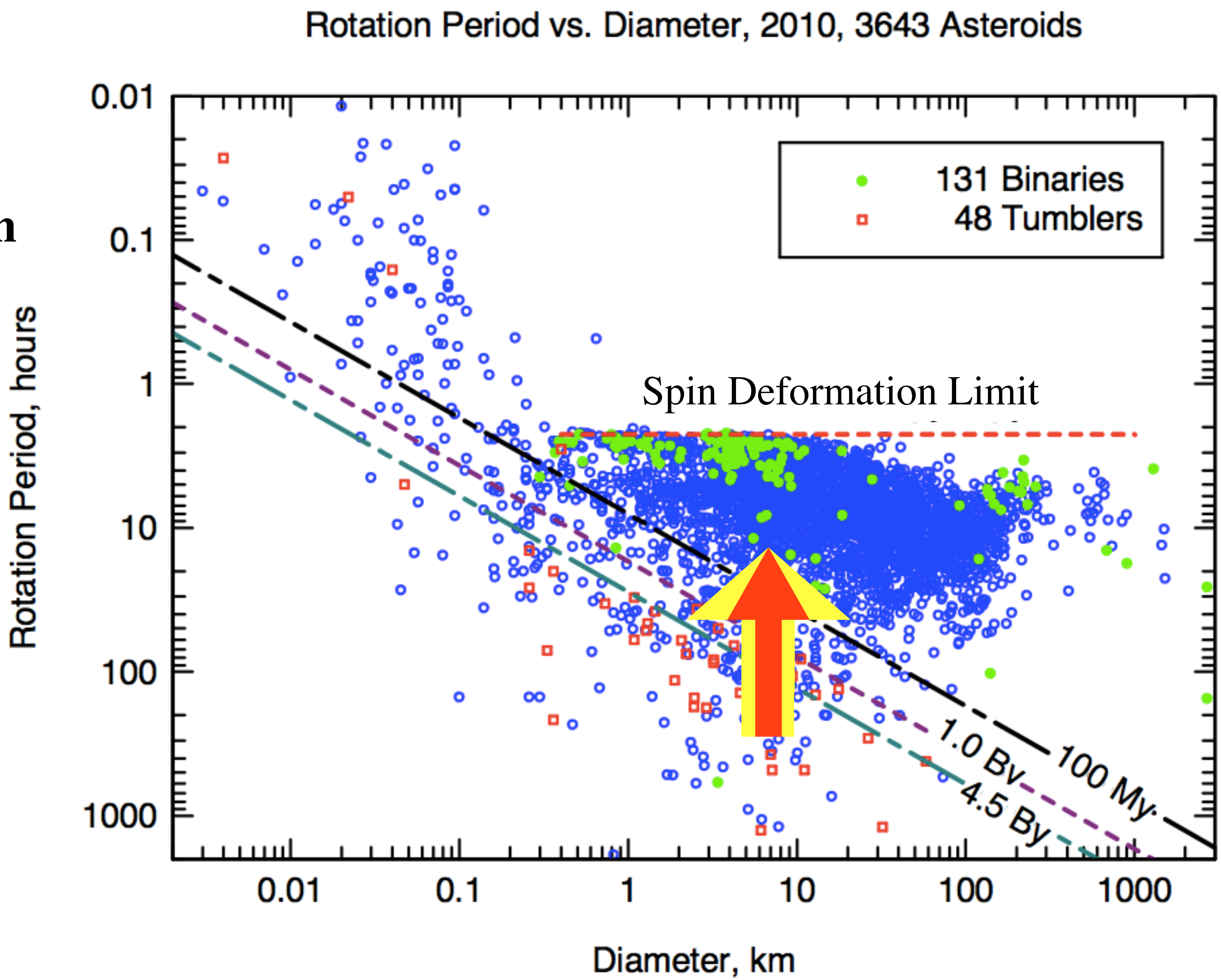
  - where ϕ is the internal friction angle and ρ is the density
- For a limiting spin rate of 0.043 °/sec (2.3 hours):

Friction Angle (deg)	Limiting Density (g/cm^3)
90°	2.06
45°	2.49
30°	3.09

Spin Deformation Limit

Angular Momentum

Angular Velocity

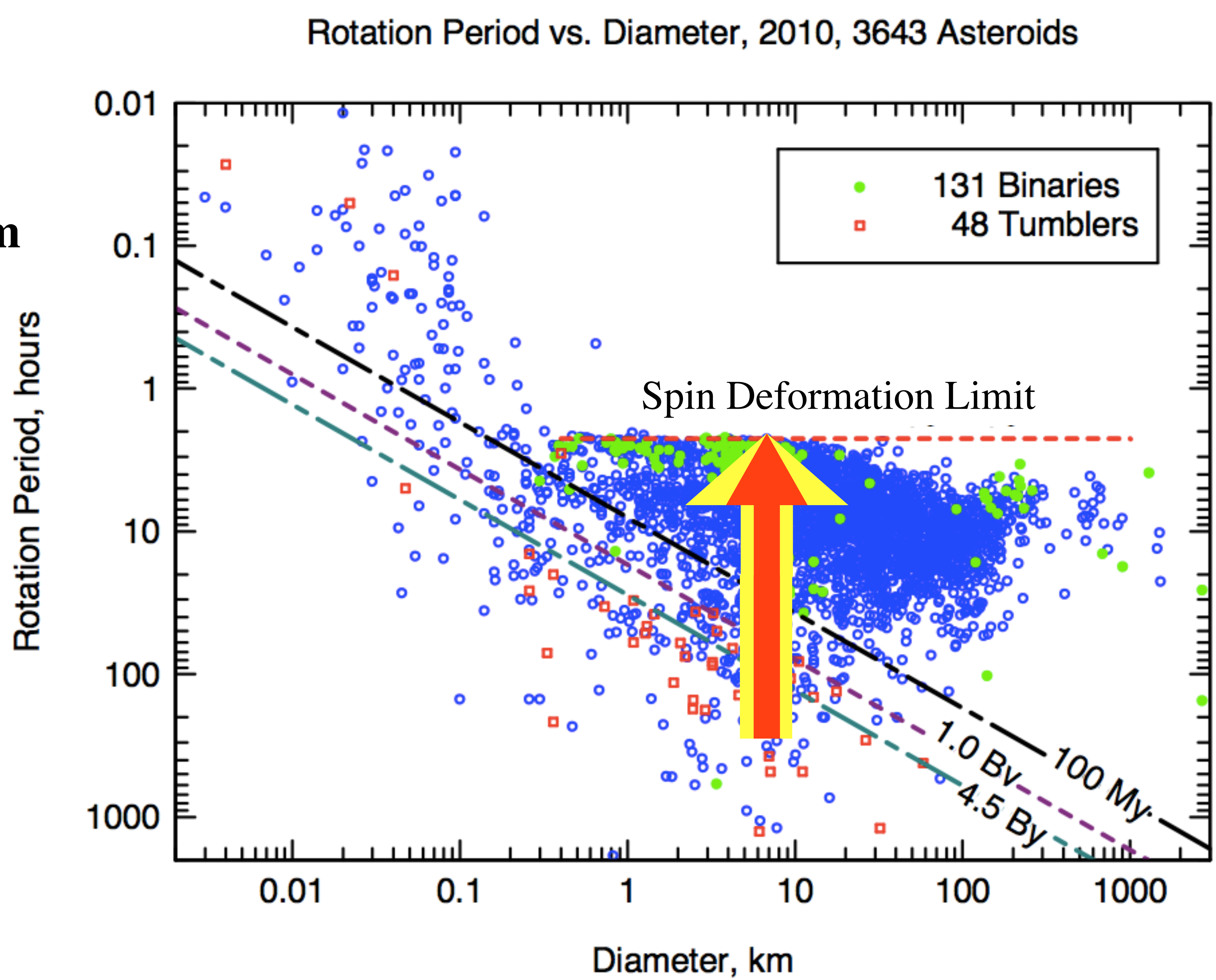


D..

Spin Deformation Limit

Angular Momentum

Angular Velocity

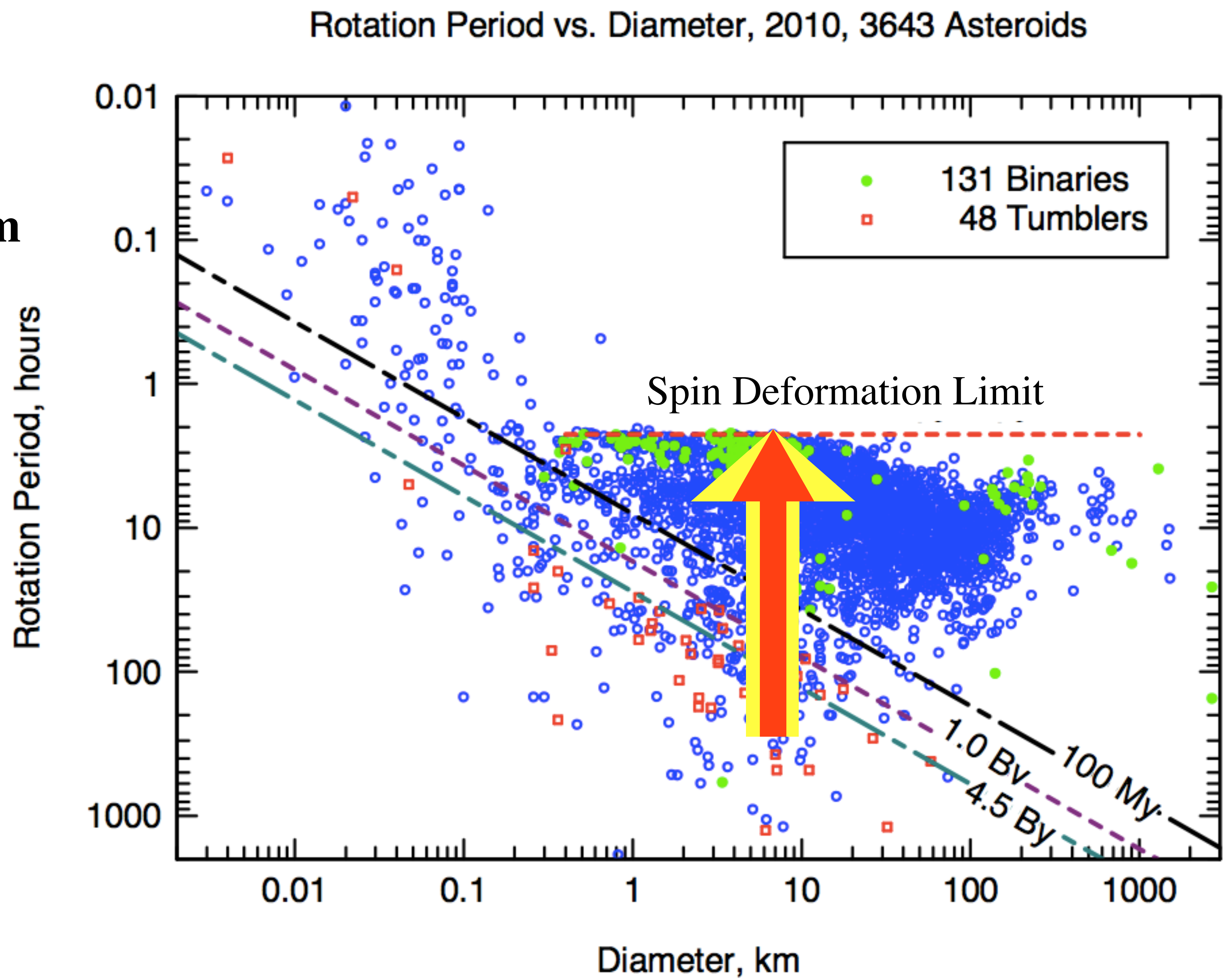


D..

Spin Deformation Limit

Angular Momentum

Angular Velocity



D..

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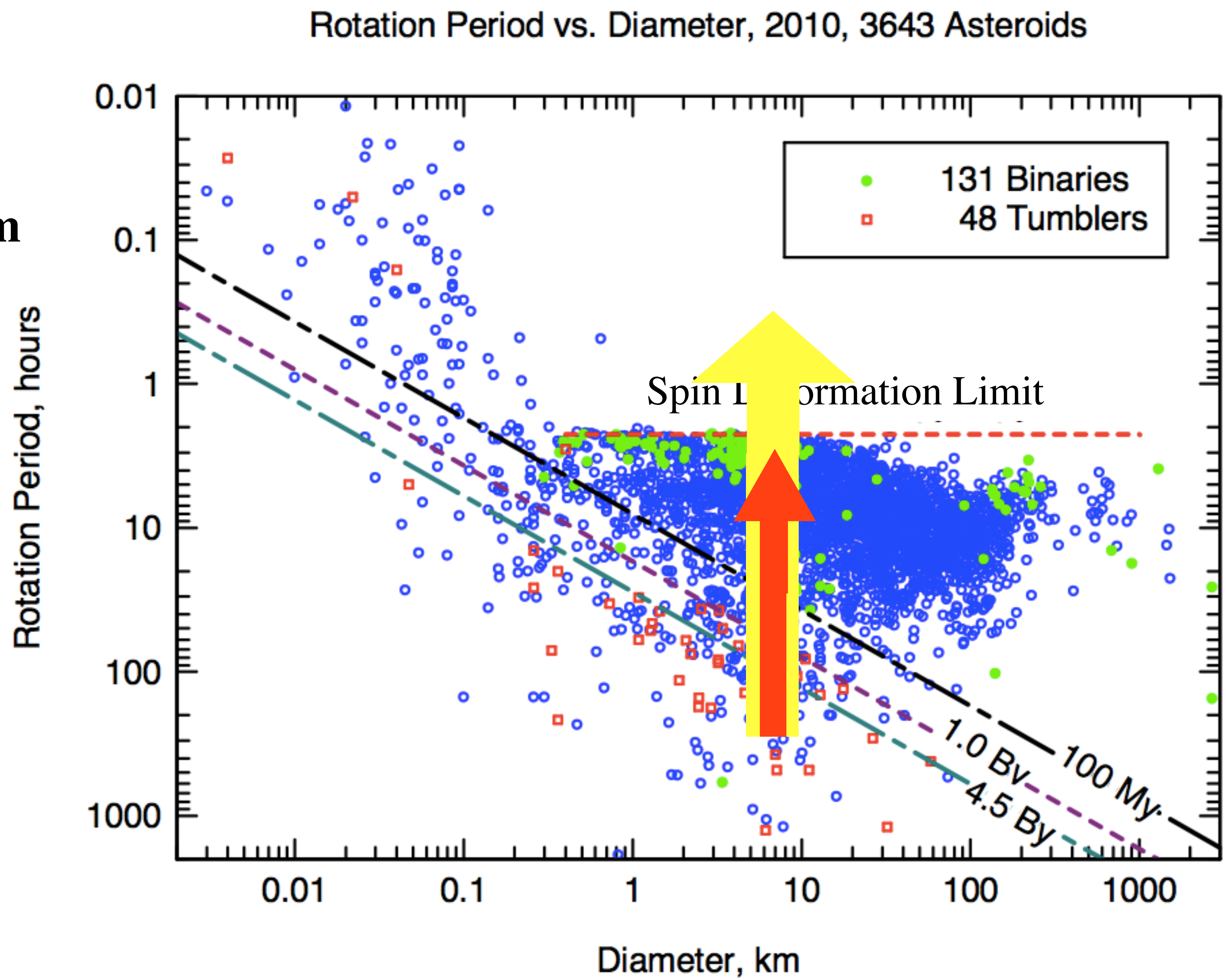
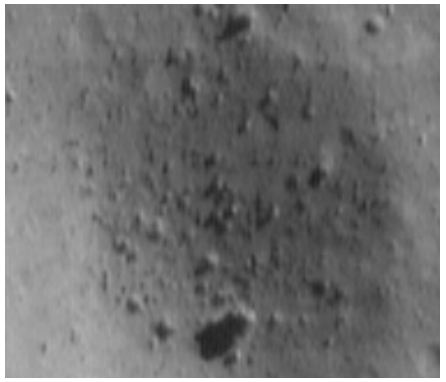
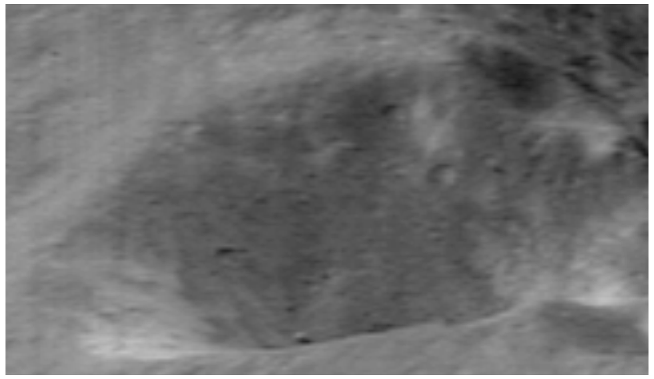


D..

What is a Rubble Pile?





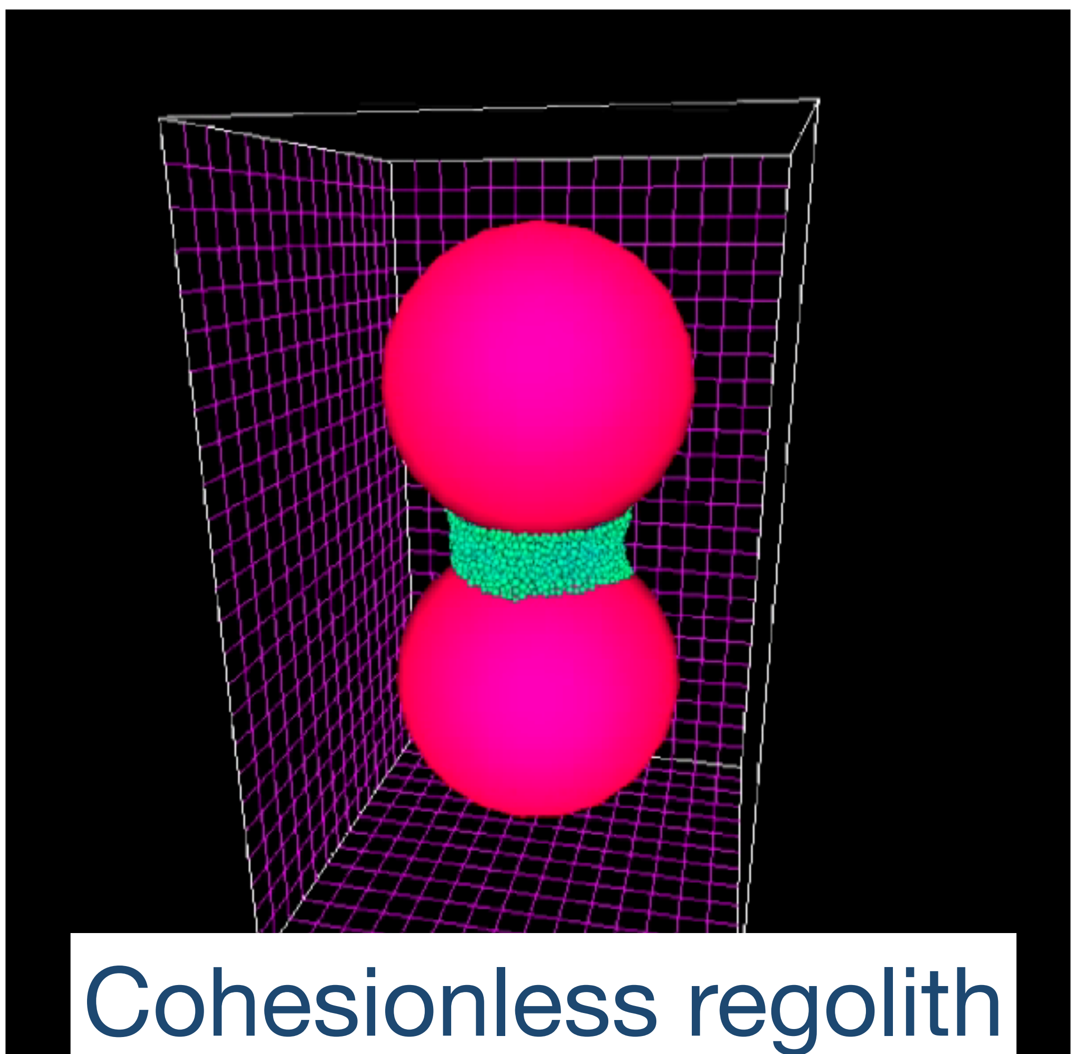
- A size distribution of boulders and grains.
  - Extends from ~ microns to a few 100 meters across
  - Measurements of Itokawa suggest:
    - $1/d^3$ from ~ centimeters to decameters
    - $1/d^3$ from ~ microns to 100 microns
- For these, and shallower distributions, fines “dominate” in number and surface area over larger grains
  - Implies that larger boulders are emplaced or covered in a matrix of finer grains
- What are the consequences of this?
  - These finer grain distributions can serve as a “matrix” that touch all larger blocks.
  - Applying basic properties of cohesive grains measured on Earth and the Moon provide predictions for cohesive strength of a rubble pile.

Physics of Micro-Gravity Bodies

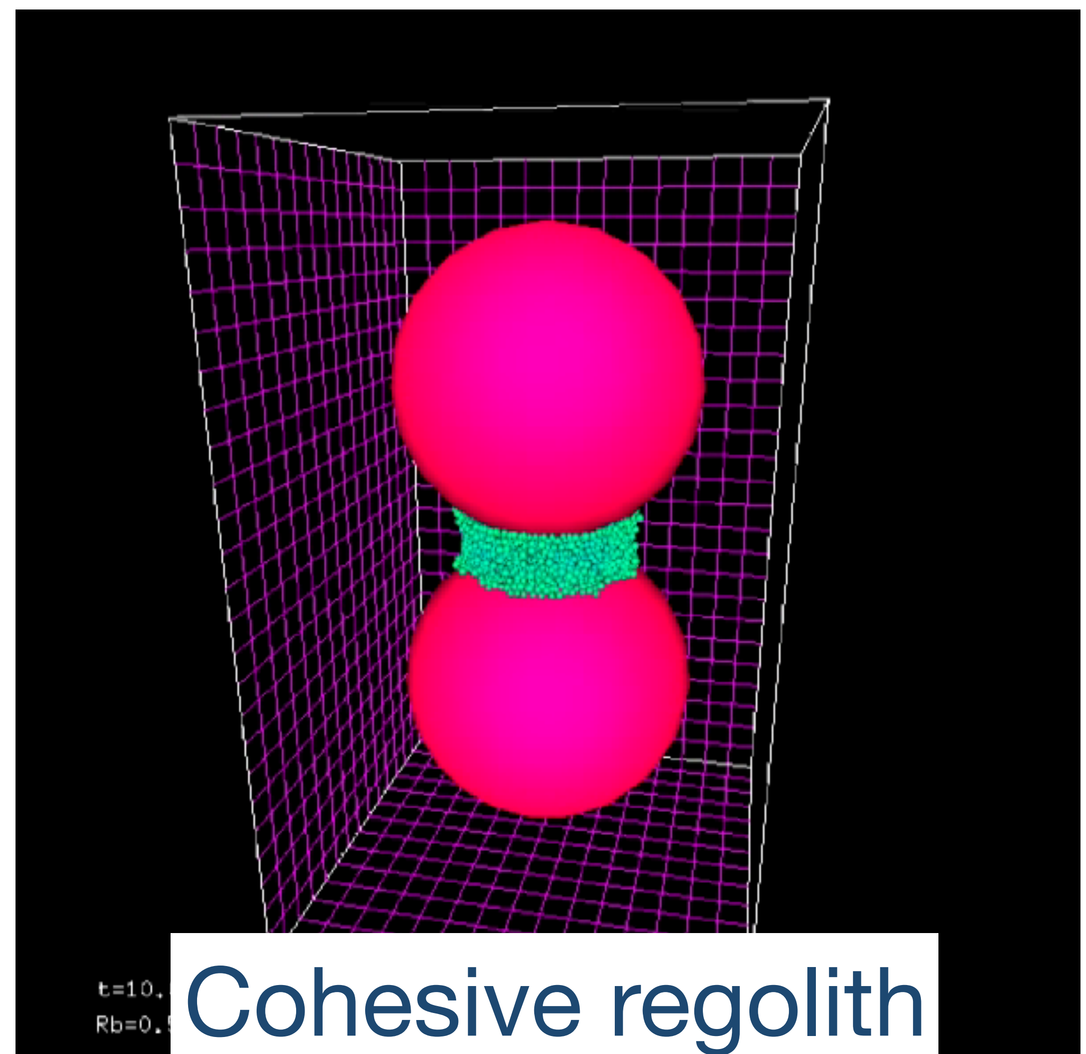
- Where does cohesive strength arise?
- Chemical bonds:
  - Are very strong and can sustain extremely high spin rates
  - Are not relevant for cohesion ***between*** gravels/rocks
- van der Waals forces:
  - In microgravity, can van der Waals forces supply enough cohesion? (Asphaug, *LPSC 2009*; Scheeres et al., *Icarus 2010*)
  - For asteroid sizes less than a few kilometers in size, van der Waals attraction between gravel-sized grains can become as significant as their weight
  - The amount of cohesion needed to keep a fast-spinning rubble pile together is very small (Holsapple, *Icarus 2007*)

How does this work?

- Cohesive van der Waals forces between smaller grains can hold larger boulders in place
- Validated with detailed granular mechanics simulations
  - 1-meter boulders with interstitial regolith with van der Waals forces
  - Equal pull forces applied to each… very different outcomes



Cohesionless regolith



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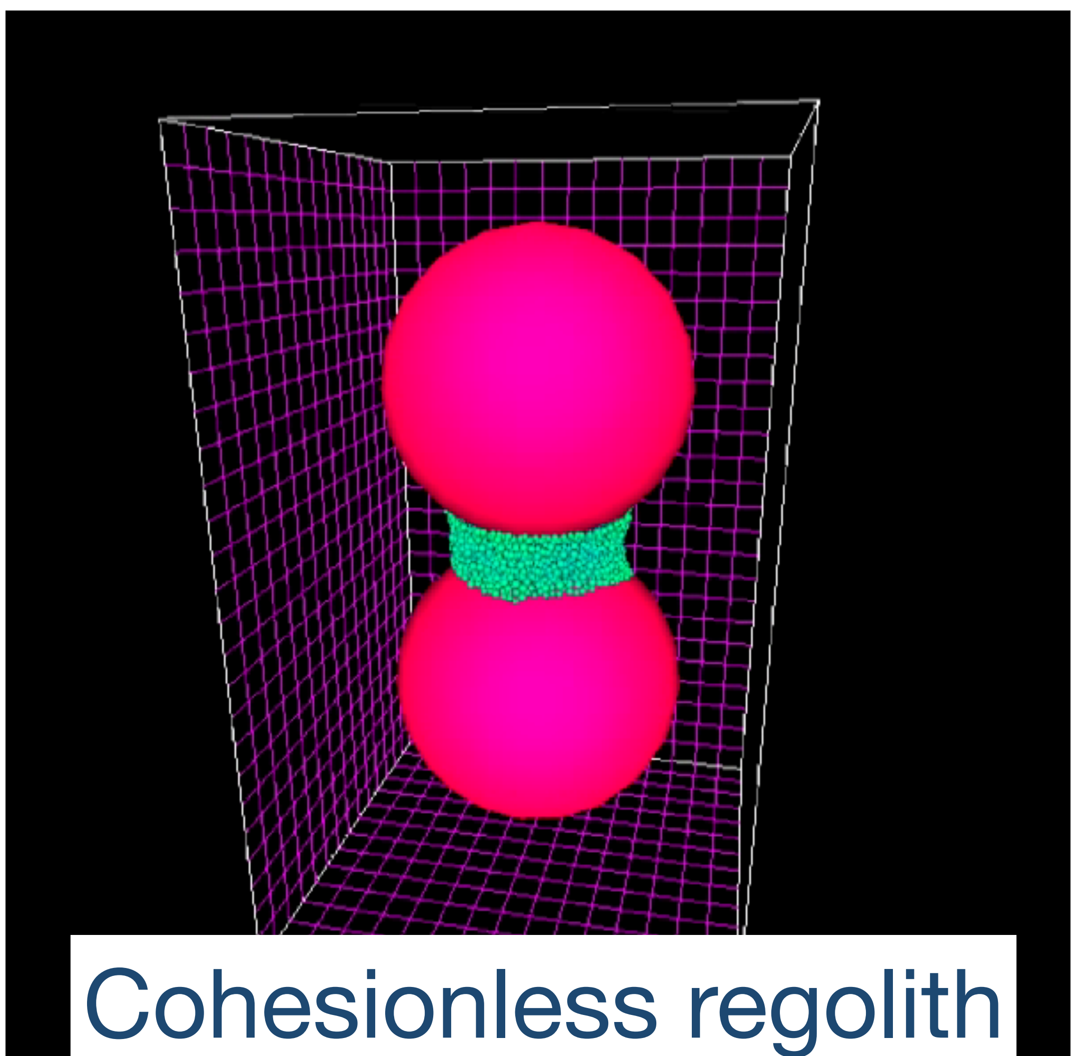

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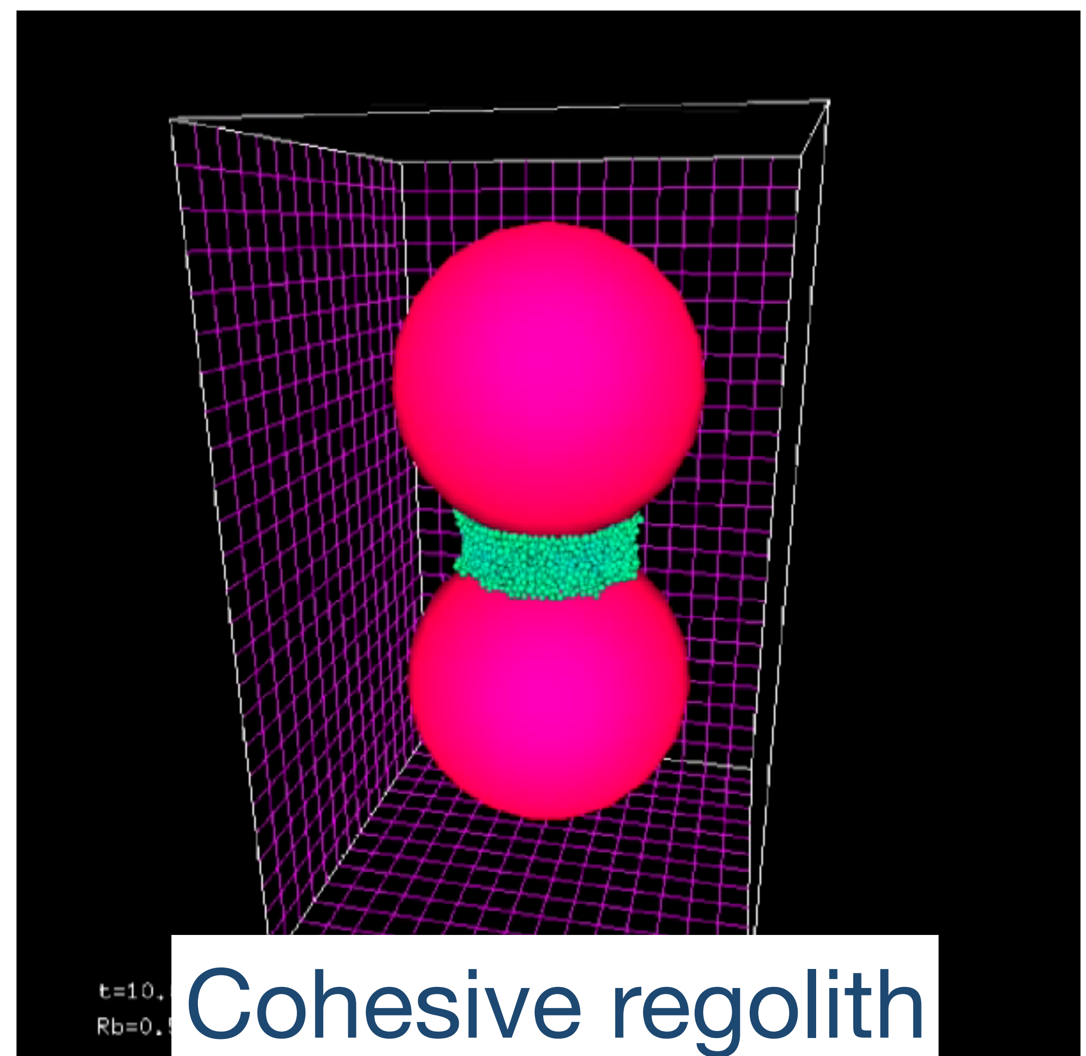
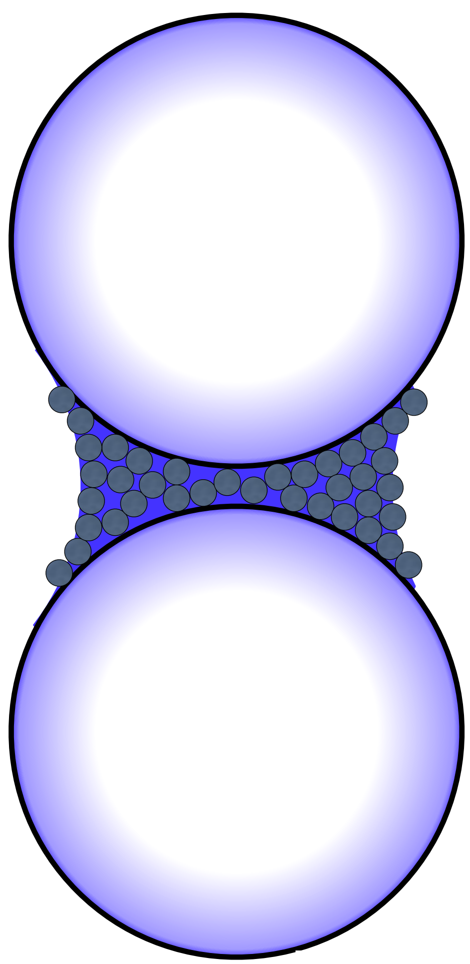


Cohesive regolith

How Strong is it?

- Predicts a cohesive strength model for asteroids dependent on fundamental physical properties and mean grain size
  - Model is consistent with measured cohesive strength properties of the upper lunar regolith: 30 – 60 Pa for particle size ~ 5-10 microns



Cohesive forces scaling factor

Coordination number

Packing fraction

Modified Hamaker constant

$$\sigma_{yy} = 1.73 \times 10^{-2} \left[\frac{A_h C_{\#} C_s \phi}{\pi^2} \right] \frac{1}{\bar{r}_p}$$

Mean particle radius - regolith

Modifications due to: orientation of the contacts, fraction of contacts in tension and magnitude distribution of cohesive forces.

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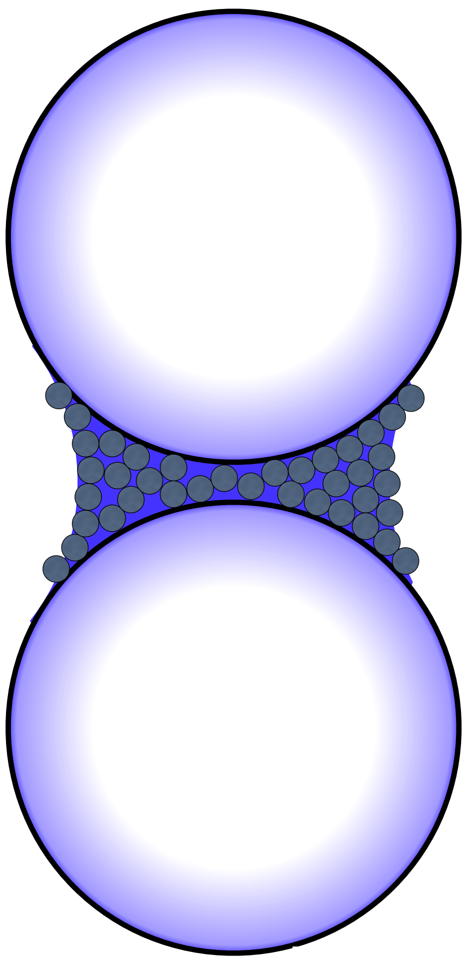

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Spin Rates of Rubble Piles with Cohesion

How do Rubble Pile Asteroids Fail?

Drucker-Prager Failure Criterion

$$\sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{6}}+s(\sigma_1+\sigma_2+\sigma_3)\leq k$$

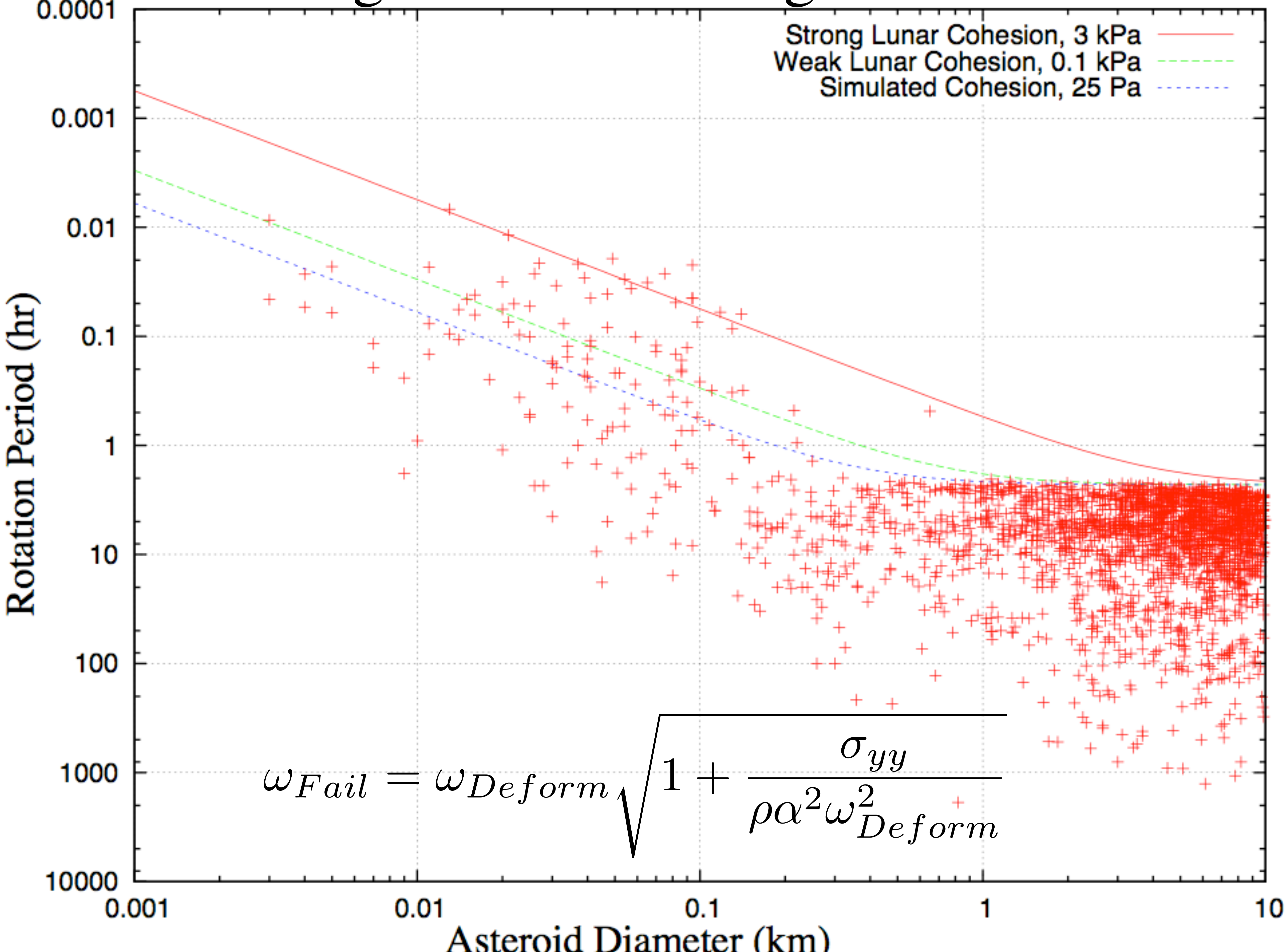
k = Cohesive Shear Stress for Failure at 0 pressure

$$(\sigma_1, \sigma_2, \sigma_3) = \text{ Principal Stresses}$$

$$s=\frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)}$$

$$\omega_{Fail}=\omega_{Deform}\sqrt{1+\frac{\sigma_{yy}}{\rho\alpha^2\omega_{Deform}^2}}$$

Cohesive Strength: Drucker-Prager Yield Criterion



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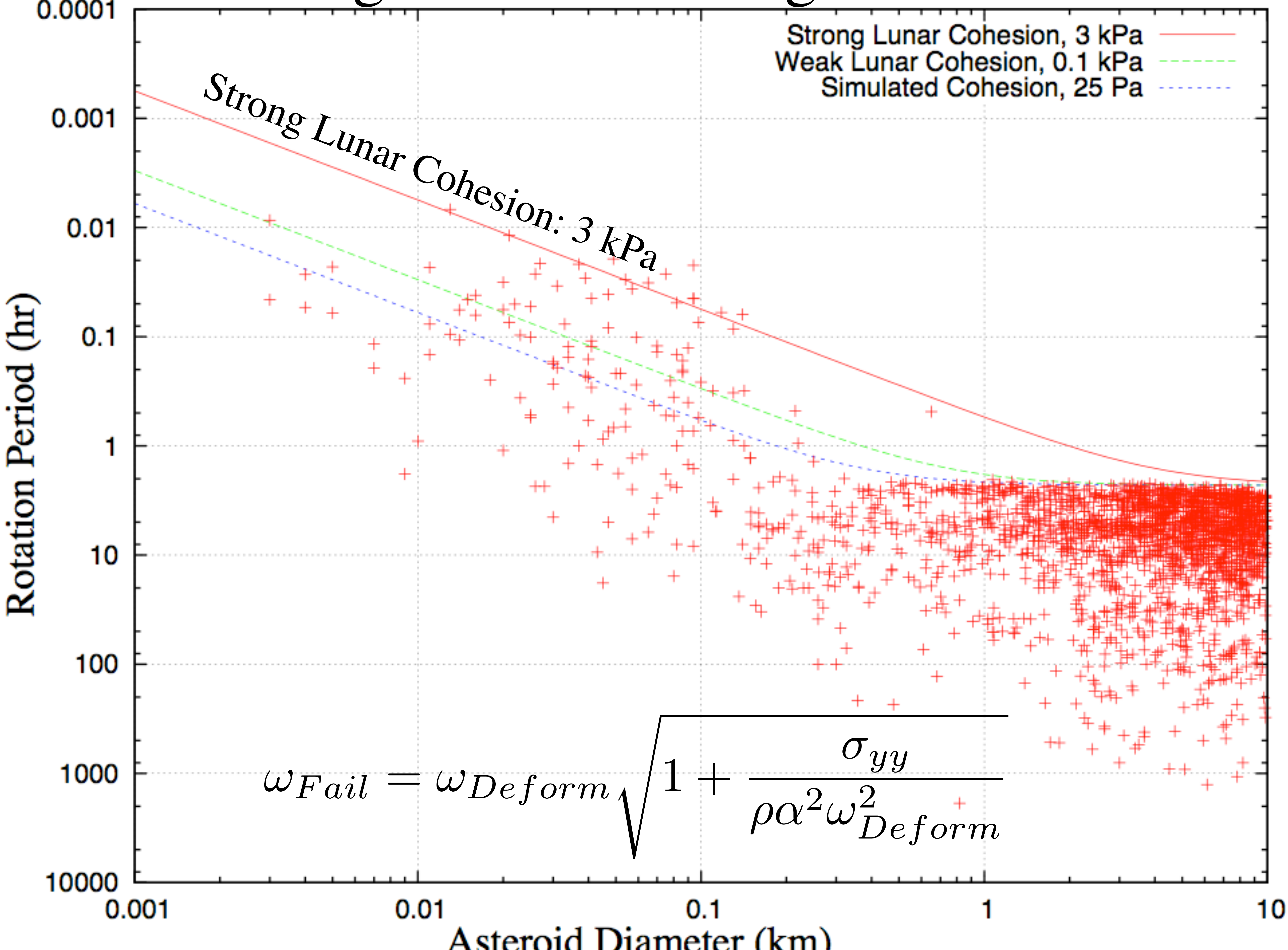

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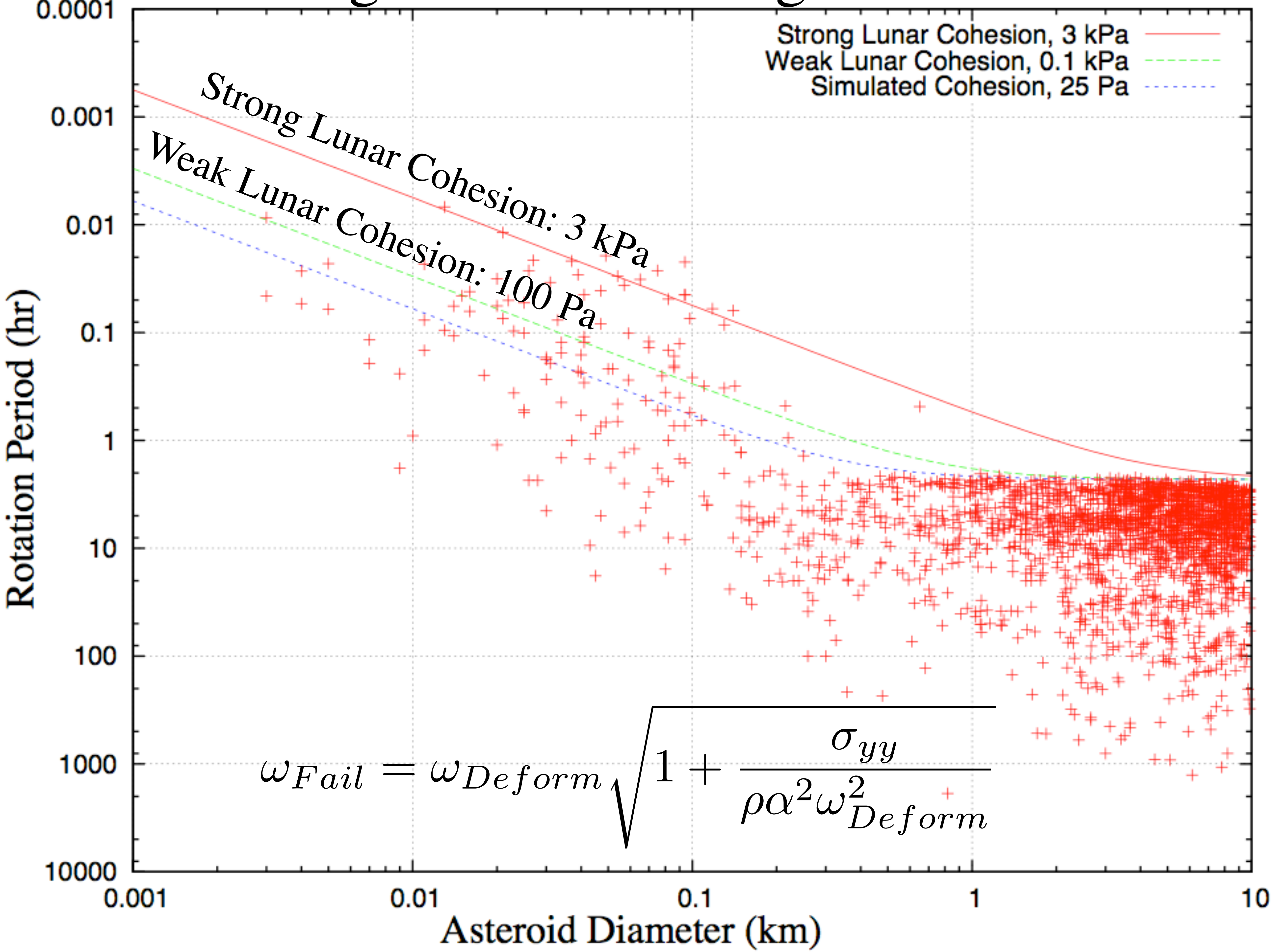

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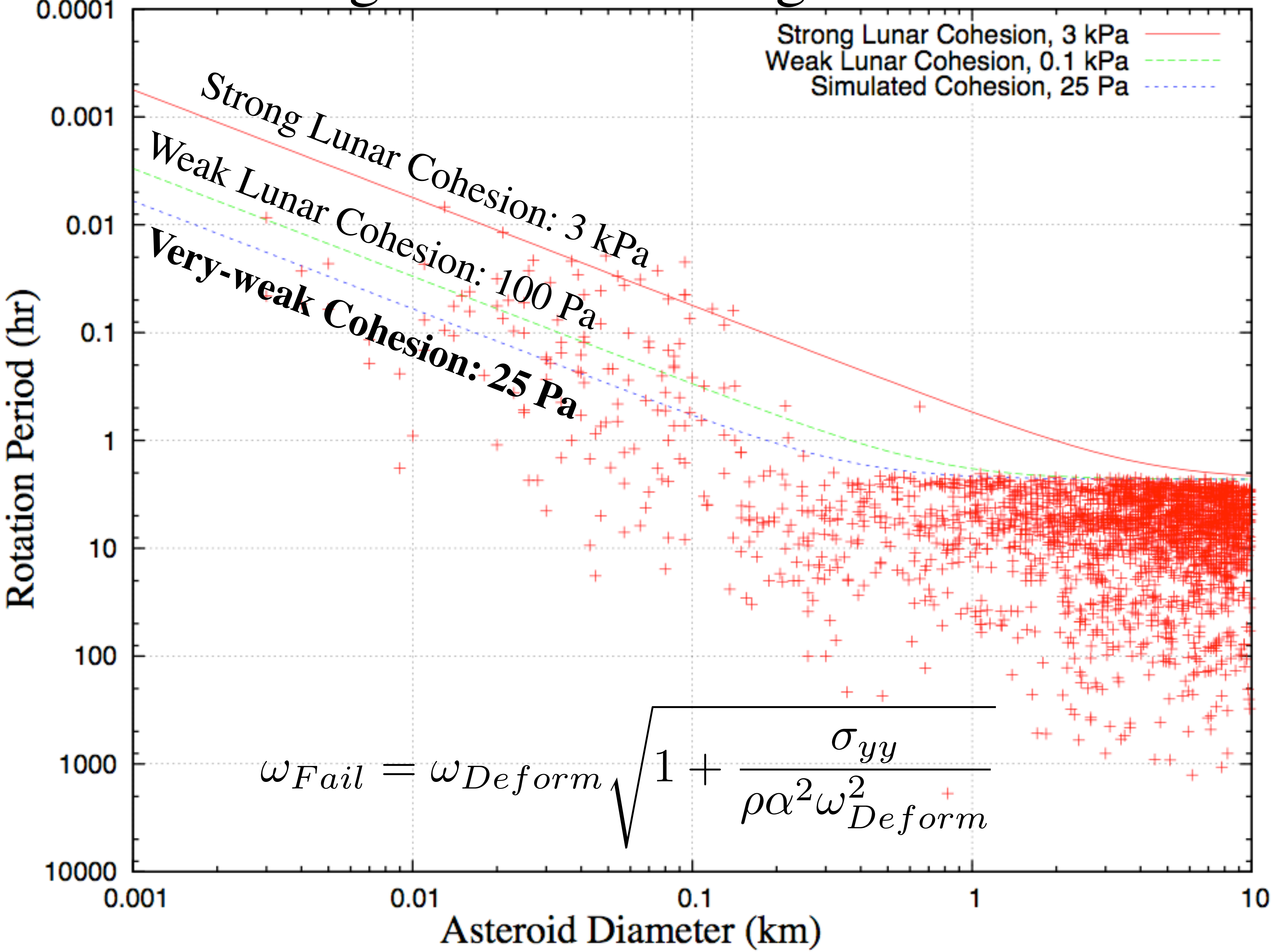

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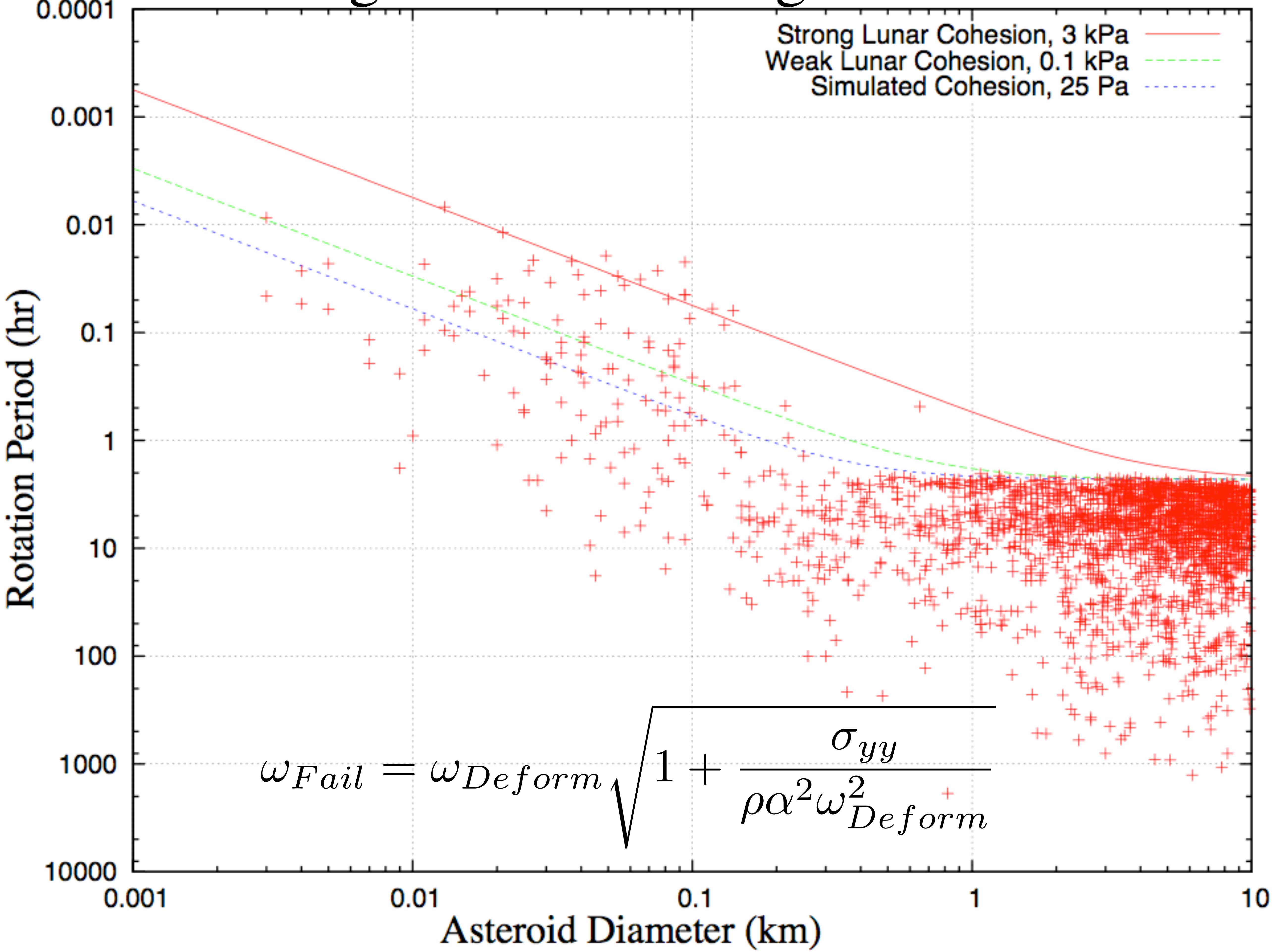
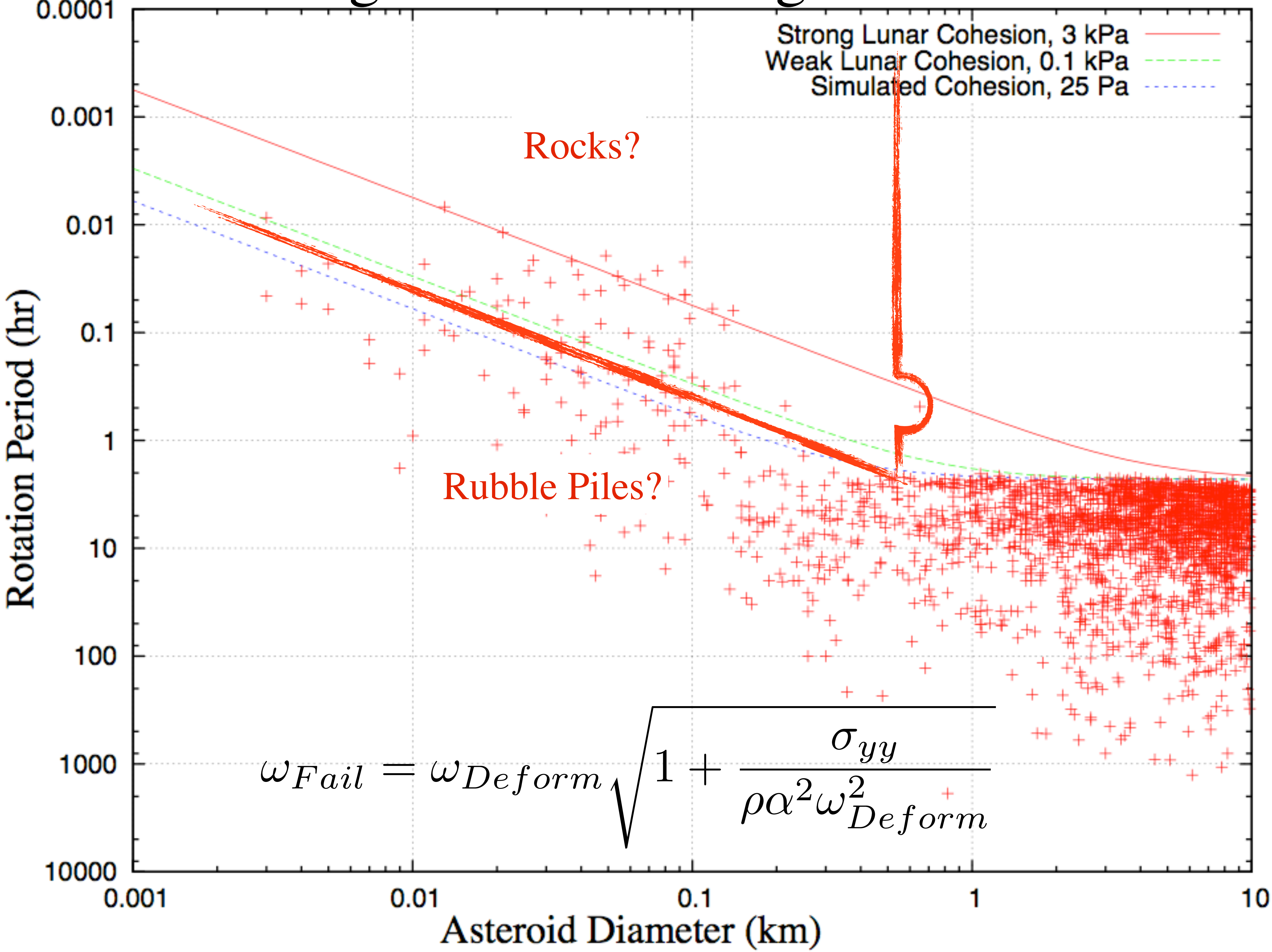


Cohesive Strength: Drucker-Prager Yield Criterion



$$\omega_{Fail} = \omega_{Deform}\sqrt{1 + \frac{\sigma_{yy}}{\rho\alpha^2\omega^2_{Deform}}}$$

Observations of Rubble Pile Cohesive Strength

Recent Observations of Specific Asteroids

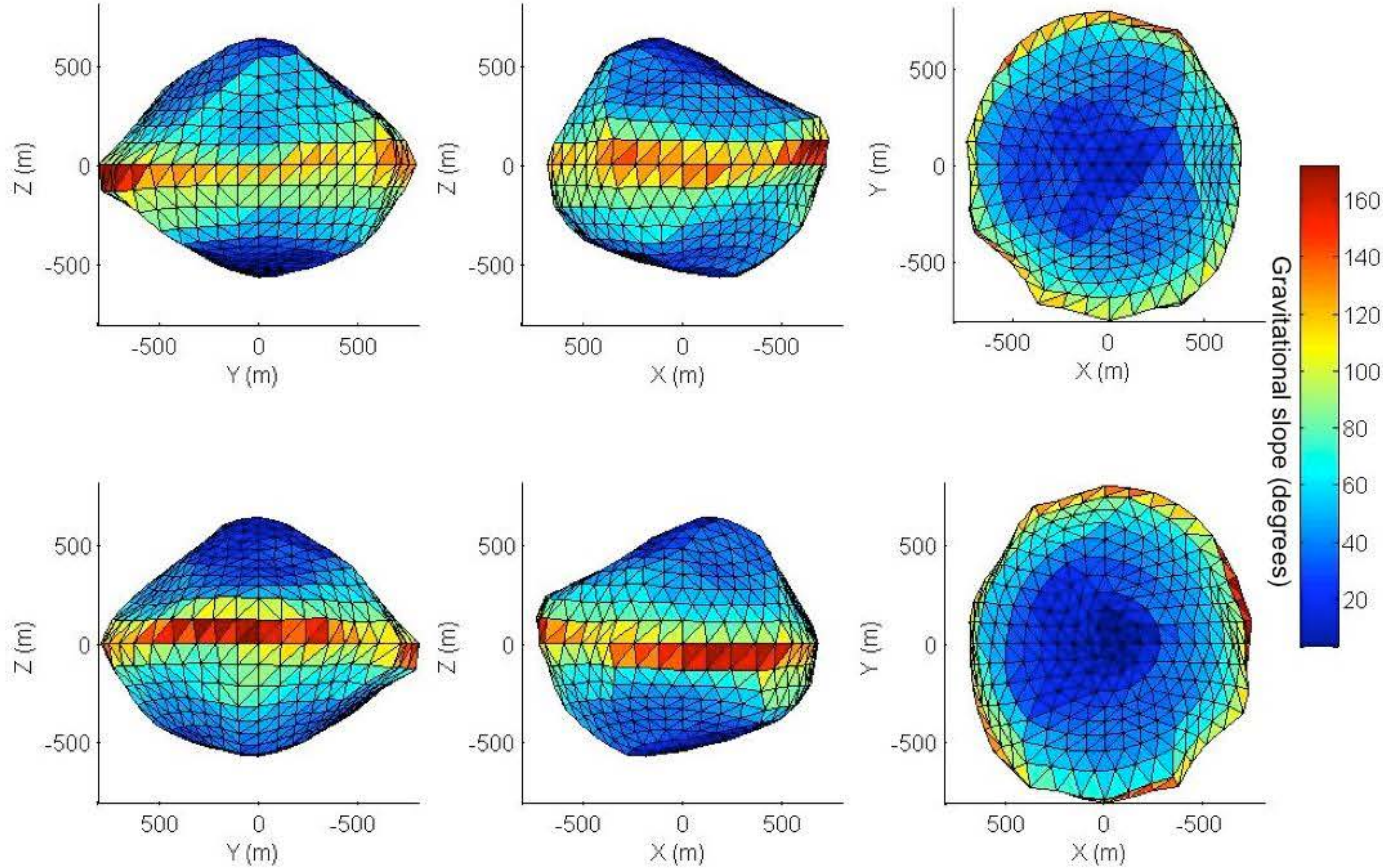
- Recent observations of active asteroids and specific fast-spinning asteroids show that cohesion must be an important aspect of asteroid geophysics
- Specific examples are:

1950 DA

2013 P/R3

2008 TC3

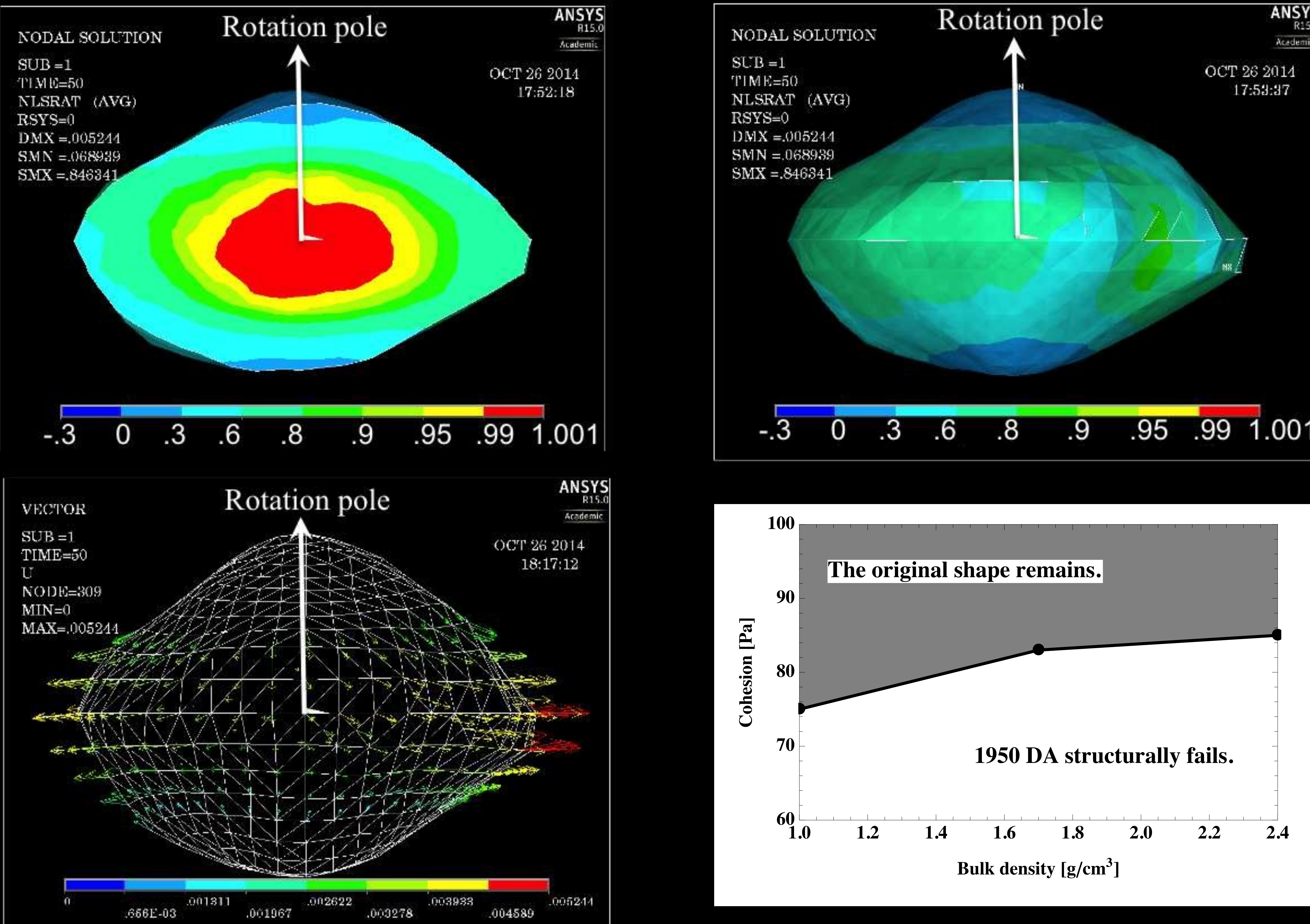
Consistent with 1950 DA

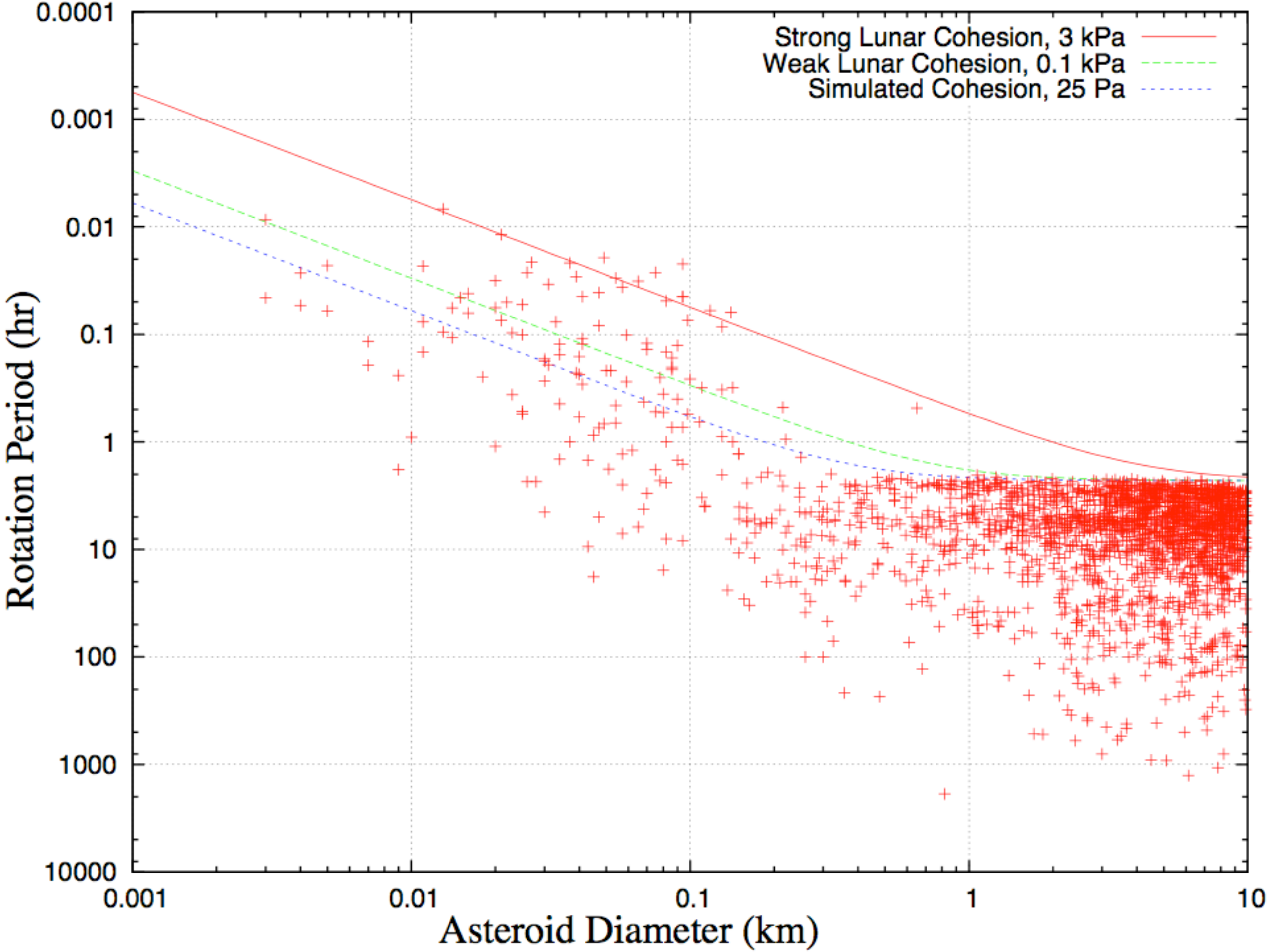


Rotation period = 2.12 hours, Impact probability 250 / 1 000 000 in Year 2880

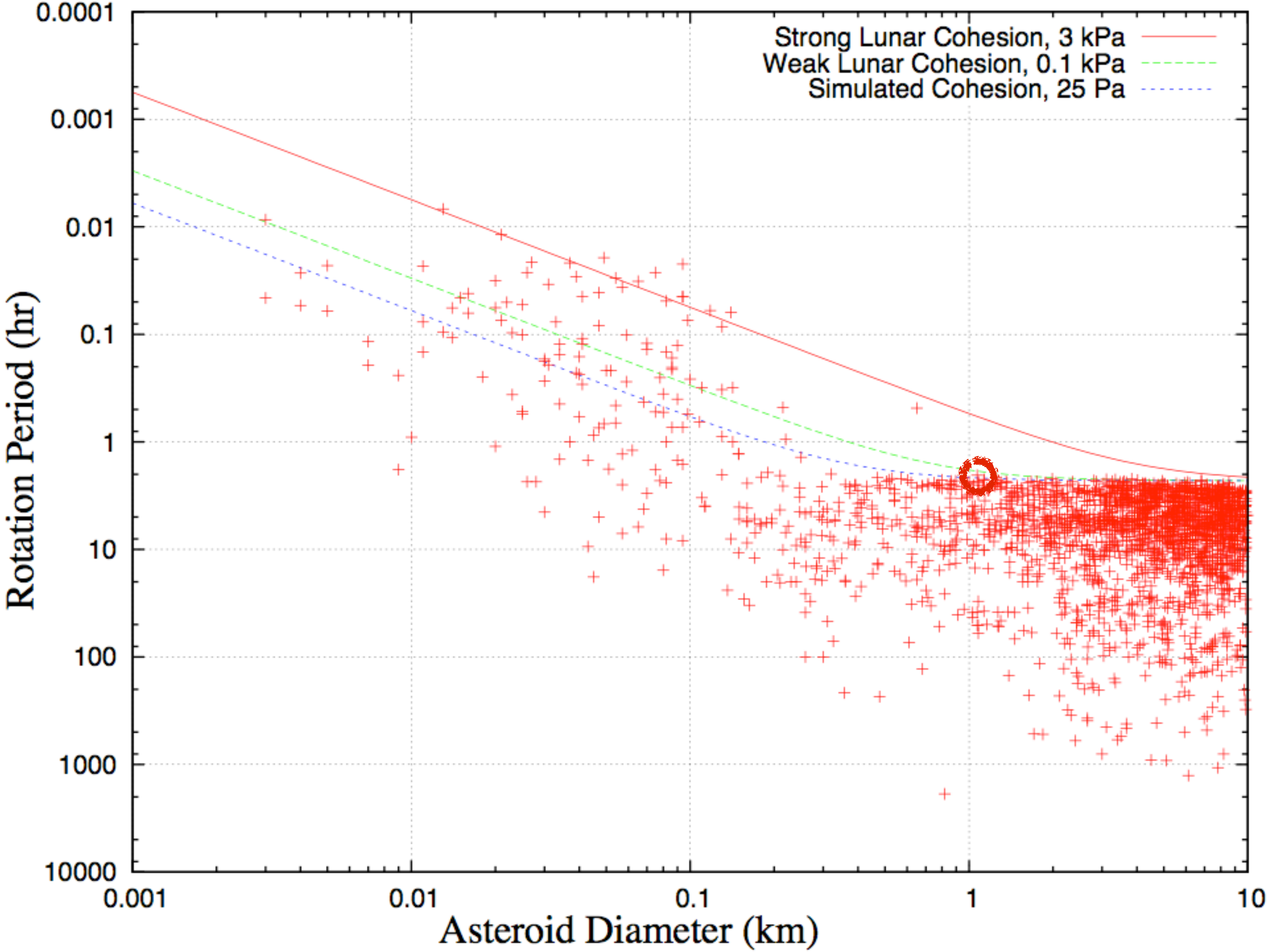
Probable strength ~ 60 Pascals (Rozitis et al. Nature)

“Stress and Failure Analysis of Rapidly Rotating Asteroid (29075) 1950 DA,” Hirabayashi & Scheeres, ApJL 2015





Strong Lunar Cohesion, 3 kPa
Weak Lunar Cohesion, 0.1 kPa
Simulated Cohesion, 25 Pa
Rotation Period (hr)
0.0001
0.001
0.01
0.1
1
10
100
1000
10000
Asteroid Diameter (km)
0.001
0.01
0.1
1
10



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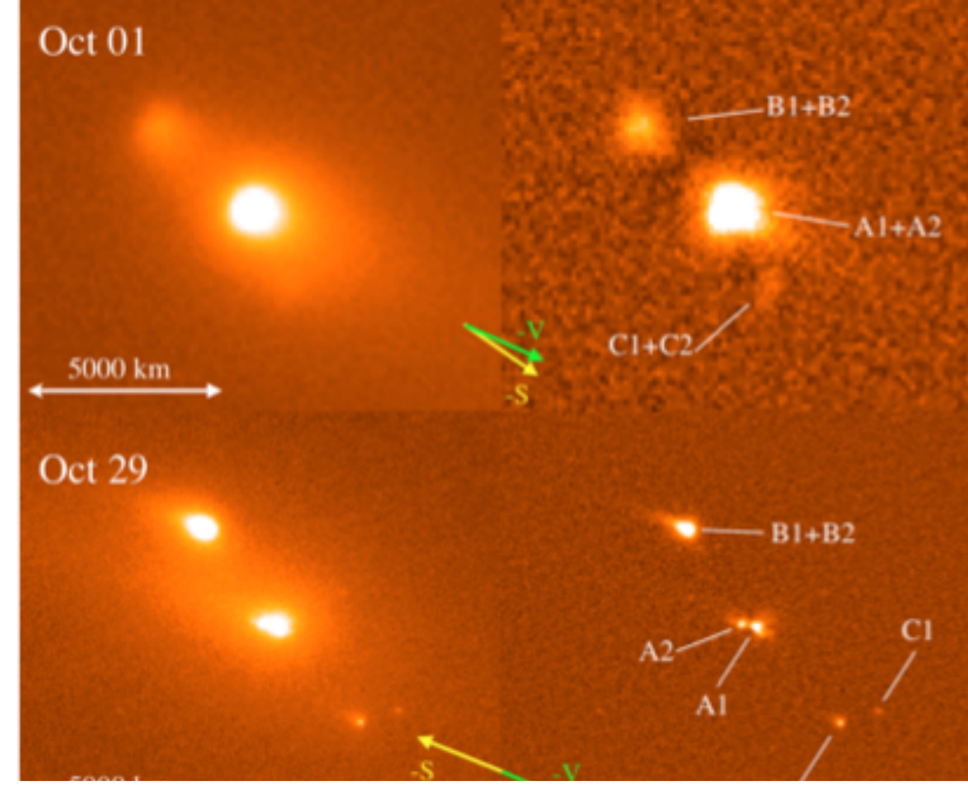
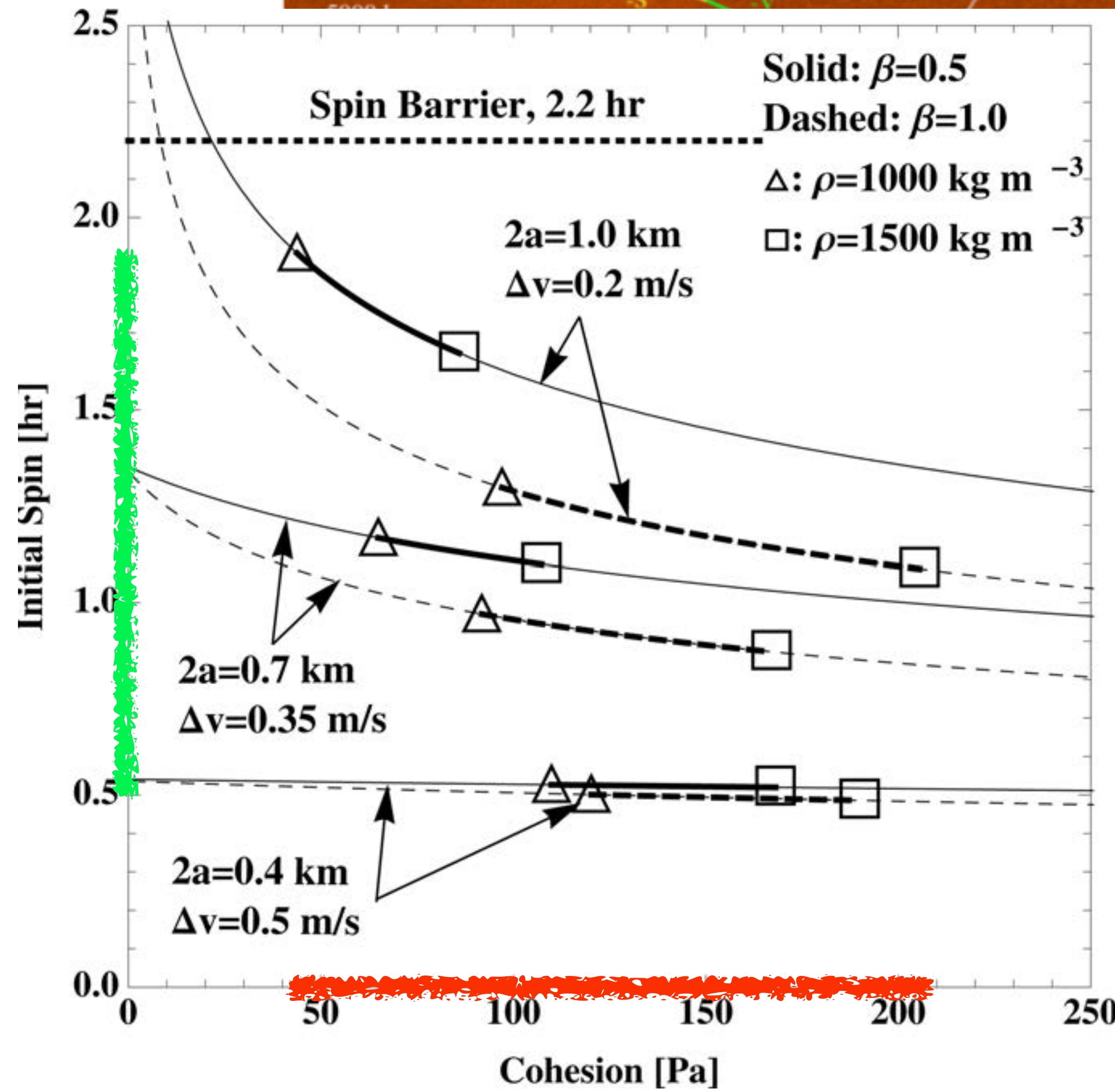
Rotation Period (hr)
0.0001
0.001
0.01
0.1
1
10
100
1000
10000
Asteroid Diameter (km)
0.001
0.01
0.1
1
10

Consistent with P/2013 R3

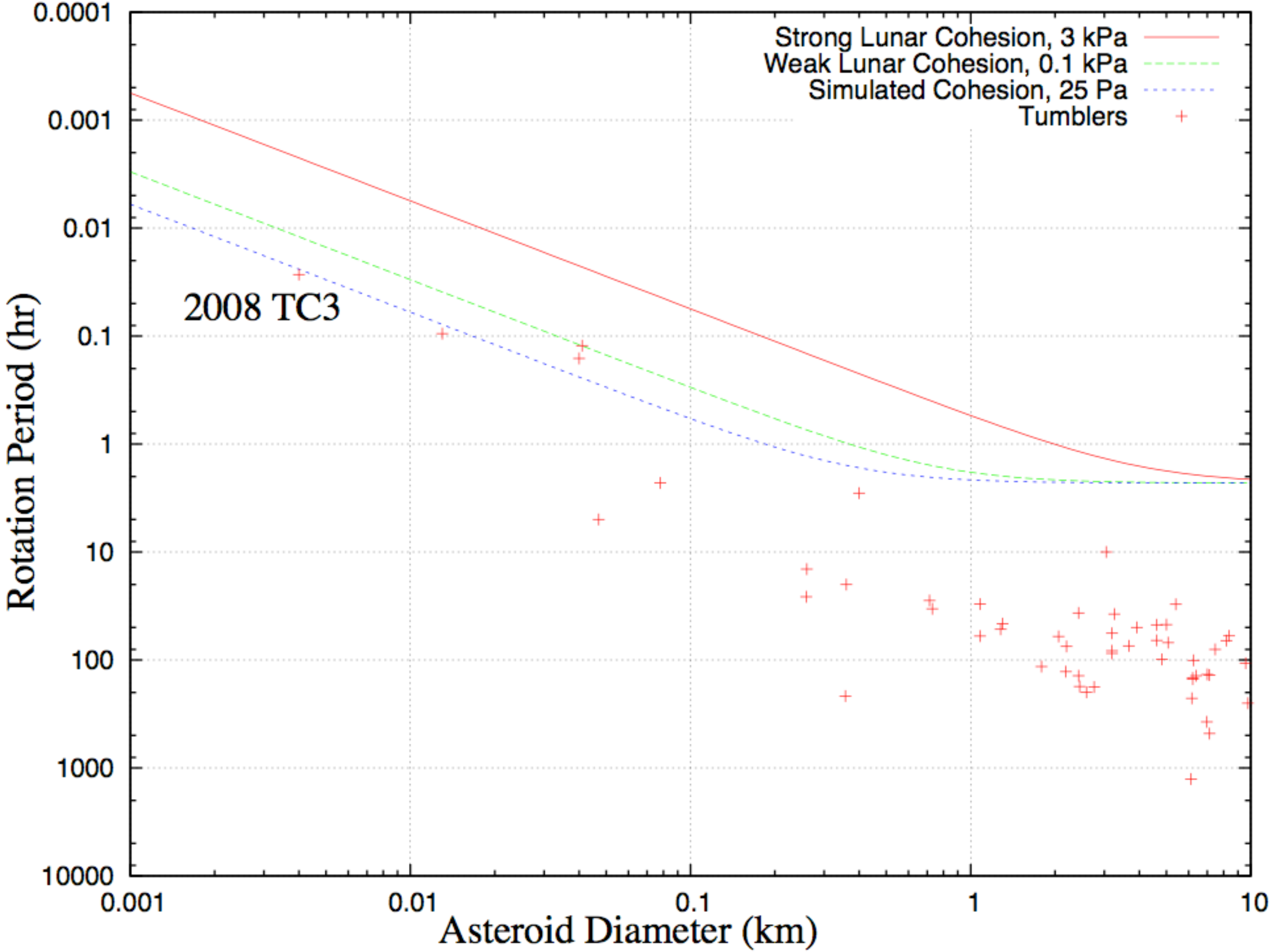
- Observed by Jewitt et al. (ApJL 2014) to be an "active asteroid" whose formation is consistent with rotational fission
- Hirabayashi et al. (ApJL 2014) show the published measurements indicate:
  - Parent body strength of **40 — 210 Pa**
  - Jewitt et al. (AJ 2017) to 5**0 — 100 Pa**
- Constraints found by:
  - Mapping observed pairs back to parent:
    - Assume either an ellipsoid or spherical parent
  - Assuming C-Type bulk density
  - Evaluating necessary cohesion with a Drucker-Prager strength model



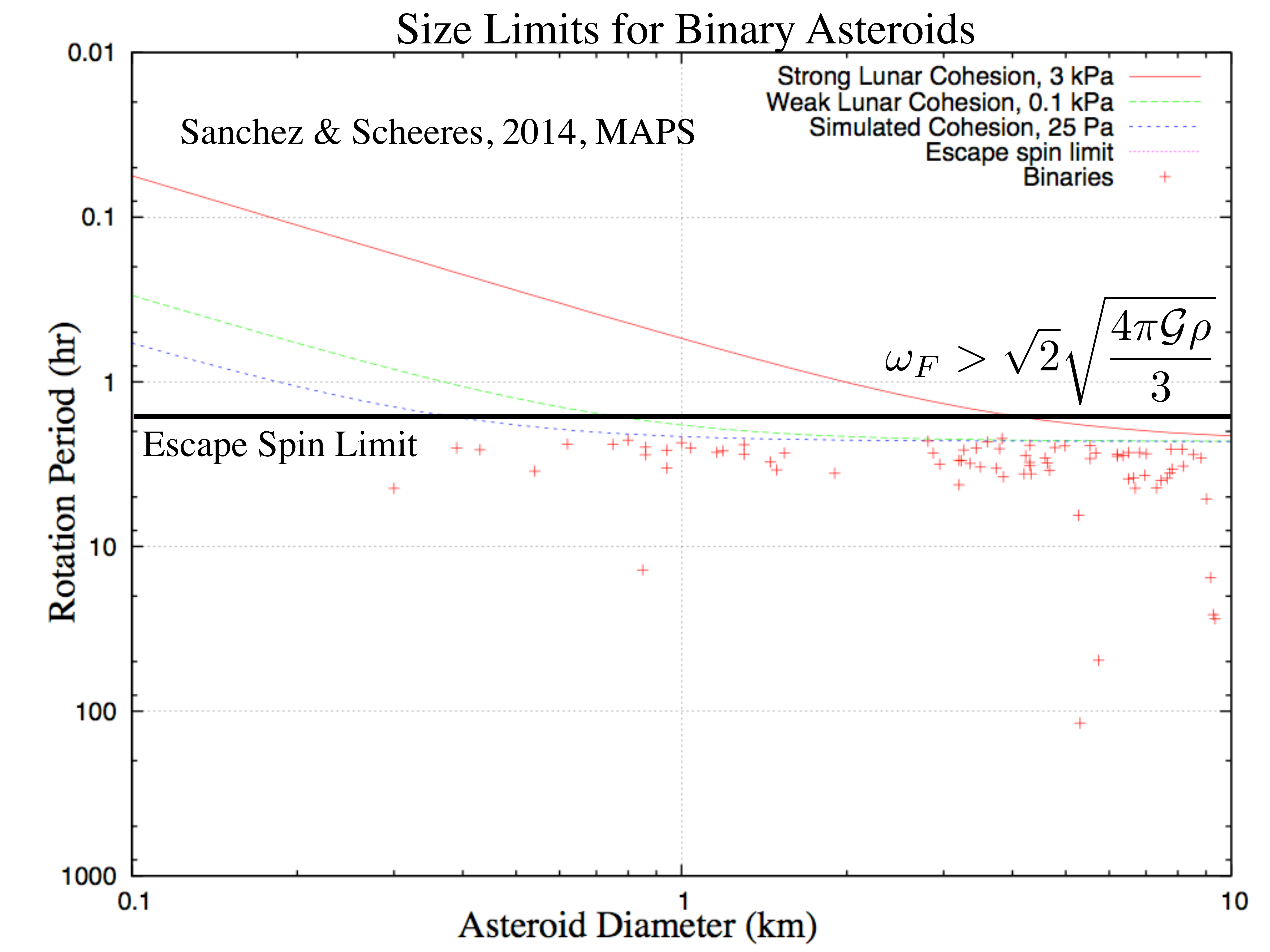


2008 TC3

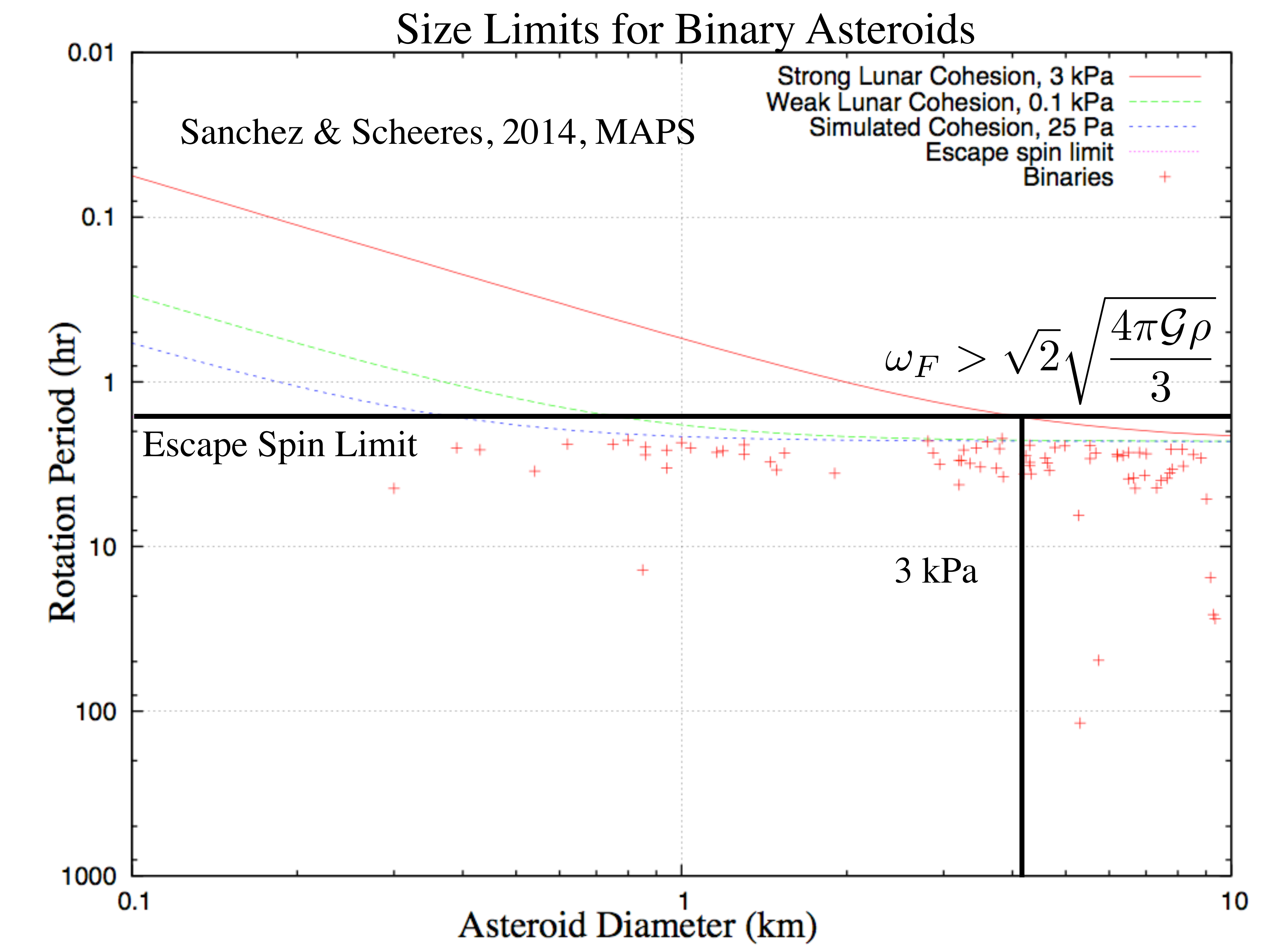
- 2008 TC3 = Almahatta Sitta Meteorite
- Pre-entry observations *(Scheirich et al., MAPS 2010)*:
  - Tumbling, fast spinning body… but only requires ~ 25 Pa of cohesion to remain a stable collection of rocks, easily provided by having the larger components embedded within a matrix of fines ~ < 10 microns
- Entry observations:
  - Significant macro-porosity (*Kohout et al., Icarus 2011)*
  - High break-up altitude, indicating a "weak" body (*Popova et al., MAPS 2011)*
  - Substantial loss of 1-10 micron material in upper atmosphere (*Borovicka & Charvat, A&A 2009)*
- Ground-fall observations *(Jenniskens et al., Nature 2009)*:
  - Was composed of mineralogically diverse components consistent with the parent body being an aggregate
- CAVEAT *(Borovicka, IAU 2015)*:
  - Break-up altitude occurred at a dynamic pressure of 50 kPa — much stronger than proposed strength… however the breakup of rubble piles in the atmosphere is not understood



Strong Lunar Cohesion, 3 kPa
Weak Lunar Cohesion, 0.1 kPa
Simulated Cohesion, 25 Pa
Tumblers
2008 TC3
Rotation Period (hr)
Asteroid Diameter (km)
0.0001
0.001
0.01
0.1
1
10
100
1000
10000
0.001
0.01
0.1
1
10



Size Limits for Binary Asteroids
Strong Lunar Cohesion, 3 kPa
Weak Lunar Cohesion, 0.1 kPa
Simulated Cohesion, 25 Pa
Escape spin limit
Binaries
Sanchez & Scheeres, 2014, MAPS
$\omega_F > \sqrt{2}\sqrt{\frac{4\pi\mathcal{G}\rho}{3}}$
Escape Spin Limit
Rotation Period (hr)
0.01
0.1
1
10
100
1000
Asteroid Diameter (km)
0.1
1
10



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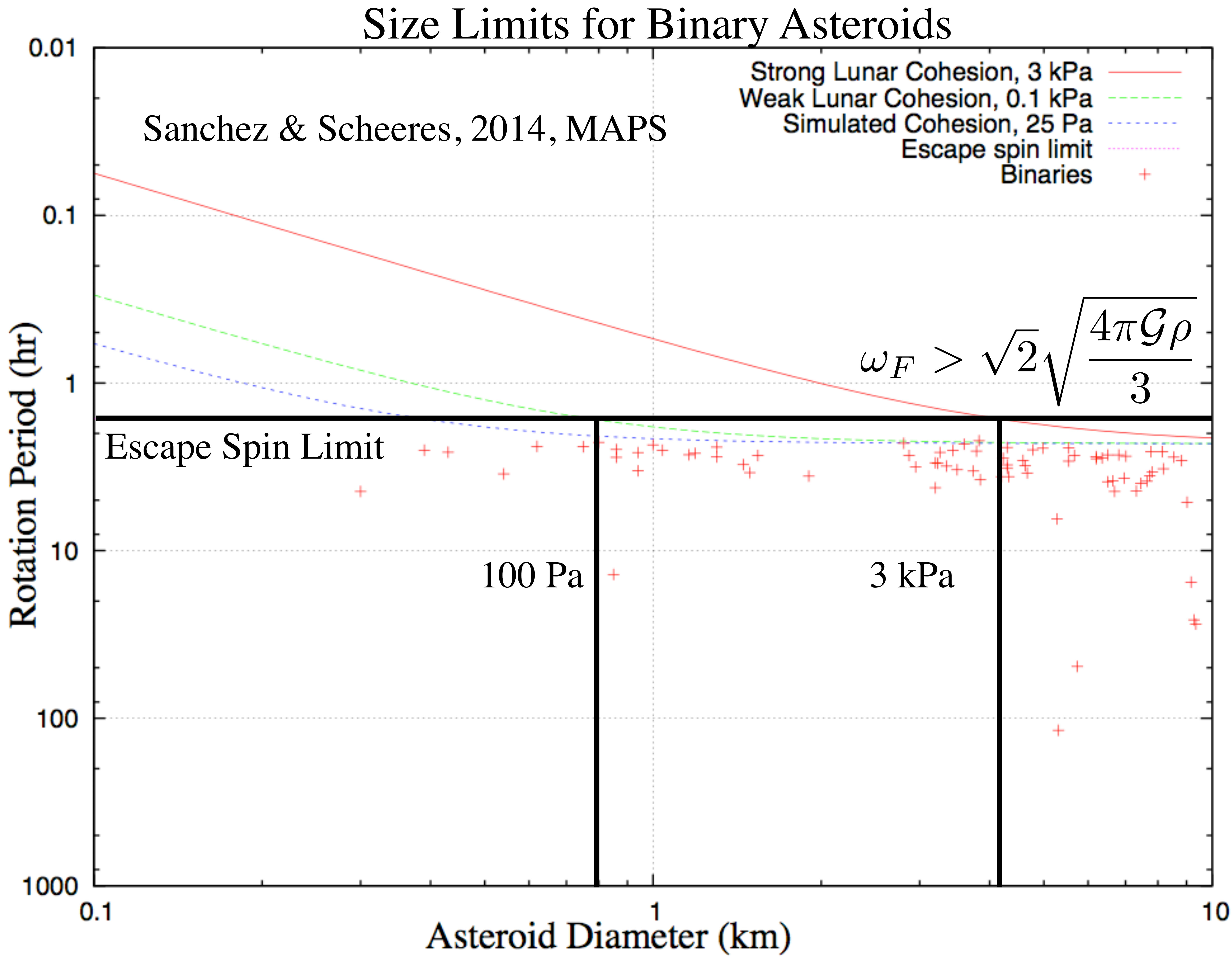
Escape Spin Limit
3 kPa
Rotation Period (hr)
Asteroid Diameter (km)
0.01
0.1
1
10
100
1000
0.1
1
10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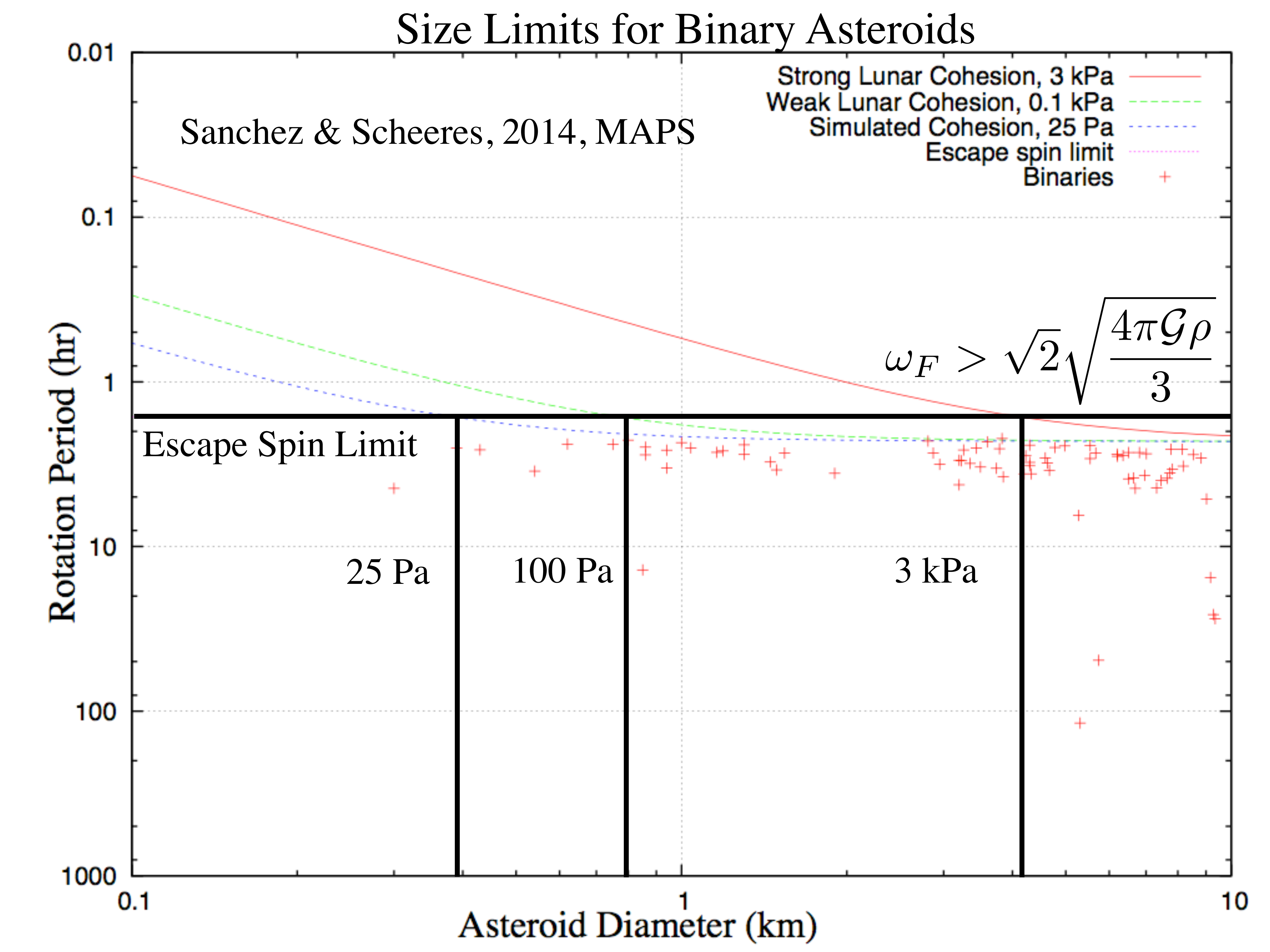
Escape Spin Limit
100 Pa
3 kPa
Rotation Period (hr)
0.01
0.1
1
10
100
1000
Asteroid Diameter (km)
0.1
1
10



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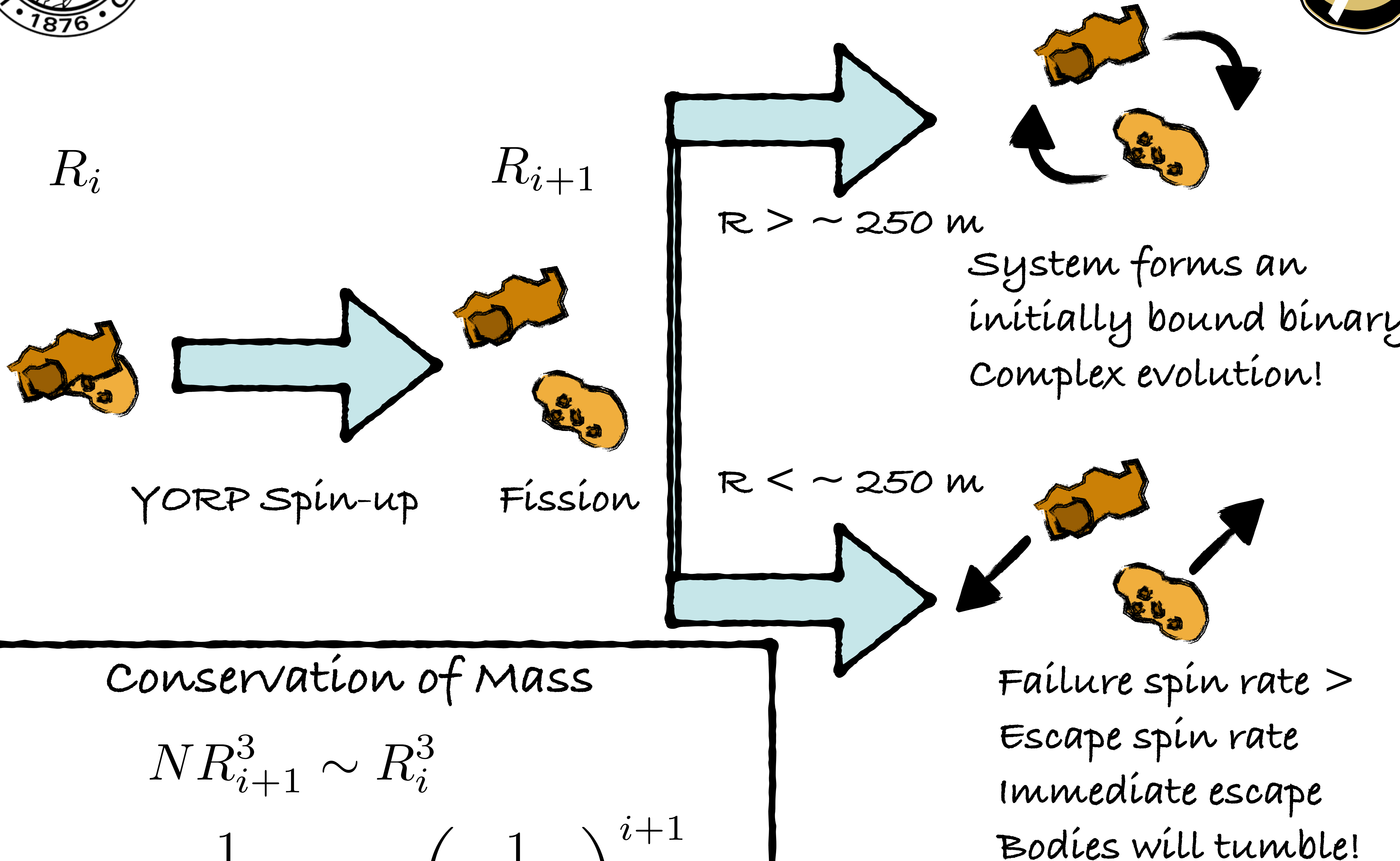
25 Pa
100 Pa
3 kPa
Rotation Period (hr)
Asteroid Diameter (km)
0.01
0.1
1
10
100
1000
0.1
1
10

A Fork in the Road

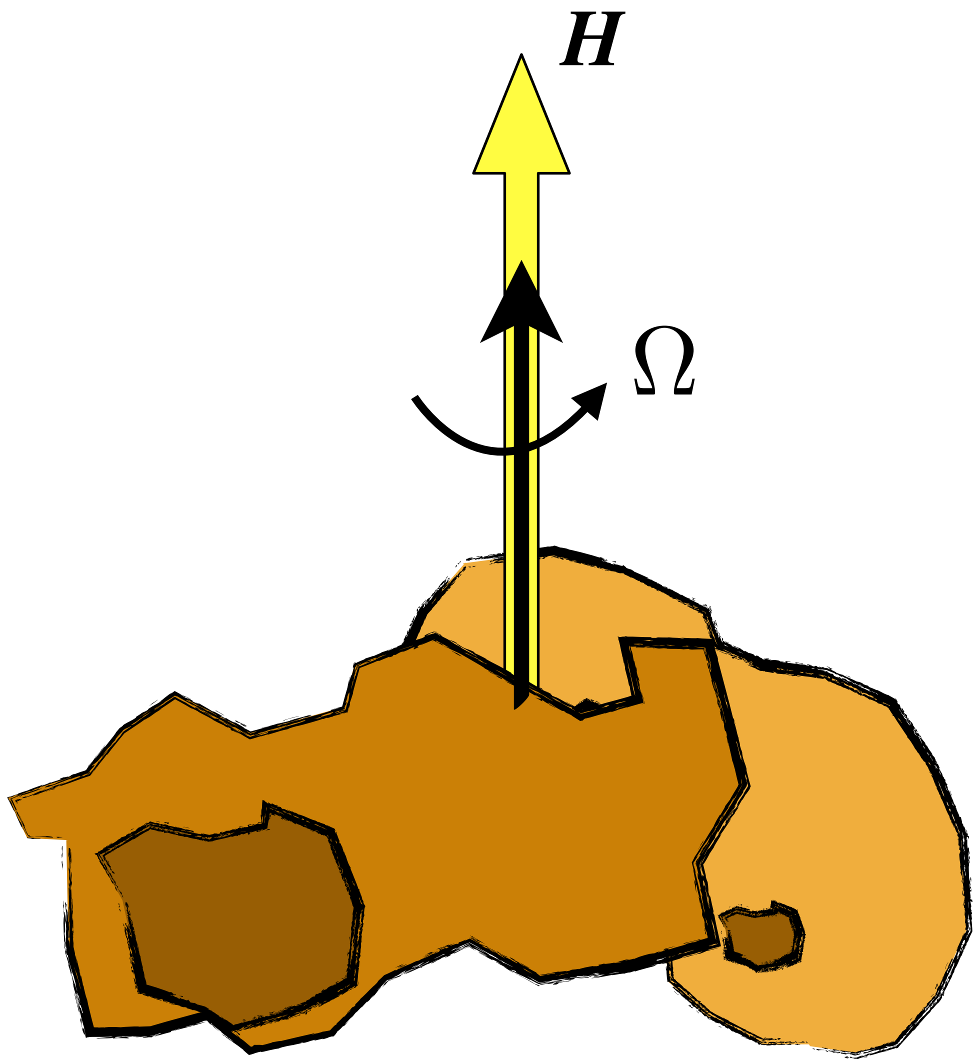


Conservation of Mass

$$N R_{i+1}^3 \sim R_i^3$$

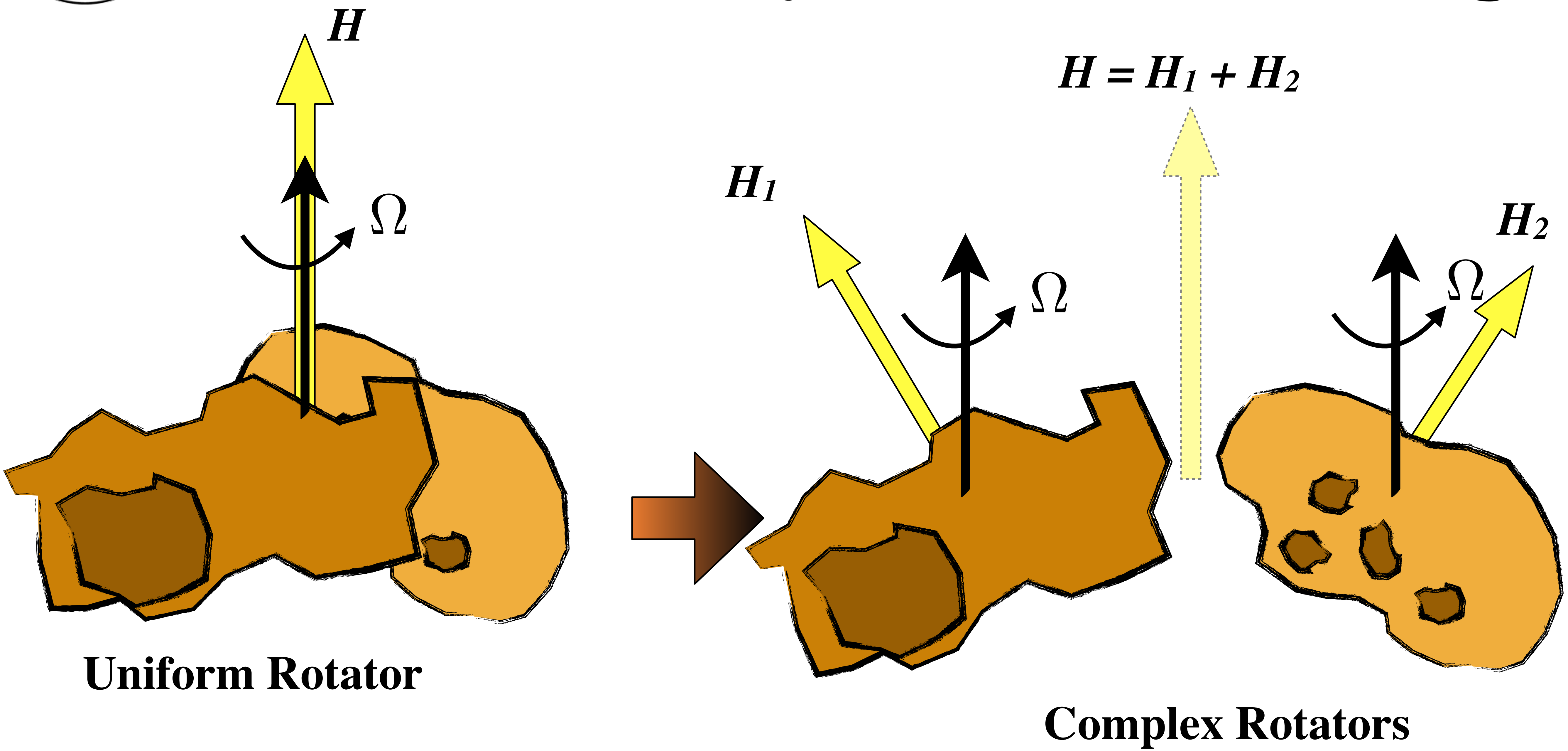
$$R_{i+1} \sim \frac{1}{N^{1/3}} R_i \sim \left(\frac{1}{N^{1/3}} \right)^{i+1} R_0$$

Fission Leads to Fast Rotating Tumblers

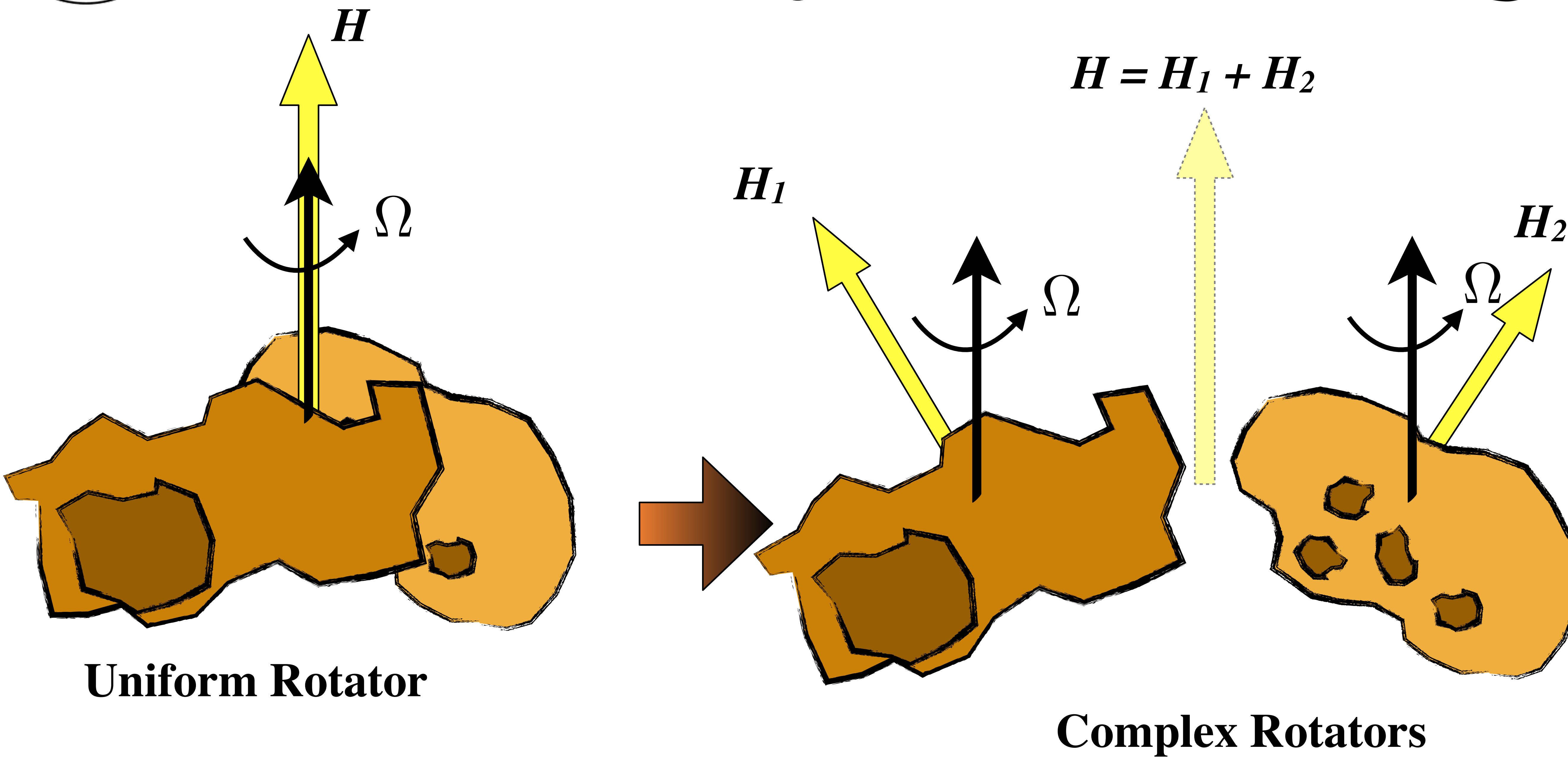


Uniform Rotator

Fission Leads to Fast Rotating Tumblers

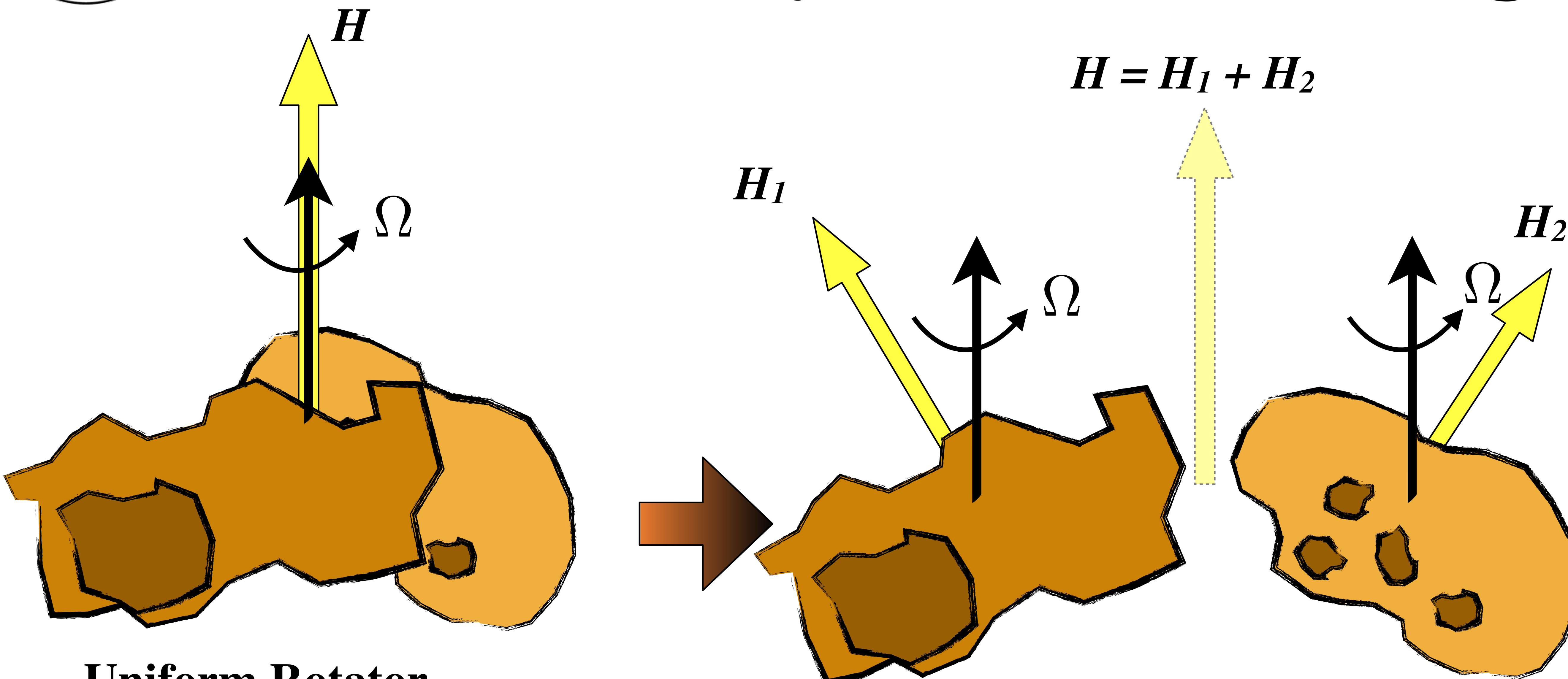


Fission Leads to Fast Rotating Tumblers



A tumbling body will likely be less affected by YORP… how fast does it relax wrt YORP timescales?

Fission Leads to Fast Rotating Tumblers



Uniform Rotator

Complex Rotators

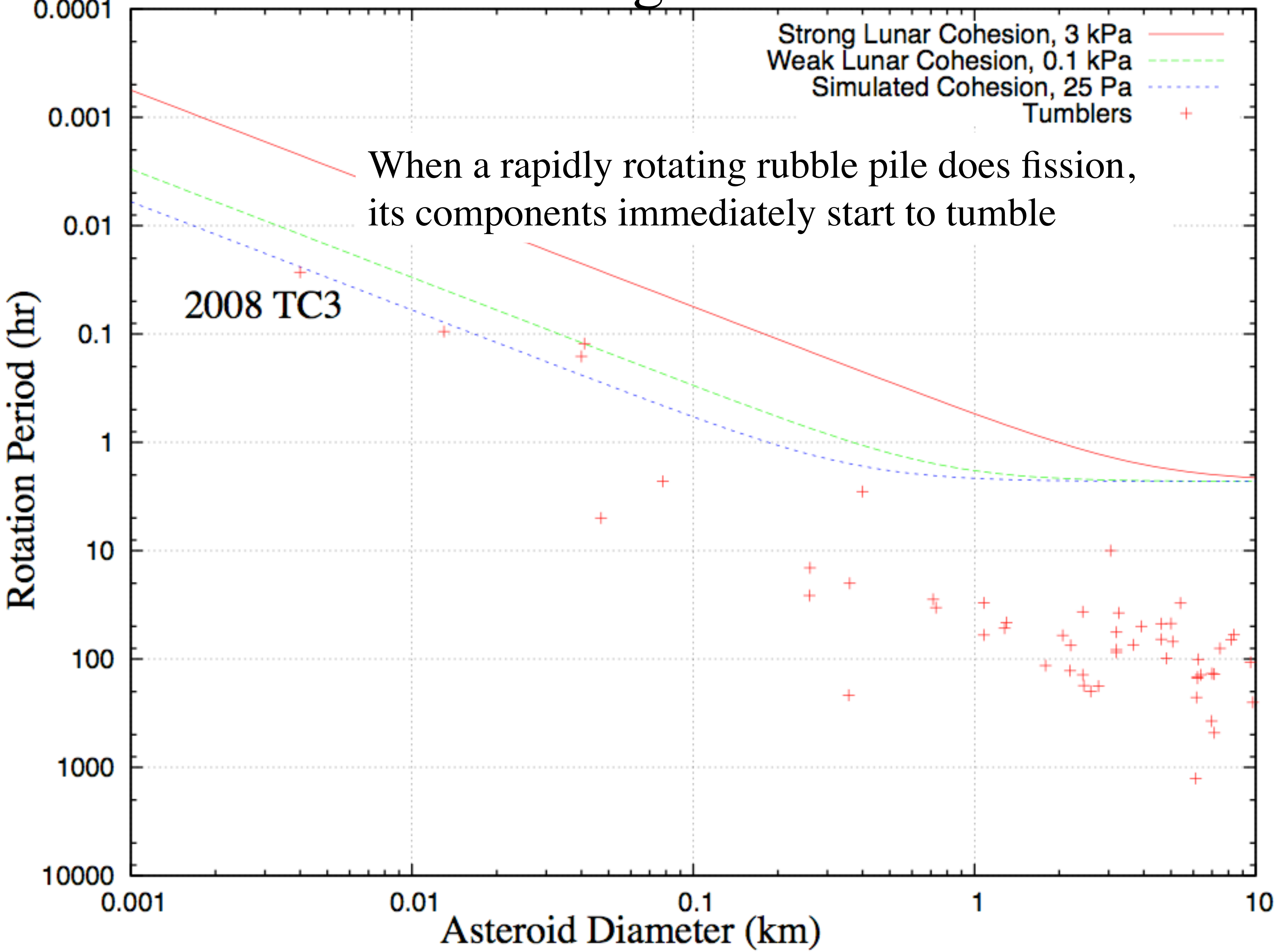
A tumbling body will likely be less affected by YORP… how fast does it relax wrt YORP timescales?

Scheirich et al. (2015)

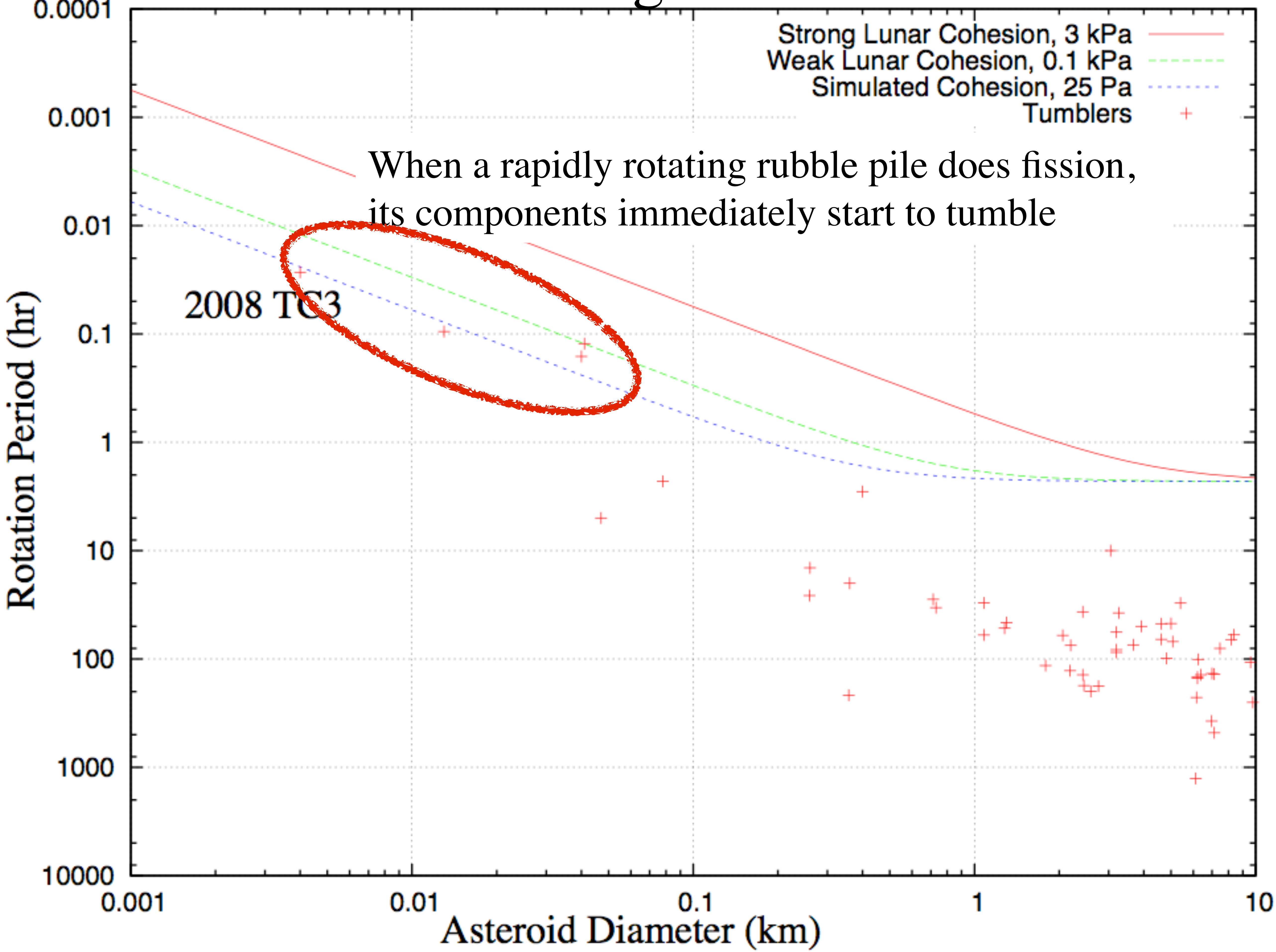
$$\tau_R/\tau_Y \sim 1 \times 10^{-5} \rightarrow 0.3$$

Margot et al. 2000

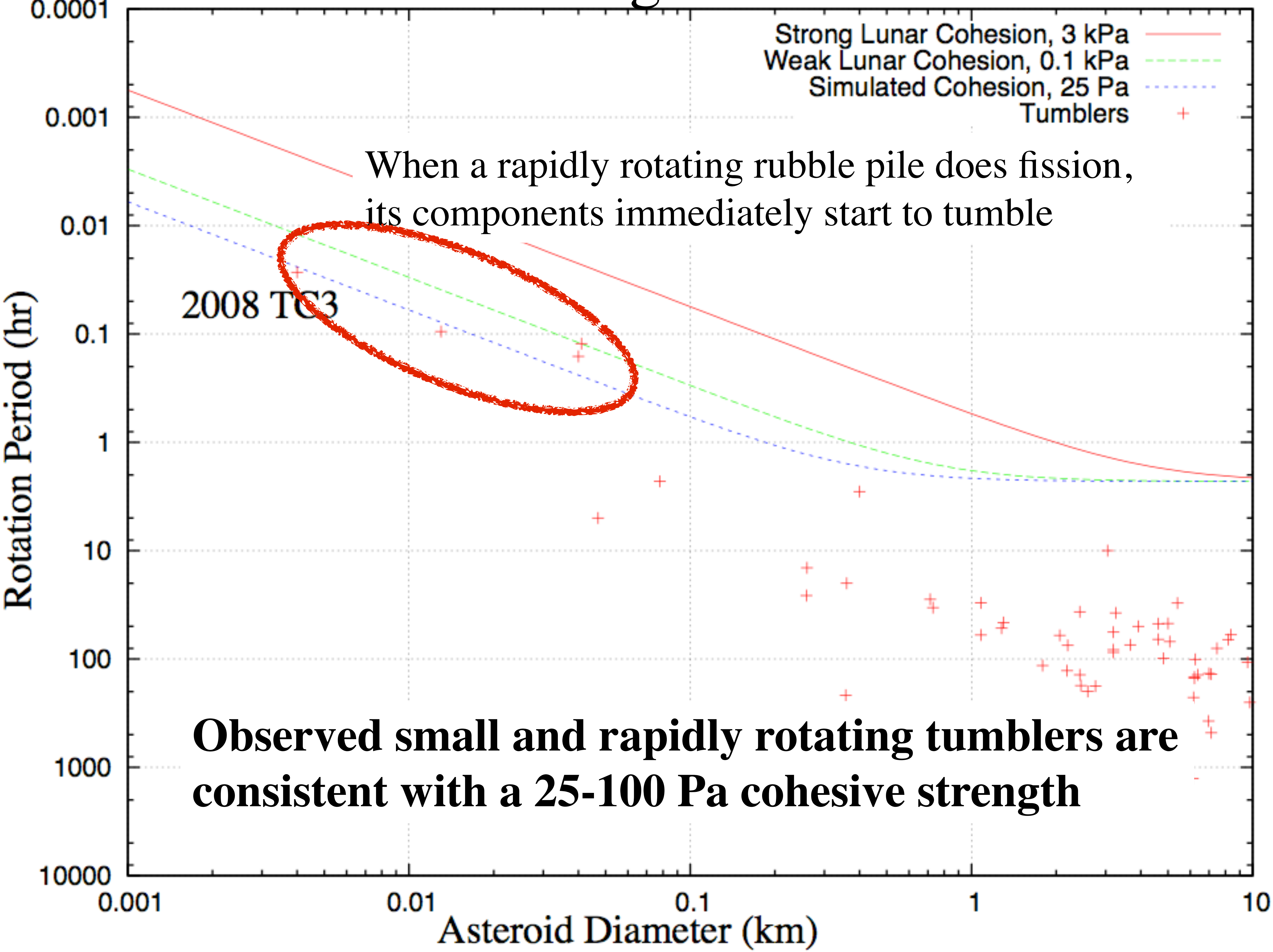
Fast Tumbling Asteroids

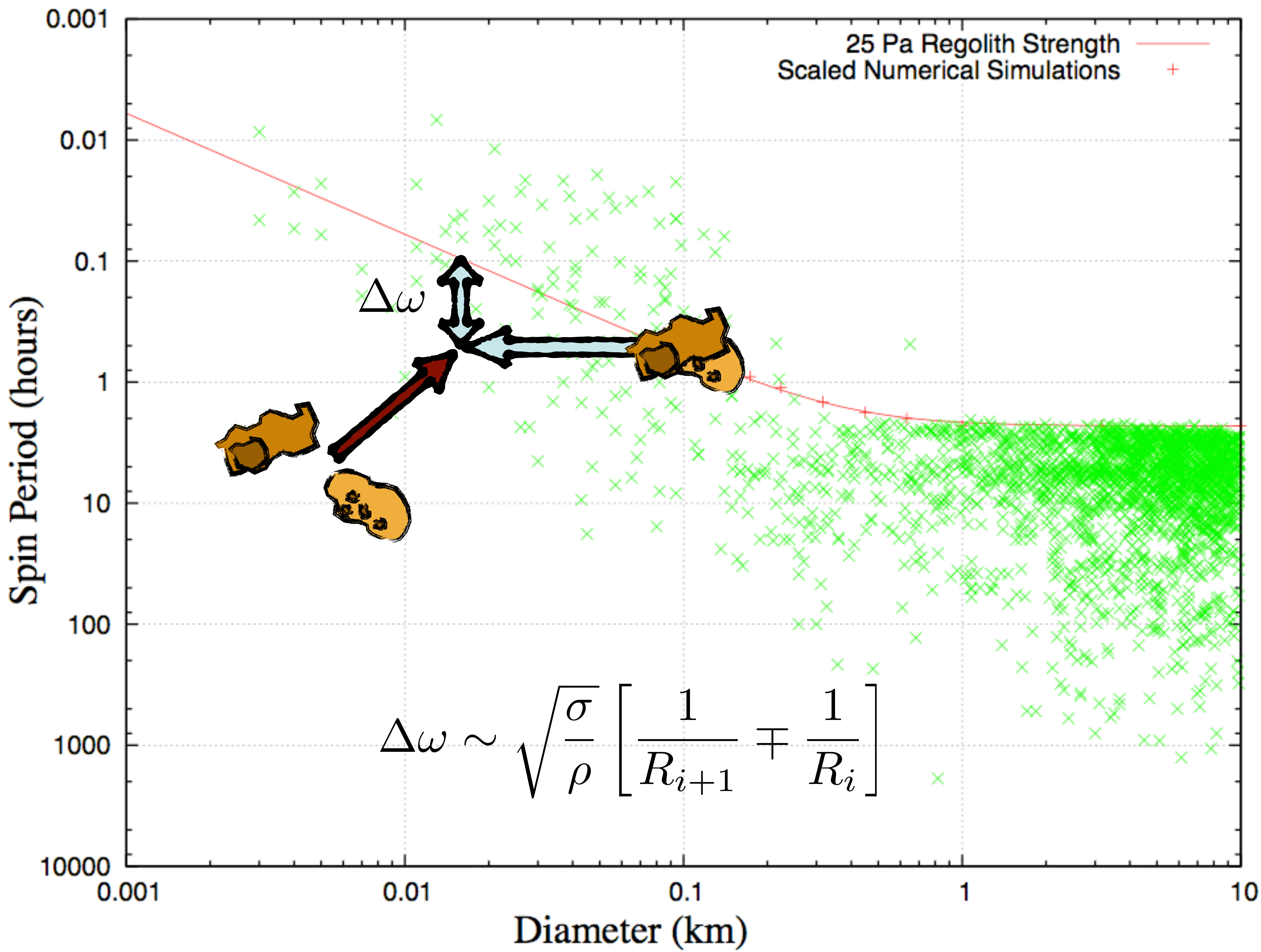


Fast Tumbling Asteroids



Fast Tumbling Asteroids





25 Pa Regolith Strength
Scaled Numerical Simulations
Spin Period (hours)
Diameter (km)
0.001
0.01
0.1
1
10
100
1000
10000
0.001
0.01
0.1
1
10
$\Delta\omega$
$\Delta\omega \sim \sqrt{\frac{\sigma}{\rho}}\left[\frac{1}{R_{i+1}} \mp \frac{1}{R_i}\right]$

Time-Scale for Next Fission

$$T_{i,i+1} \sim \frac{\Delta\omega_{i,i+1}}{\dot{\omega}_{i+1}} \sim \frac{4\pi A^2}{3\Phi}\frac{\sqrt{\sigma\rho}}{\mathcal{C}}\left[1 \mp \frac{1}{N^{1/3}}\right] R_{i+1}$$

Lifetime Once Abrupt Fission Begins

$$T < \frac{4\pi A^2}{3\Phi}\frac{\sqrt{\sigma\rho}}{\mathcal{C}}\frac{1 \mp \frac{1}{N^{1/3}}}{1 - \frac{1}{N^{1/3}}} R_0$$

$R_0 \sim 250$ m

$\sigma \sim 100$ Pa

$N = 2 \quad \Phi \sim 1 \times 10^{17}$ kg-m/s^2 $\quad \rho \sim 3000$ kg/m^3 $\quad A = 1AU$

$$T < \begin{cases} 0.4 \rightarrow 4 \text{ MY} & \text{Spins up in the same direction} \\ 3.5 \rightarrow 35 \text{ MY} & \text{Spins up in the reverse direction} \end{cases}$$

Strong YORP *Weak YORP*

What is the Lifetime of a Small Rubble Pile Asteroid?

- For 100 Pa of strength, bodies ~ 500 m or less enter an end-of-life phase that continues till the body is disaggregated into its component pieces
- In the absence of unrealized sinks, lifetime is finite:
  – At 1 AU as short as 0.4 MY for a 500 m body
  – For 1998 TC3 (at ~ 5 m in size) from 4,000 -> 350,000 years
  – In the main belt a factor of 10 times longer
- Caveats / Future Work:
  – How does tidal dissipation work for a small, cohesive rubble pile?
  – For how long can a rubble pile be trapped at a zero spin rate?
  – What role does size distribution play?
  – What are realistic values of the normalized YORP coefficient?
  – Which observations can verify this?

Small Rubble Pile Summary

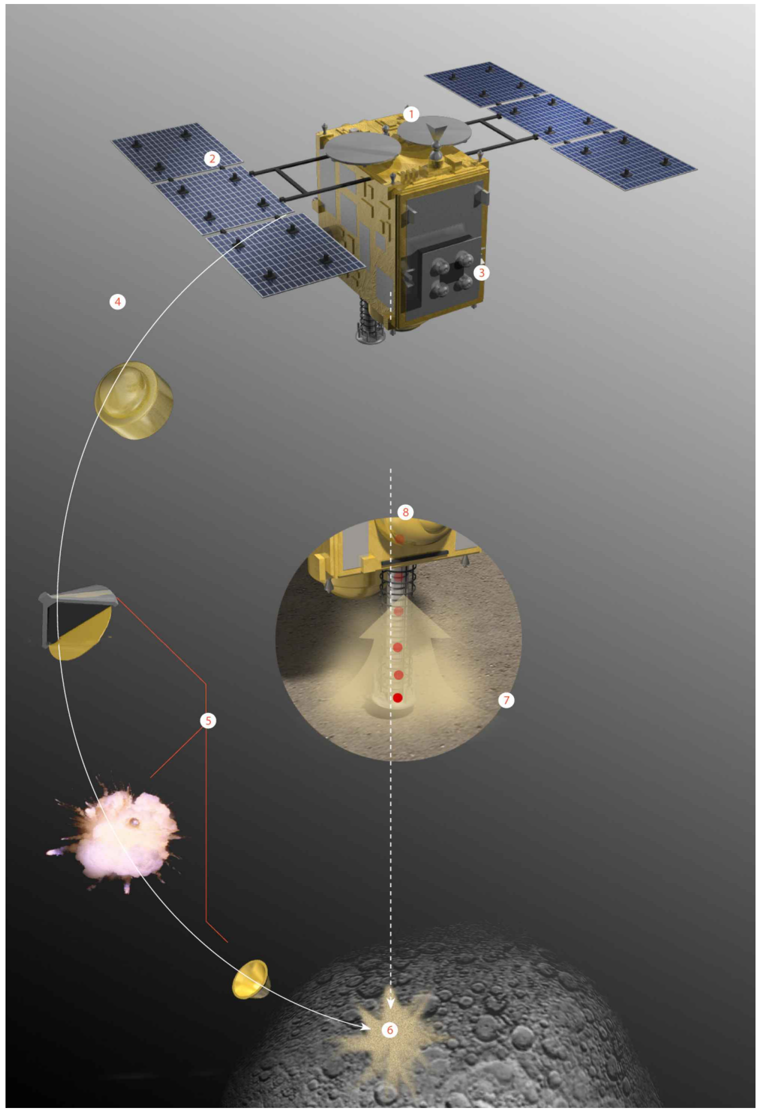
- Rubble pile asteroids can be strengthened by cohesive forces between the smallest grains in their size distributions
- Simulation and theoretical predictions are consistent with the measured strength of the upper lunar regolith
  - Fitting strength to the observed population assuming a Drucker-Prager Yield criterion predicts ~ 25-100 Pa
    - Based on: Overall spin/size curve, binary small size cut-off, small tumbling asteroids
  - Consistent with a mean grain size of ~ 2-8 microns for a lunar-type regolith, agrees with Itokawa measurements
- Recent observations are consistent with these limits
  - 1950 DA found to need > 60 Pa cohesion to hold together
  - P/2013 R3 found to be consistent with a strength of 40-210 Pa
  - 2008 TC3 requires a strength at least > 25 Pa to hold together

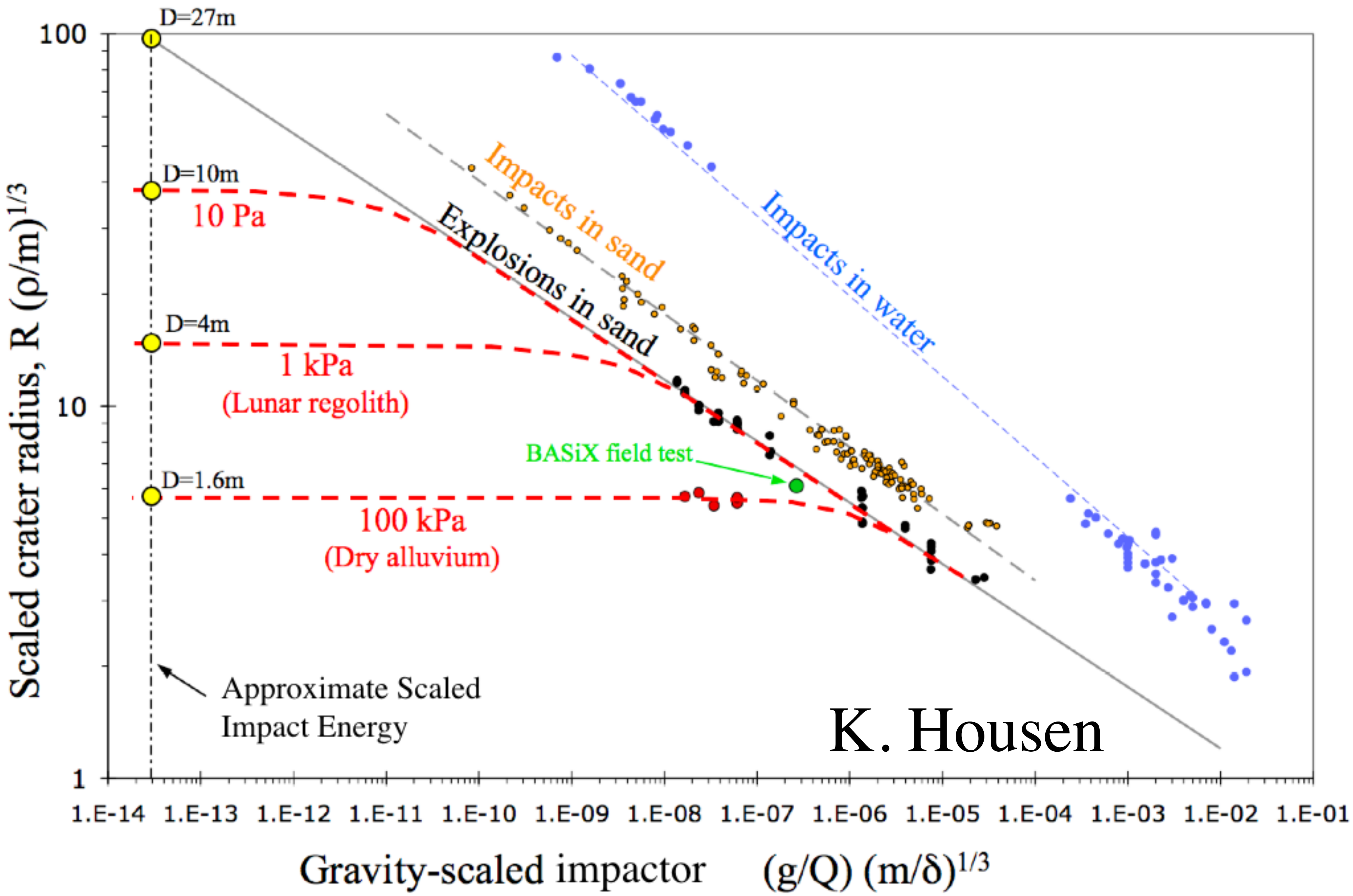
When & How Can we Test?

- In addition to Earth observations, the sample return space missions of Hayabusa2 and OSIRIS-REx will provide specific observations and opportunities to determine the level of strength within rubble pile bodies
- Sample return enables the detailed properties of "unprocessed" asteroid material to be determined
- Hayabusa2
  - Impact experiment will enable direct measurement of regolith strength
- OSIRIS-REx
  - Accurate tracking of the spacecraft will enable internal density inhomogeneities to be mapped, testing theories of morphology

Hayabusa2 Tests

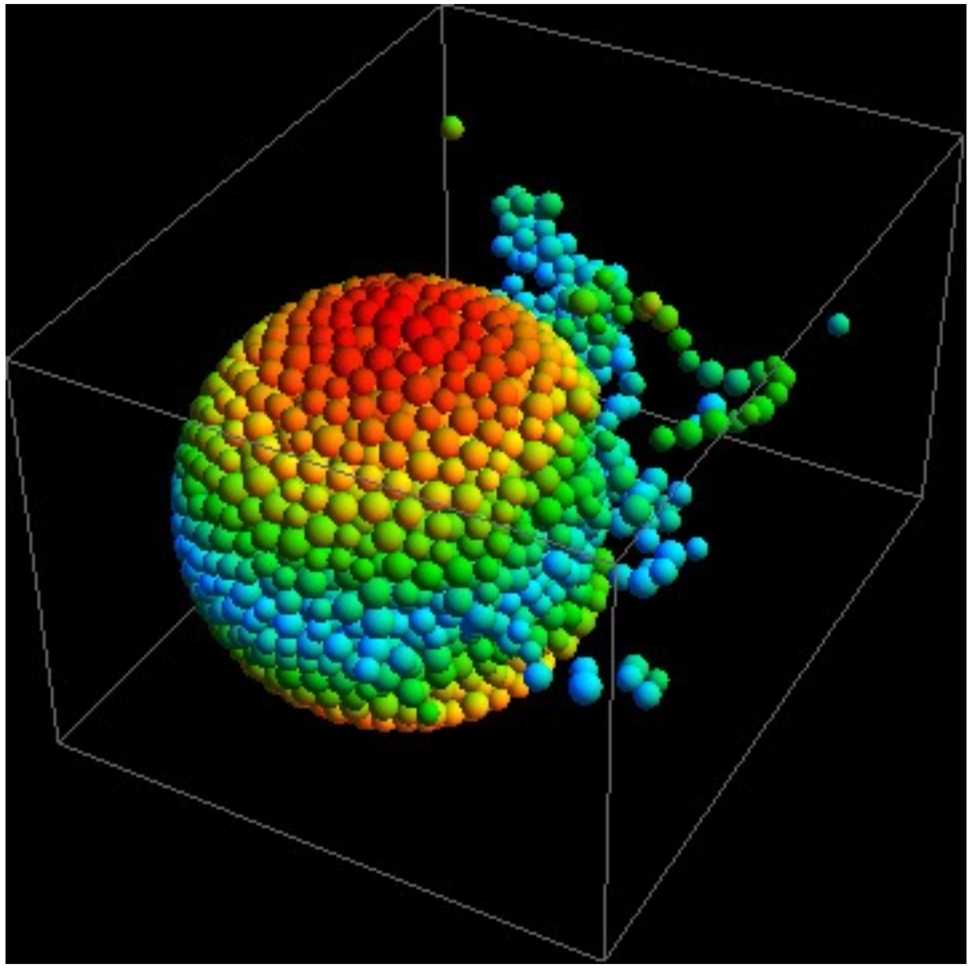
- Direct measurement of the crater size from the H2 impactor will correlate with regolith strength



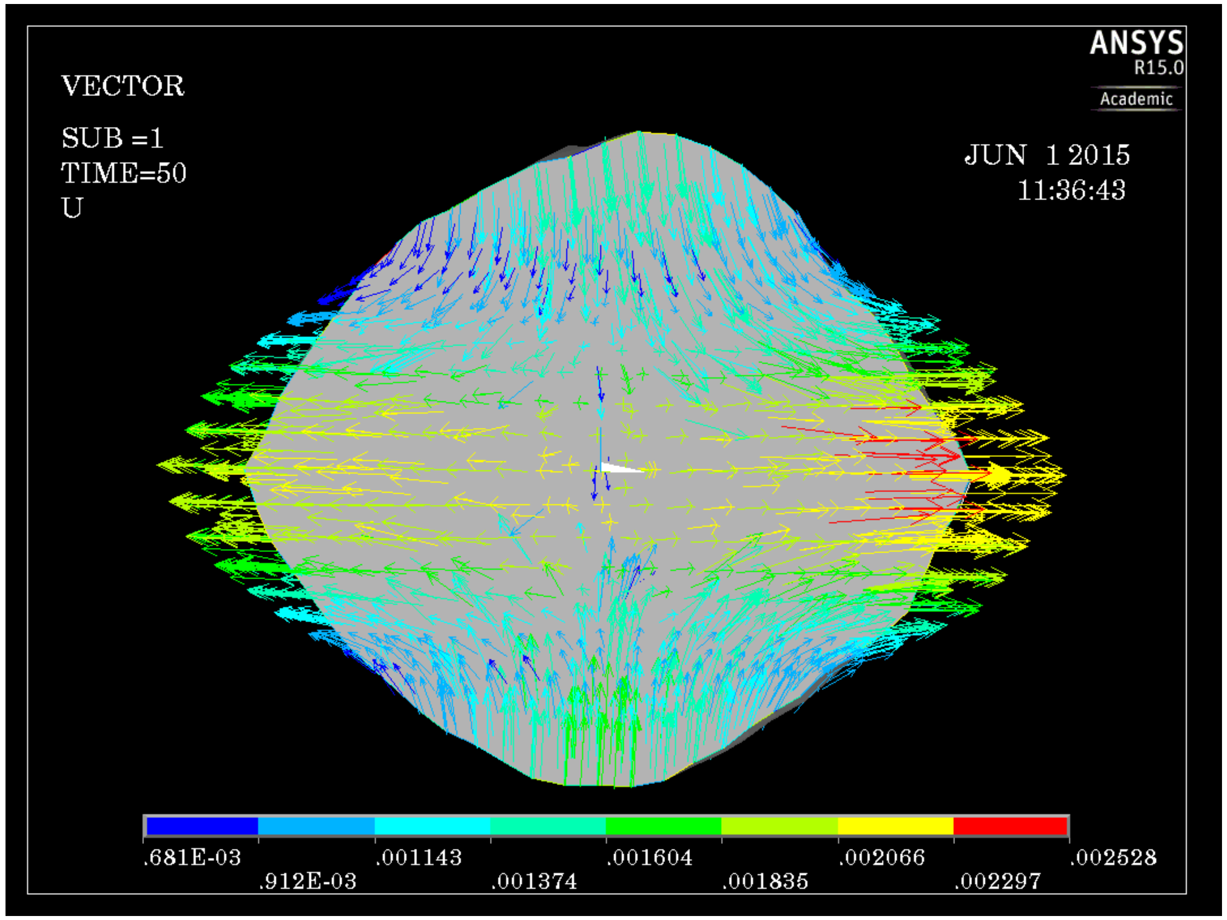


OSIRIS-REx Tests

- The internal distribution of strength strongly controls how a rubble pile asteroid will fail, and will leave signatures in its mass distribution



Strong Core = Surface Shedding

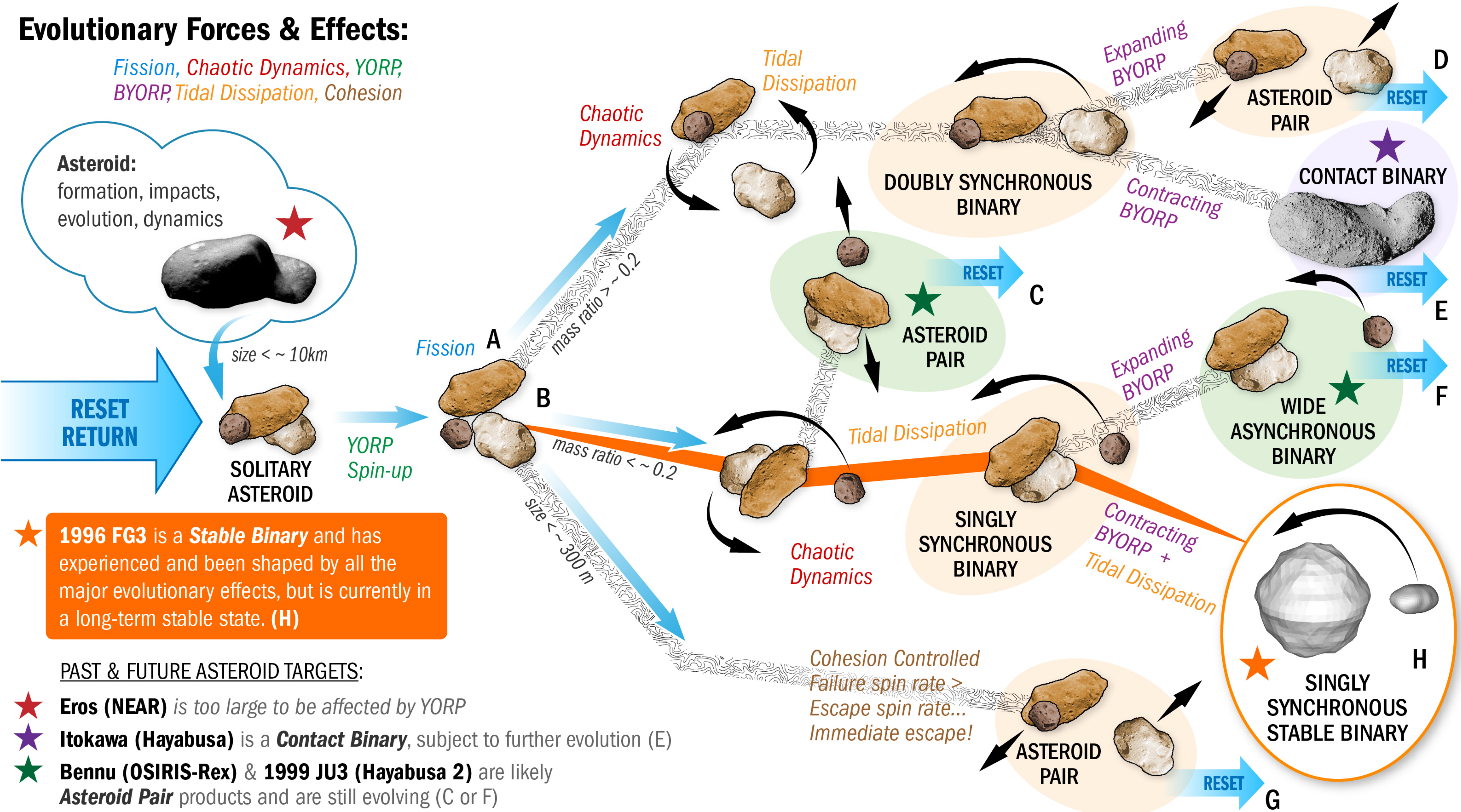


Uniform Strength
= Internal Failure

Summary / Conclusions

- The Celestial Mechanics of rubble pile asteroids — rotating collections of rigid bodies resting on each other — can explain and predict many phenomenon observed in the asteroid population
- These problems also pose interesting and open questions about how systems with coupled orbital and rotational motion dynamically evolve
- The size of the smallest grains seem to matter! Provide an intersection between classical and quantum physics.

Evolutionary Pathways of Asteroids



A: Scheeres (2007); B: Jacobson & Scheeres (2011a); C: Pravec et al. (2010); D: Fang & Margot (2012); E: Scheeres et al. (2007); F: Jacobson et al. (2014); G: Scheeres et al. (2010); H: Jacobson & Scheeres (2011b)