Multi-Objective Optimal Control

Massimiliano Vasile

Aerospace Centre of Excellence, Department of Mechanical and Aerospace Engineering
University of Strathclyde, Glasgow (United Kingdom)

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Pareto Dominance and Efficiency

Pareto Dominance

Consider the vector functions $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^m$, with $\mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_i(\mathbf{x}), ..., f_m(\mathbf{x})]^T$, $\mathbf{g} : \mathbb{R}^n \to \mathbb{R}^q$, with $\mathbf{g}(\mathbf{x}) = [g_1(\mathbf{x}), g_2(\mathbf{x}), ..., g_j(\mathbf{x}), ..., g_q(\mathbf{x})]^T$ and problem

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{F} \\ &s.t. \\ &\mathbf{g}(\mathbf{x}) \le 0 \end{aligned} \tag{MOP}$$

Given the feasible set $X = \{\mathbf{x} \in \mathbb{R}^n | \mathbf{g}(\mathbf{x}) \le 0\}$ and two feasible vectors $\mathbf{x}, \hat{\mathbf{x}} \in \mathbf{x}$, we say that $\mathbf{x}$ is dominated by $\hat{x}$ if $f_i(\hat{\mathbf{x}}) \le f_i(\mathbf{x})$ for all $i = 1, ..., m$ and there exists a k so that $f_k(\hat{\mathbf{x}}) \neq f_k(x)$. We use the relation $\hat{\mathbf{x}} \prec \mathbf{x}$ that states that $\hat{\mathbf{x}}$ dominates $\mathbf{x}$.

Pareto Efficiency

A vector $\mathbf{x}^* \in X$ will be said to be Pareto efficient, or optimal, with respect to Problem (MOP) if there is no other vector $\mathbf{x} \in X$ dominating $\mathbf{x}^*$ or:

$$\mathbf{x} \nprec \mathbf{x}^*, \qquad \forall \mathbf{x} \in X$$

Pareto Set

All non-dominated decision vectors in X form the Pareto set X_P and the corresponding image in criteria space is the Pareto front.

Karush-Khun-Tucker Optimality Conditions

[Cha08]

Necessary condition for $\mathbf{x}^*$ to be locally optimal.

Theorem (KKT)

If $\mathbf{x}^* \in X$ is an efficient solution to problem MOP, then there exist vectors $\eta \in \mathbb{R}^m$ and $\lambda \in \mathbb{R}^q$ such that:

$$\sum_i^m \eta_i \nabla f_i(\mathbf{x}^*) + \sum_j^q \lambda_j \nabla g_j(\mathbf{x}^*) = 0 \tag{1}$$

$$g_j(\mathbf{x}^*) = 0, \quad j = 1, ..., q \tag{2}$$

$$\lambda_j \geq 0, j = 1, ..., q \tag{3}$$

$$\eta_i \geq 0, i = 1, ..., m \tag{4}$$

$$\exists \eta_i > 0 \tag{5}$$

Pareto Set and Front

In the unconstrained case KKT optimality conditions reduce to:

$$\sum_{i}^{m} \eta_i \nabla f_i(\mathbf{x}^*) = 0 \tag{6}$$

$$\eta_i \geq 0, i = 1, ..., m \tag{7}$$

$$\exists \eta_i > 0 \tag{8}$$

Condition 6 leads to an interesting result (Hillermeier2001 [Hil01]) that the Pareto set is an $m - 1$ dimensional manifold. This also implies that the Pareto set has zero measure in $\mathbb{R}^n$ with $m \leq n$.

Multi-Objective Optimal Control

Consider the following multi-objective optimal control problem (MOCP):

$$
\begin{aligned}
&\min_{\mathbf{u}} \mathbf{F} \\
&s.t. \\
&\dot{\mathbf{x}} = \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) \\
&\mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) \leq 0 \\
&\boldsymbol{\psi}(\mathbf{x}_0, \mathbf{x}_f, t_0, t_f) \leq 0 \\
&t \in [t_0, t_f]
\end{aligned}
\tag{MOCP}
$$

where $\mathbf{F}$ is a vector function of the state variables $\mathbf{x} : [t_0, t_f] \rightarrow \mathbb{R}^n$, control variables $\mathbf{u} \in L^{\infty}$, time t and some static parameters $p \in \mathbb{R}^q$. Functions $\mathbf{x}$ belong to the Sobolev space $W^{1,\infty}$, objective functions are $f_i : \mathbb{R}^{n+2n} \times \mathbb{R}^p \times [t_0, t_f] \longrightarrow \mathbb{R}$, $\mathbf{h} : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \times [t_0, t_f] \longrightarrow \mathbb{R}^n$, algebraic constraints $\mathbf{g} : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \times [t_0, t_f] \longrightarrow \mathbb{R}^s$, and boundary conditions $\mathbb{R}^{2n+2} \longrightarrow \mathbb{R}^q$. Note that problem (MOCP) is generally non-smooth and can have many local minima.

MOCP: How to Solve it?

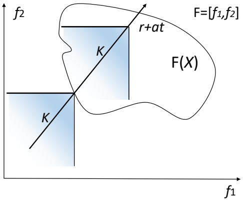
- Option 1 is to attempt the solution of the problem in vector form.
- Option 2 is to find a suitable form of scalarisation and then use the existing machineries to solve single objective optimal control problems.
- Option 3 is to use a mix of Option 1 and Option 2.

In the following we will introduce some suitable scalarisation techniques and we will then show how to combine Option 1 and Option 2 into a single method with some desirable theoretical and computational properties.

Pascoletti-Serafini Scalarisation[Eic08]

The scalarisation of Pascoletti-Serafini is based on the idea of descent cones K. An optimal (K-minimal) solution to problem MOP is solution to the following constrained single objective optimisation problem:

$$\begin{aligned} &\min_t \; t \\ &s.t. \\ &\mathbf{a}t - \mathbf{F}(\mathbf{x}) + \mathbf{r} \in K \\ &\mathbf{g}(\mathbf{x}) \leq 0 \end{aligned} \tag{9}$$

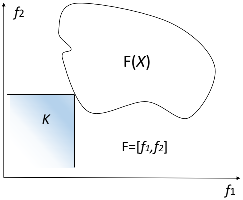


or, in a more computationally friendly, form:

$$\begin{aligned} &\min_s \; s \\ &s.t. \\ &w_j(f_j(\mathbf{x}) - z_j) \leq s \quad \forall j = 1, ..., m \\ &\mathbf{g}(\mathbf{x}) \leq 0 \end{aligned} \tag{PS}$$

A point is K-minimal when:

$$(\bar{\mathbf{F}} - K) \cap \mathbf{F}(X) = \{\bar{\mathbf{F}}\}$$

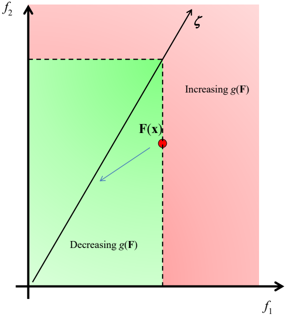


From this definition one can understand that a K-minimal point is Pareto efficient.

Chebyshev Scalarisation[Eic08]

Chebyshev scalarisation is based on the idea of descent directions identified by the weights $\mathbf{w}$:

$$\begin{array}{l} \min_{\mathbf{x}\in X} \max_{j\in\{1,\dots,m\}} w_j(f_j(\mathbf{x}) - z_j) \\ s.t. \\ \mathbf{g}(\mathbf{x}) \leq 0 \end{array} \qquad \text{(CS)}$$



Theorem (CS)

A point $(s, \mathbf{x}) \in \mathbb{R} \times X$ is a minimal solution of problem (PS) with $\mathbf{z} \in \mathbb{R}^m$, $z_j < \min_{\mathbf{x}\in X} f_j(\mathbf{u})$, $j = 1, ..., m$, and $\mathbf{w} \in int(\mathbb{R}^m_+)$, if and only if $\mathbf{x}$ is a solution of problem (CS).

From theorem CS one can expect that the solution of the PS and CS problems are equivalent. This is an important property when designing algorithms because, in some cases, the solution of PS translates into the solution of CS.

(Scalar) Pontryagin Maximum Principle

Given the following optimal control problem in Mayer's form:

$$
\begin{aligned}
&\min f(\mathbf{x}_f, t_f)\\
&s.t\\
&\dot{\mathbf{x}} = \mathbf{h}(\mathbf{x}, \mathbf{u}, t)\\
&\mathbf{g}(\mathbf{x}, \mathbf{u}, t) \geq 0\\
&\boldsymbol{\psi}(\mathbf{x}_0, \mathbf{x}_f, t_0, t_f) \geq 0\\
&t \in [t_0, t_f]
\end{aligned}
$$

If $\mathbf{u}^*$ is a locally optimal solution for problem (PSOCP) then there exist a vector $\eta \in \mathbb{R}^m$, $\lambda \in \mathbb{R}^n$ and a vector $\mu \in \mathbb{R}^q$ such that:

$$
\begin{aligned}
&\mathbf{u}^* = \underset{\mathbf{u} \in U}{\operatorname{argmin}}(\lambda^T \mathbf{h}(\mathbf{x}^*, \mathbf{u}, t) + \mu^T \mathbf{g}(\mathbf{x}^*, \mathbf{u}, t))\\
&\lambda^T \nabla_x \mathbf{h}(\mathbf{x}^*, \mathbf{u}^*, t) + \mu^T \nabla_x \mathbf{g}(\mathbf{x}^*, \mathbf{u}^*, t) + \dot{\lambda} = 0\\
&\lambda \geq 0; \mu \geq 0
\end{aligned}
$$

with transversality conditions:

$$
\begin{aligned}
&\nabla_x f + \nu^T \nabla_x \psi = \lambda_x(t_f)\\
&\nu \geq 0
\end{aligned}
$$

Pascoletti-Serafini Scalarised MOCP

Consider each objective function to be $f_j(\mathbf{x}_f, t_f)$ and the scalarised Multi-Objective Optimal Control problem:

$$
\begin{array}{ll}
\min_{s_f} s_f & \\
s.t. & \\
w_j(f_j(\mathbf{x}_f, t_f) - z_j) - s_f \leq 0 & \forall j = 1, ..., m \\
\dot{\mathbf{x}} = \mathbf{h}(\mathbf{x}, \mathbf{u}, t) & \\
\mathbf{g}(\mathbf{x}, \mathbf{u}, t) \geq 0 & \\
\psi(\mathbf{x}_0, \mathbf{x}_f, t_0, t_f) \geq 0 & \\
t \in [t_0, t_f] &
\end{array}
\tag{PSOCP}
$$

If s is a slack variable with final condition s_f and zero time variation $\dot{s} = 0$, then problem (PSOCP) presents itself in a form similar to Mayer's problem. The major difference is the mixed boundary constraint on x_f, t_f and s_f for every $j = 1, ..., m$.

Necessary Conditions for Local Optimality

Theorem (Vasile2017)

Consider the function $H = \lambda^T \mathbf{h}(\mathbf{x}, \mathbf{u}, t) + \mu^T \mathbf{g}(\mathbf{x}, \mathbf{u}, t)$. If $\mathbf{u}^*$ is a locally optimal solution for problem (PSOCP), with associated state vector $\mathbf{x}^*$, and H is Frechet differentiable at $\mathbf{u}^*$, then there exist a vector $\eta \in \mathbb{R}^m$, $\lambda \in \mathbb{R}^n$ and a vector $\mu \in \mathbb{R}^q$ such that:

$$\mathbf{u}^* = \underset{\mathbf{u} \in U}{\operatorname{argmin}} \, \lambda^T \mathbf{h}(\mathbf{x}^*, \mathbf{u}, t) + \mu^T \mathbf{g}(\mathbf{x}^*, \mathbf{u}, t)$$
$$\lambda^T \nabla_x \mathbf{h}(\mathbf{x}^*, \mathbf{u}^*, t) + \mu^T \nabla_x \mathbf{g}(\mathbf{x}^*, \mathbf{u}^*, t) + \dot{\lambda} = 0$$
$$\dot{\lambda}_s = 0$$
$$\lambda \geq 0; \mu \geq 0$$

with transversality conditions:

$$1 - \sum_j^m \eta_j = \lambda_s(t_f)$$
$$\eta^T \nabla_x \mathbf{F} + \nu^T \nabla_x \psi = \lambda_x(t_f)$$
$$\eta > 0; \nu \geq 0$$

Example

Consider the very simple one-dimensional controlled dynamical system with constant control acceleration and two objectives on the terminal states:

$$
\begin{aligned}
&\min s_f \\
&-w_1 x_f < s_f \\
&w_2 v_f < s_f \\
&\dot{x} = v; \dot{v} = -u; \dot{s} = 0; \\
&\dot{\lambda}_s = 0; \dot{\lambda}_x = 0; \dot{\lambda}_v = -\lambda_x; \\
&x(t_0) = 0; v(t_0) = 1; \\
&0 \leq u \leq 1 \\
&s_f \geq 0
\end{aligned}
$$

with $x_f = x(t_f), v_f = v(t_f), s_f = s(t_f)$ and terminal conditions:

$$
\begin{aligned}
&\lambda_s(t_f) = 1 - \eta_1 - \eta_2; \\
&\lambda_x(t_f) = -\eta_1 w_1; \lambda_v(t_f) = \eta_2 w_2;
\end{aligned}
$$

Example

The solution of the controlled dynamics is given by:

$$x = \frac{t^2}{2} + t \quad t \in [t_0, t_1]$$
$$v = t + 1 \quad t \in [t_0, t_1]$$
$$x = v_f t + x_1 \quad t \in [t_1, t_f]$$
$$v = v_1 = v_f \quad t \in [t_1, t_f]$$
$$x_1 = x(t_1); v_1 = v(t_1)$$

In this case it is easy to demonstrate that the Pareto front is given by the following second order algebraic equation:

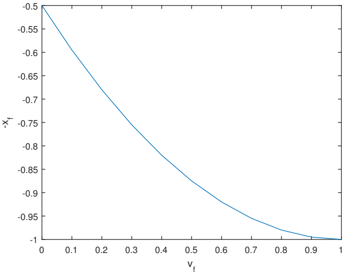
$$x_f = \frac{1 + 2v_f - v_f^2}{2}$$

We want to show that all the points along the front satisfy the optimality conditions and represent a minimum for s_f.

Example

Consider first the extreme values:

$$
\begin{aligned}
& \min s_f \\
& -x_f \leq s_f \\
& x = \frac{t^2}{2} + t \quad t \in [t_0, t_1] \\
& v = t + 1 \quad t \in [t_0, t_1] \\
& x = v_f t + x_1 \quad t \in [t_1, t_f] \\
& v = v_1 = v_f \quad t \in [t_1, t_f] \\
& x_1 = x(t_1); v_1 = v(t_1)
\end{aligned}
$$



By imposing the continuity conditions at t_1 we get a simple algebraic problem:

$$
\begin{aligned}
& \min s_f \\
& x_1 = -\frac{t_1^2}{2} + t_1 \\
& v_f = -t_1 + 1 \\
& -s_f = v_f t_f + x_1
\end{aligned}
$$

$$
\begin{aligned}
& \min s_f \\
& -s_f = 1 - \frac{t_1^2}{2} \\
& 0 \leq t_1 \leq 1
\end{aligned}
$$

$$
\begin{aligned}
& s_f = -1; t_1 = 0 \\
& s_f = -1/2; t_1 = 1
\end{aligned}
$$

Example

We now need to verify that we can find a suitable set of Lagrange multipliers that satisfy the necessary conditions:

$$H = \lambda_x v - \lambda_v u + \mu_1(u-1) - \mu_2 u$$
$$\frac{\partial H}{\partial u} = -\lambda_v + \mu_1 - \mu_2$$
$$\lambda_v = -\lambda_x(t - t_f) + \lambda_v(t_f)$$

$$\lambda_v(t_f) = 0$$
$$\lambda_x(t_f) = -\eta_1$$
$$\lambda_v < 0 \quad \forall t \in [t_0, t_f]$$

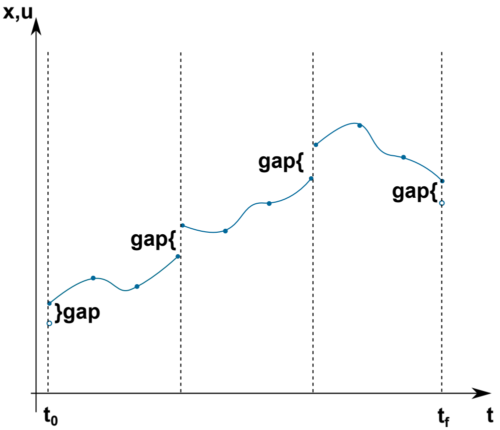
These equations confirm that there is a single switching point for the control u^*.
The conditions on the multipliers associated to the slack variable s_f are always satisfied.

Transcription of the MOCP

We now have a scalarised and a vector from of the MOCP. In both cases the formulation contains a mix of algebraic and differential equations (DAE).
The next step is to transcribe the infinite dimensional system of DAE into a finite dimensional Nonlinear Programming Problem that can be solved numerically.
The transcription technique proposed here is based on Finite Elements in time on spectral basis.

Direct Finite Element Transcription

(Vasile 2000, 2003, 2010, [VF00][VBZ03][Vas10])



Transcription of the optimal control problem (PSOCP) into a finite dimensional Nonlinear Programming Problem. We start from decomposing the time domain in a finite number of time elements.

Bi-discontinuos Integral Form

The differential constraints are recast in weak form and integrated by parts, leading to:

$$\int_{t_0}^{t_f} \dot{\mathbf{w}}^T \mathbf{x} + \mathbf{w}^T \mathbf{h}(\mathbf{x}, \mathbf{u}, t) dt - \mathbf{w}_f^T \mathbf{x}_f^b + \mathbf{w}_0^T \mathbf{x}_0^b = 0 \qquad (10)$$

where $\mathbf{w}$ are generalised weight functions and $\mathbf{x}^b$ are the boundary values of the states, that may be either imposed or free.

State and Control Transcription

Given the partition of the time domain:

$$D = \bigcup_{j=1}^{N} D_j(t_{j-1}, t_j) \tag{11}$$

one can parametrise, over each D_j, the states, controls and weight functions as

$$\mathbf{x}(t) = \mathop{\mho}_{j=1}^{N} \mathbf{X}_j = \mathop{\mho}_{j=1}^{N} \sum_{s=0}^{l} \phi_{sj}(t)\overline{\mathbf{x}}_{sj} \tag{12}$$

$$\mathbf{u}(t) = \mathop{\mho}_{j=1}^{N} \mathbf{U}_j = \mathop{\mho}_{j=1}^{N} \sum_{s=0}^{m} \gamma_{sj}(t)\overline{\mathbf{u}}_{sj} \tag{13}$$

$$\mathbf{w}(t) = \mathop{\mho}_{j=1}^{N} \mathbf{W}_j = \mathop{\mho}_{j=1}^{N} \sum_{s=0}^{l+1} \theta_{sj}(t)\overline{\mathbf{w}}_{sj} \tag{14}$$

where $\mathop{\mho}_{j=1}^{N}$ denotes the juxtaposition of the polynomials defined over each sub-interval, $\phi_{sj}(t)$, $\gamma_{sj}(t)$ and $\theta_{sj}(t)$ indicate the s-th polynomial over element j and are chosen among the space of polynomials of degree l, m and $l+1$ respectively, while $\overline{\mathbf{x}}_{sj}$, $\overline{\mathbf{u}}_{sj}$ and $\overline{\mathbf{w}}_{sj}$ denote the nodal values of the states, control and test functions.

State and Control Transcription (2)

It is practical to define each D_j over the normalised interval $[-1, 1]$ through the transformation

$$\tau = 2\frac{t - \frac{t_j - t_{j-1}}{2}}{t_j - t_{j-1}} \tag{15}$$

This way it's easy to express the polynomials $\phi_{sj}(t)$, $\gamma_{sj}(t)$ and $\theta_{sj}(t)$ as the Lagrange interpolation on the Gauss nodes in the normalised interval:

$$\phi_{sj}(t) = \widetilde{\phi}_{sj}(\tau) = \prod_{k=0, k\neq s}^{l} \frac{\tau - \tau_k}{\tau_s - \tau_k} \tag{16}$$

where τ_* indicates a Gauss node, and similarly can be done for γ_{sj} and θ_{sj}. Different Gauss nodes will lead to schemes with slightly different characteristics.

Transcribed Problem

(Vasile 2000, 2003, 2010, [VF00][VBZ03][Vas10])

$$\begin{aligned}\tilde{f}_i =& \alpha_i f_i\left(\mathbf{X}_0^b, \mathbf{X}_f^b, \mathbf{U}_0^b, \mathbf{U}_f^b, t_0, t_f\right) + \\ & \beta_i \sum_{j=1}^{N} \sum_{k=1}^{l+1} \sigma_k L_i\left(\mathbf{X}_j(\tau_k), \mathbf{U}_j(\tau_k), \tau_k\right) \frac{\Delta t_j}{2}\end{aligned} \tag{17}$$

and for the variational constraints leads for each element j to the system

$$\begin{aligned}c_j = \sum_{k=1}^{l+1} \sigma_k \left[\dot{\mathbf{W}}_j(\tau_k)^T \mathbf{X}_j(\tau_k) + \mathbf{W}_j(\tau_k)^T \mathbf{h}_j(\tau_k) \frac{\Delta t_j}{2}\right] \\ - \mathbf{W}_{p+1,j}^T \mathbf{X}_j^b + \mathbf{W}_{1,j}^T \mathbf{X}_{j-1}^b = 0\end{aligned} \tag{18}$$

DFET Transcribed PSOCP

Vasile and Ricciardi 2016[VR16]

Once the differential equations have been transcribed into a set of nonlinear algebraic equations, the original optimal control problem can be casted in the Pascoletti-Serafini form and solved with a standard NLP solver:

$$
\begin{aligned}
&\min_{s_f} s_f \\
&s.t. \\
&w_j(f_j(\mathbf{Y}, t) - z_j) - s_f \leq 0 \qquad \forall j = 1, ..., m \\
&\mathbf{c}(\mathbf{Y}, t) \geq 0 \\
&t \in [t_0, t_f]
\end{aligned}
\tag{PSDFET}
$$

where $\mathbf{Y} = [\mathbf{X}, \mathbf{U}, \mathbf{X}_0^b, \mathbf{X}_f^b, t_0, t_f]$ is the decision vector of the NLP problem and boundary conditions $\psi(\mathbf{X}_0^b, \mathbf{X}_f^b, t_0, t_f) \geq 0$ are included in the constraint vector $\mathbf{c}$.

Necessary Optimality Conditions of the Transcribed Problem

Theorem

If Y^* is a locally optimal solution for problem (PSDFET), then there exist a vector $\eta \in \mathbb{R}^m$, $\lambda \in \mathbb{R}^n$ such that:

$$\lambda^T \nabla_Y \mathbf{c}(\mathbf{Y}, t) + \eta^T \nabla_Y \mathbf{w}^T \mathbf{F}(\mathbf{Y}, t) = 0$$

$$1 - \sum \eta_j = 0$$

$$\lambda \geq 0; \eta > 0$$

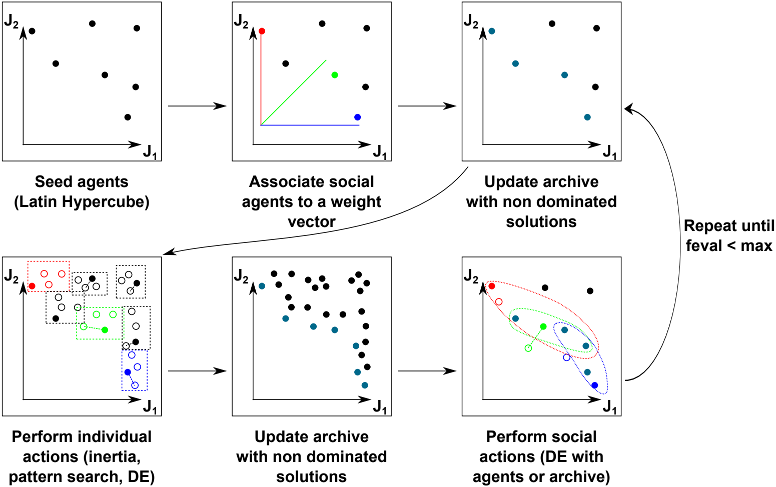
From the definition of Pascoletti-Serafini scalarisation we know that a K-efficient solution of problem (PSDFET) is locally optimal and satisfies the above theorem.
To be noted that this theorem is equivalent to the KTT conditions previously defined. In the following we will distinguish between the optimisable parameters $\mathbf{p} = [\mathbf{U}, \mathbf{X}_0^b, \mathbf{X}_f^b, t_0, t_f]^T$ and the state parameters $\mathbf{x} = \mathbf{X}$.

Solving the Transcribed MOCP

The transcribed PSOCP suggests that a standard NLP solver can be used to find a Pareto efficient solution.
A direct application of this technique will provide a single point on the Pareto front unless a strategy is implemented to change the weight vector **w** (see for example the Normal Boundary Intersection strategy).
Alternatively we can use a method devised to solve vector MOO problems and generate a population of solutions each associated to a different **w**.
We can then use the PSOCP to locally converge to a Pareto efficient point from each solution in the population.
In the following we will explain how.

Multi-Agent Collaborative Search (1)

[Vas][VZ11][ZV13][RV15]



Multi-Agent Collaborative Search (2)

During the exploration phase no gradient information is used and the agents converge, in parallel, towards the Pareto set using a series of sampling heuristics. From Theorem 2 we know that the solution of problem (PS) is also solution of problem (CS). Therefore, when agents do not implement any gradient-based local search approach, they solve the following problem:

$$\min_{\mathbf{p}\in X} \max_{j} w_j(f_j - z_j) \tag{19}$$

The assumption is that the control vector **p** is in the feasible set, or, in other words, that all constraints are satisfied.

Bi-level Formulation

Vasile and Ricciardi 2016 - [RVM16][RVTM16][VR16]

In order to maintain feasibility, problem (19) is solved with the following bi-level formulation:

$$
\begin{array}{l}
\min_{\mathbf{p}^c \in X} \max_j w_j(f_j(\mathbf{x}^*, \mathbf{p}^*) - z_j) \\
s.t. \\
(\mathbf{x}^*, \mathbf{p}^*) = \operatorname{argmin}_{\mathbf{p}}\{(\mathbf{p} - \mathbf{p}^c)^T(\mathbf{p} - \mathbf{p}^c) | \mathbf{c}(\mathbf{x}, \mathbf{p}) \geq 0\} \\
f_j(\mathbf{x}^*, \mathbf{p}^*) = \begin{cases} f_j(\mathbf{x}^*, \mathbf{p}^*) & \text{if } \mathbf{c}(\mathbf{x}^*, \mathbf{p}^*) \geq 0 \\ \mathbf{L} + \mathbf{k}nf & \text{otherwise} \end{cases}
\end{array}
\tag{20}
$$

Once an agent decides to trigger a local search with a gradient method an NLP solver is invoked and directly applied to problem (PSDFET).

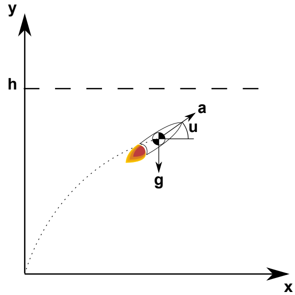
Goddard Rocket - (Ricciardi and Vasile 2016)

Physical model and constraints

- Constant gravity acceleration g
- Constant thrust acceleration a
- Control parameters: thrust angle u
- At final time, altitude must be h
- At final time, vertical velocity must be 0

Numerical settings

- 10000 fun evals, 10 agents
- 10 solutions in the archive
- 160 variables



Objectives

- Minimise mission time
- Maximise horizontal velocity

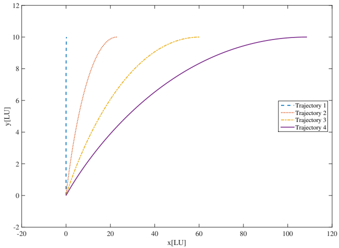
Goddard Rocket

Ricciardi and Vasile 2016 - [RVM16]

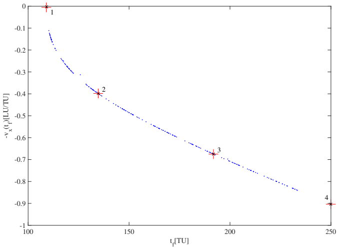
$$\min_{t_f, u} [f_1, f_2]^T = [t_f, -v_x(t_f)]^T \tag{21}$$

$$\begin{cases} \dot{x} & = v_x \\ \dot{v}_x & = a \cos u \\ \dot{y} & = v_y \\ \dot{v}_y & = -g + a \sin u \end{cases}$$

$$\begin{cases} x(0) = 0; & v_x(0) = 0 \\ y(0) = 0; & v_y(0) = 0 \\ y(t_f) = \overline{h}; & v_y(t_f) = 0 \end{cases}$$

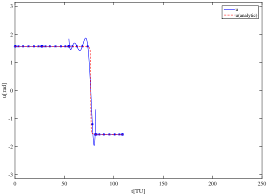


Trajectories

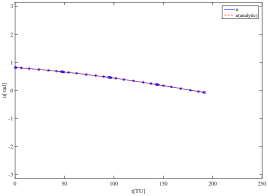


Pareto front

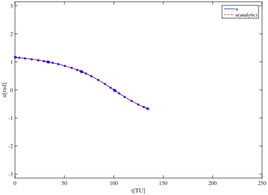
Goddard Rocket - (Ricciardi and Vasile 2016)



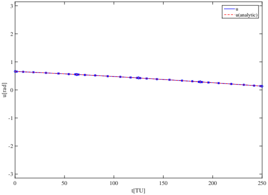
Control law for point 1



Control law for point 3



Control law for point 2



Control law for point 4

Optimal Descent - (Ricciardi and Vasile 2017)

Multi-objective version of a problem proposed in [BK02].

$$\min[-\theta(t_f), q_U]^T$$

$$s.t.$$

$$\dot{h} = v \sin\gamma$$

$$\dot{\phi} = \frac{v}{r}\cos\gamma \sin\psi / \cos\theta$$

$$\dot{\theta} = \frac{v}{r}\cos\gamma\cos\psi$$

$$\dot{v} = -\frac{D}{m} - g\sin\gamma$$

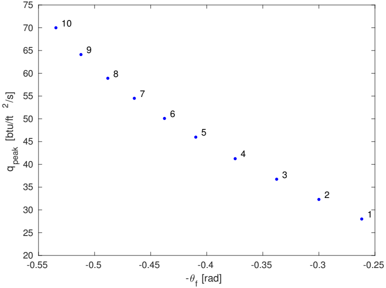
$$\dot{\gamma} = \frac{L}{mv}\cos\beta + \cos\gamma\left(\frac{v}{r} - \frac{g}{v}\right)$$

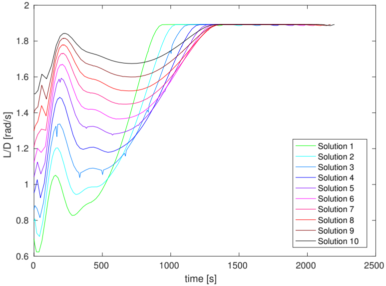
$$\dot{\psi} = \frac{1}{mv\cos\gamma} L\sin\beta + \frac{v}{r\cos\theta}\cos\gamma\sin\psi\sin\theta$$

$$q \leq q_U$$

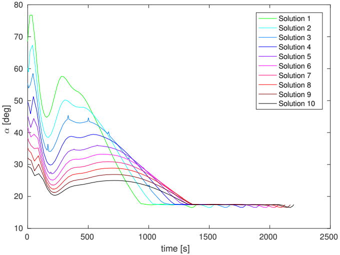
Optimal Descent - (Ricciardi and Vasile 2017)

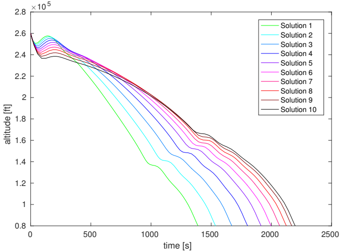
The extreme values correspond to the single objective solutions in [BK02].

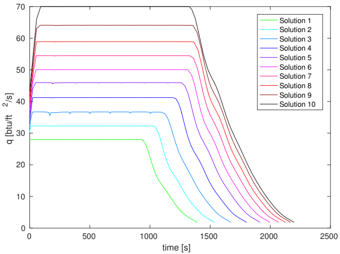


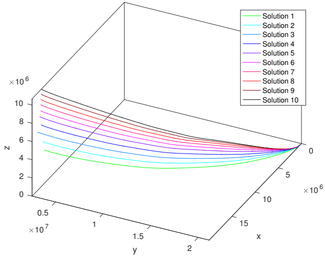


Optimal Descent - (Ricciardi and Vasile 2017)









Final Remarks I

- A multi-objective optimal control problem can be reformulated with an appropriate scalarisation approach into a constrained single objective problem.
- Necessary conditions for optimality were derived for the scalarised MOCP.
- The scalarisation technique called Pascoletti-Serafini is equivalent to a weighted Chebyshev scalerisation.
- The transcribed PSOCP can be solved with a memetic algorithm providing globally efficient solutions.

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