TIDAL EVOLUTION OF CLOSE-IN SATELLITES AND EXOPLANETS

Lecture 1. Darwin theory

S.Ferraz-Mello

SDSM 2017

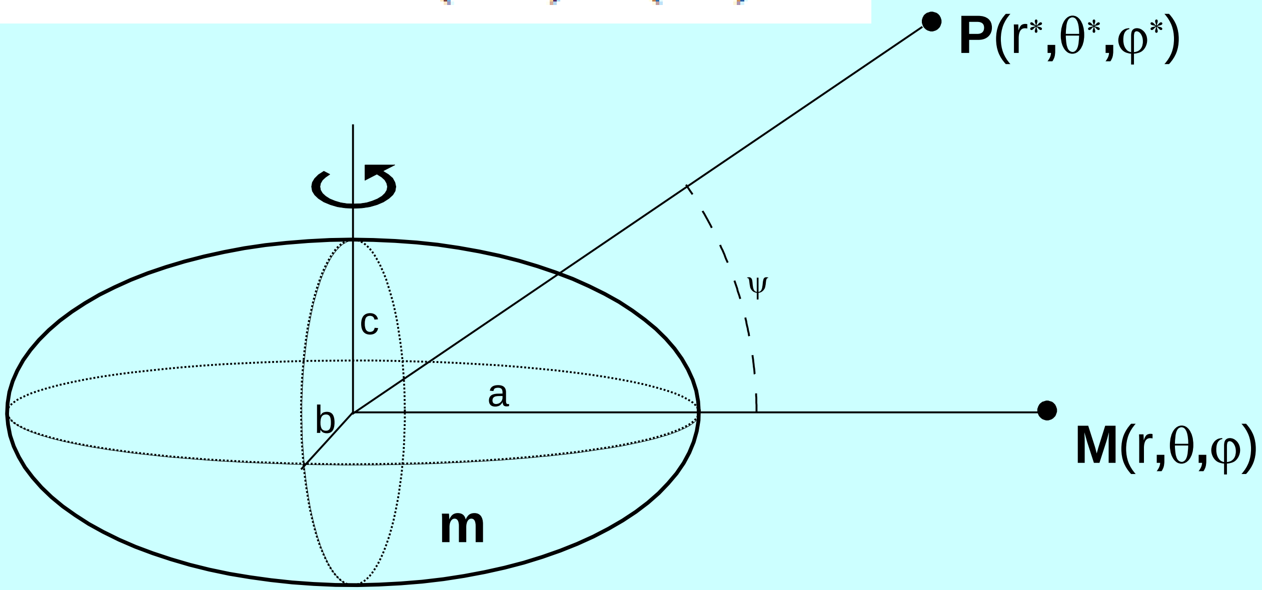
Reference: S.Ferraz-Mello et al.
Cel.Mech.Dyn.Ast.**101**,171,2008
(astro-ph 0712-1156 **v3**)

Darwin, 1880

Goldreich, 1963
Kaula, 1964
Alexander, 1973
Zahn, 1977 &c
Mignard, 1979
Hut, 1981
Eggleton &c, 1998
Mardling & Lin, 2002
Efroimsky &c, 2007 &c
Ferraz-Mello, 2013 &c
Correia &c, 2014
Ragazzo & Ruiz, 2016
etc.

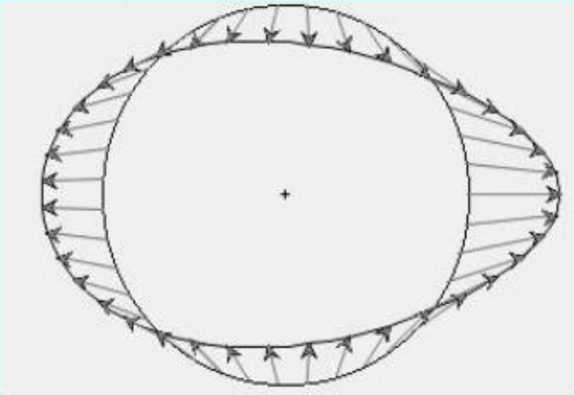
Static Equilibrium Tide
(Jeans prolate homogeneous spheroid)

$$\epsilon = \frac{a}{b} - 1 = \frac{15}{4}\left(\frac{M}{m}\right)\left(\frac{R}{r}\right)^3$$



Actually
a>b>c

But .... If
too close



Potential in the point P

$$U = -\frac{Gm}{r^*}\left(1 + \frac{B-A}{2mr^{*2}}(3\cos^2\Psi - 1)\right)$$

$k_f = 15A/4mR^2$ (so-called fluid Love number)

$$= -\frac{Gm}{r^*} - \frac{k_f GMR^5}{2r^3 r^{*3}}(3\cos^2\Psi - 1);$$

Remark – It is independent on the rotational state of the body

Tidal decomposition of U (Fourier)

$$= -\frac{3k_f GMR^5}{4a^3 r^{*3}}\Bigg[-\frac{2}{3} - e^2 + \left(1 + \frac{3}{2}e^2 - \frac{1}{2}S^2\right)P^2$$

$$+\left(1 - \frac{5}{2}e^2 - \frac{1}{2}S^2\right)P^2\cos(2\varphi^* - 2\ell - 2\omega) + \frac{7}{2}$$

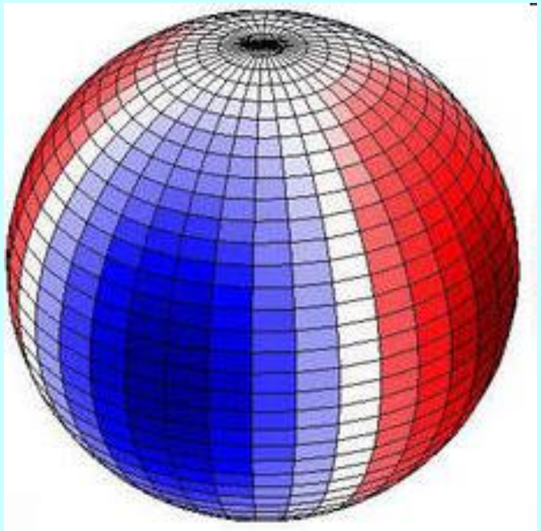
$$-\frac{1}{2}eP^2\cos(2\varphi^* - \ell - 2\omega) + \frac{17}{2}e^2P^2\cos(2\varphi^* -$$

$$-\left(2 - 3P^2\right)e\cos\ell - \left(3 - \frac{9}{2}P^2\right)e^2\cos 2\ell$$

Semi-diurnal harmonics

$$(2\varphi^* - 2\ell - 2\omega)$$

when $\Omega \neq n$



Semi-diurnal frequency

$$\nu = (2\Omega - 2n)$$

Main frequencies

#	frequency	$\Omega \gg n$	$\Omega \simeq n$
0	$2\Omega - 2n$	semi-diurnal	-
1	$2\Omega - 3n$	semi-diurnal	monthly
2	$2\Omega - n$	semi-diurnal	monthly
3	$2\Omega - 4n$	semi-diurnal	semi-monthly
4	2Ω	semi-diurnal	semi-monthly
5	n	monthly (radial)	monthly (radial)

N.B. (a) freq. #1 = - freq. #2

(b) Radial = Zonal (i.e. longitude independent)

Tidal Evolution (action on M)

$$
\begin{aligned}
F_1 &= -\frac{GmM}{r^2} - \frac{3GM}{2r^4}(A+B+C) + \frac{9GMA}{2r^4} \\
F_2 &= 0 \\
F_3 &= 0
\end{aligned}
$$

The force is radial, its torque is equal to zero

In addition, it is conservative

$$
\mathbf{F}\cdot\mathbf{v} = \mathcal{F}(r)\mathbf{r}\cdot\mathbf{v} = \frac{1}{2}\mathcal{F}(r)d(\mathbf{r}^2)/dt =
$$

$$
= \frac{1}{2}\mathcal{F}(r)d(r^2)/dt = \mathcal{F}(r)r\,dr/dt
$$

Darwin dynamic equilibrium tide
(introduces ad hoc tidal lags)

Darwin (1880)

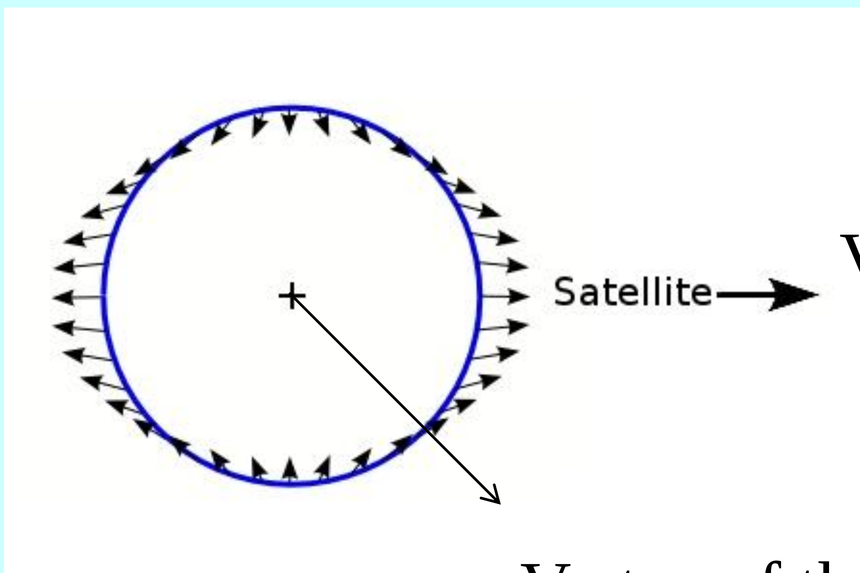
$$\cos \Phi_i$$

$$\cos(\Phi_i - \varepsilon_i) = \cos \Phi_i + \varepsilon_i \sin \Phi_i .$$

$$= \cos \Phi_i + \underbrace{\varepsilon_i \cos(\Phi_i - 90^\circ)}_{\text{ANELASTIC TIDE}} .$$

Elastic and anelastic tides

$$\cos(\Phi_i - \varepsilon_i) = \cos\Phi_i + \varepsilon_i \cos(\Phi_i - 90^\circ).$$



Vertex of the elastic tide

Vertex of the anelastic tide

Tidal forces acting on a mass M* @ P

$$\mathbf{F} = -M^*\mathrm{grad}_{\mathbf{r}*}U_2 = \underbrace{-M^*\frac{\partial U_2}{\partial r^*}}_{F_1}\,\hat{\mathbf{r}^*} \underbrace{-\frac{M^*}{r^*}\frac{\partial U_2}{\partial \theta^*}}_{F_2}\,\hat{\theta^*} \underbrace{-\frac{M^*}{r^*\sin\theta^*}\frac{\partial U_2}{\partial \varphi^*}}_{F_3}\,\hat{\varphi^*}$$

<u>Tidal forces acting on M</u>

Drop all * **AFTER**
the gradient is taken

$$F_1(\mathbf{r}) = \frac{-9k_d G M^2 R^5}{8a^7}\Bigg[e\boxed{(8\varepsilon_0 - 7\varepsilon_1 - \varepsilon_2 + 2\varepsilon_5)}\sin\ell$$

$$+e^2(21\varepsilon_0 - 4\varepsilon_2 - 17\varepsilon_3 + 4\varepsilon_5 + 3\varepsilon_6)\sin 2\ell - S^2(\varepsilon_0 - \varepsilon_4 - \varepsilon$$

The coefficients are formed by several different lags

$$F_2(\mathbf{r}) = 3k \quad \ldots \quad 4\varepsilon_9 + 6\varepsilon_{15} +$$

$$-eS(8\varepsilon_0 - 7\varepsilon_1 - \varepsilon_2 + 6\varepsilon_5 + 4\varepsilon_8 - 12\varepsilon_9 - 6\varepsilon_{14} + 14\varepsilon_{17})\cos(2\ell$$

$$F_3(\mathbf{r}) = \frac{3k_d G M^2 R^5}{8a^7}\Bigg[(4 - 14e^2 - 3S^2)\varepsilon_0 + 56e^2\varepsilon_1 + 2S^2(\varepsilon_8 + \varepsilon_9)$$

$$+e^2(44\varepsilon_0 - 8\varepsilon_2 + 34\varepsilon_3)\cos 2\ell + S^2(\varepsilon_0 + 2\varepsilon_4 - 2\varepsilon_8 - 2\varepsilon_9)$$

Torque and Angular Momentum

$$\mathcal{M} = \mathbf{r} \times \mathbf{F}$$

$$\mathcal{M} \equiv (0, M_2, M_3) = -rF_3\hat{\theta}^* + rF_2\hat{\varphi}^*.$$

$$\dot{\mathcal{L}} = \mathcal{M} \qquad \dot{\mathcal{L}}_{\mathrm{rot}} = -\mathcal{M} \qquad \mathcal{L}_{\mathrm{rot}} = C\Omega\hat{\mathbf{k}}$$

The <u>conservation of the angular momentum</u> allows us to obtain an equation for the variation of the rotation speed.

Averaged result:

$$< \dot{\Omega} > = -\frac{3k_d G M^2 R^5}{8Ca^6}\left[4\varepsilon_0 + e^2\boxed{(-20\varepsilon_0 + 49\varepsilon_1 + \varepsilon_2)} + \ldots\right.$$

The coefficients are formed by several different lags

We can continue without imposing a law for the lags

(as done by Kaula, FM etal, etc)

Usual choices for ε:
(a.k.a. Rheologies)

1. Lags proportional to frequencies **(Darwin)**

- a.k.a. CTL (constant time lag) theories

No.	frequency	$\Omega \gg n$
0	$2\Omega - 2n$	semi-diurnal
1	$2\Omega - 3n$	semi-diurnal
2	$2\Omega - n$	semi-diurnal
3	$2\Omega - 4n$	semi-diurnal
4	2Ω	semi-diurnal
5	n	monthly

- used in many theories: Mignard, Hut, Eggleton, Mardling & Lin, &c.

2. All semi-diurnal harmonics have **equal lags.**

- implicit in some theories. Ex: Earth-Moon
- a.k.a. CPL (constant phase lag) theories

		$\Omega \gg n$
0	$2\Omega - 2n$	semi-diurnal
1	$2\Omega - 3n$	semi-diurnal
2	$2\Omega - n$	semi-diurnal
3	$2\Omega - 4n$	semi-diurnal
4	2Ω	semi-diurnal
5	n	monthly

3. Inverse power law (**Efroimsky & Lainey**)

- stiff bodies

$$\varepsilon_k \sim \text{cte} . \{\text{frequency}_k\}^{-\alpha}$$

$$\alpha \sim 0.4$$

4. **MacDonald**

- Unphysical geometric lag.
  Does not define a rheology

Low-order Darwin theory is friendly.

Easily adapted to different models
(ex: core/mantle bodies,
effects due to response attenuation etc.)

High-order Darwin theory is feasible.

However, given our ignorance of the actual rheology of celestial bodies, the accuracy of expansions to higher-orders may be illusory.

Synchronization (Darwin's CTL rheology)

- $\varepsilon_0 \sim 0 \ (<< \varepsilon_2)$
- $\varepsilon_2 = -\varepsilon_1 \quad (>0)$

$$< \dot{\Omega} > \simeq -B \cdot \left[\varepsilon_0 - 12\varepsilon_2 e^2\right]$$ **≠ 0**

Stationary Solution

$$\left[\varepsilon_0 - 12\varepsilon_2 e^2\right] = 0$$

$$\Omega_{\mathrm{stat}} = n(1 + 6e^2)$$

Synchronous Solutions

CANNOT EXIST WITHOUT AN EXTRA COUNTERACTING TORQUE

! FREQUENT SOURCE OF ERROR

Two possibilities

(1) Supersynchronous stationary rotation

(2) Spin-Orbit resonance forced by counteracting torque (Ex: Axial Asymmetry)

VARIATION OF THE ELEMENTS

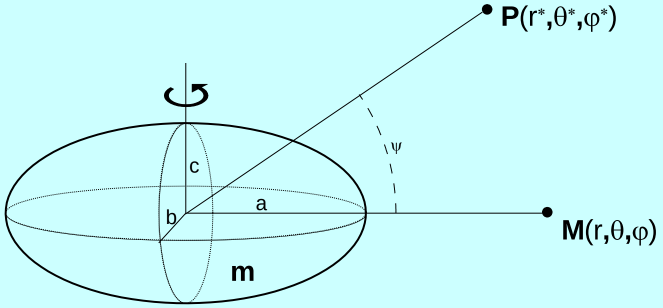
(Lagrange variational equations)

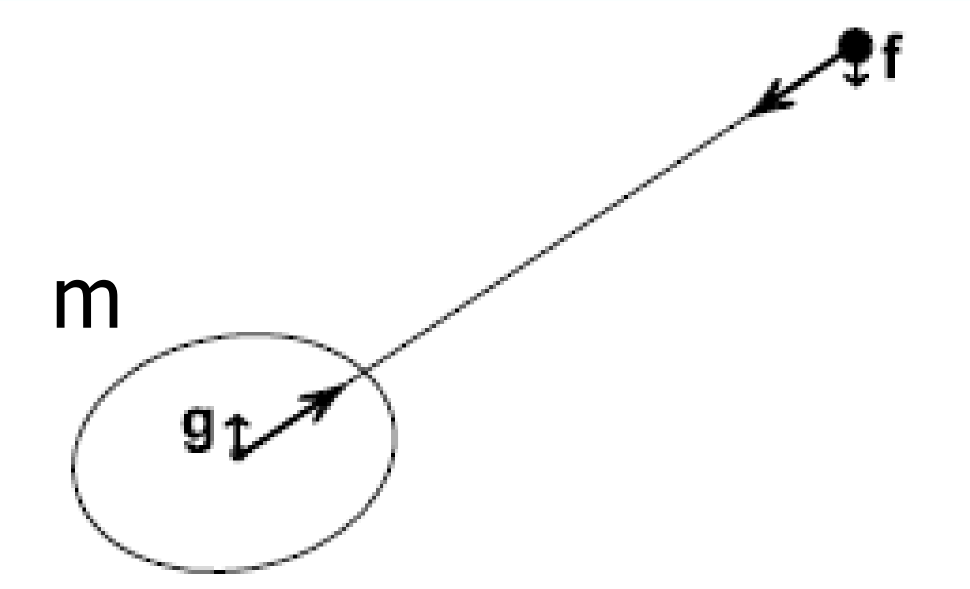
$$\dot{a} = \frac{2}{na}\frac{\partial \mathcal{R}}{\partial \ell}$$

$$\dot{e} = -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial \mathcal{R}}{\partial \omega} + \frac{1-e^2}{na^2 e}\frac{\partial \mathcal{R}}{\partial \ell}$$

(or the equivalent Gauss equations)

N.B. $$\frac{\partial \mathcal{R}}{\partial \ell} = \left[\frac{\partial \mathcal{R}}{\partial \ell^*}\right]_{\ell^* = \ell}$$





Newton's Third Law

$$\mathbf{g} = -\mathbf{f}$$

Relative equation of motion (refered to m)

$$\ddot{\mathbf{r}} = -\frac{G(m_0 + m_1)\mathbf{r}}{r^3} + \frac{(m_0 + m_1)\mathbf{f}}{m_0 m_1},$$

$$\boxed{\mathcal{R} = -(1 + M/m)\delta U}$$

Hence

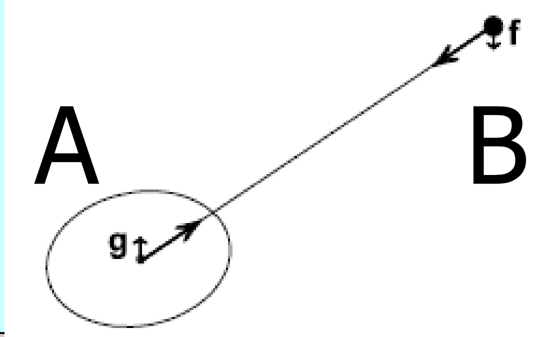
$$< \dot{n} > = -\frac{3n}{2a} < \dot{a} > =$$

$$-\frac{9n^2 k_d M R^5}{8ma^5}\left[4\varepsilon_0 - e^2(20\varepsilon_0 - \frac{147}{2}\varepsilon_1 - \frac{1}{2}\varepsilon_2 + 3\varepsilon_5) + \ldots\right.$$

and

$$< \dot{e} > = -\frac{3nek_d M R^5}{8ma^5}(2\varepsilon_0 - \frac{49}{2}\varepsilon_1 + \frac{1}{2}\varepsilon_2 + 3\varepsilon_5) + \ldots$$

The continuation depends on the chosen rheology



Example

Peale's formulas

fast rotating A + synchronous B

equal lags (independent of frequencies)

$$< \dot{n} > = -\frac{9n^2 k_{dA} m_B R_A^5 \varepsilon_{0A}}{2 m_A a^5}\left(1 + \boxed{\frac{51}{4}} e^2 - D(7e^2 \qquad)\right)$$

$$< \dot{e} > = \frac{3nek_{dA} m_B R_A^5 \varepsilon_{0A}}{2 m_A a^5}\left(\boxed{\frac{19}{4}} - 7D\right)$$

$$D = \frac{k_{dB}}{k_{dA}} \frac{\varepsilon_{2B}}{\varepsilon_{0A}} \left(\frac{m_A}{m_B}\right)^2 \left(\frac{R_B}{R_A}\right)^5 .$$

with Darwin lags 51 → 54; 19 → 22

tides in close-in exoplanets
(generally $\Omega << n$)

ε_2	annual	(<0)	$(2\Omega - n)$
ε_0	semi-annual	(<0)	$(2\Omega - 2n)$
ε_1	tierce-annual	(<0)	$(2\Omega - 3n)$
ε_5	annual (radial)	(>0)	(n)

When $\Omega_A << n$
(small planet around slow rotating star – only the tides on the planet are considered)

$$< \dot{n} > = -\frac{3n}{2a} < \dot{a} > = -\frac{9n^2 k_d M R^5}{2ma^5}\left[\varepsilon_0 - e^2(5\varepsilon_0 - \frac{147}{8}\varepsilon_1 - \frac{7}{8}\varepsilon_2)\right]$$

$$< \dot{e} > = -\frac{3nek_d M R^5}{4ma^5}\left(\varepsilon_0 - \frac{49}{4}\varepsilon_1 - \frac{5}{4}\varepsilon_2\right)$$

For exoplanets it is recommended to use the general formulas with explicit Ω_A.
See S.F.-M. et al. 2008.

<u>Application</u>

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -21\,\frac{k_\mathrm{p}}{Q_\mathrm{p}}\frac{m_0}{m_\mathrm{p}}\left(\frac{R_\mathrm{p}}{a}\right)^5 n a e^2$$

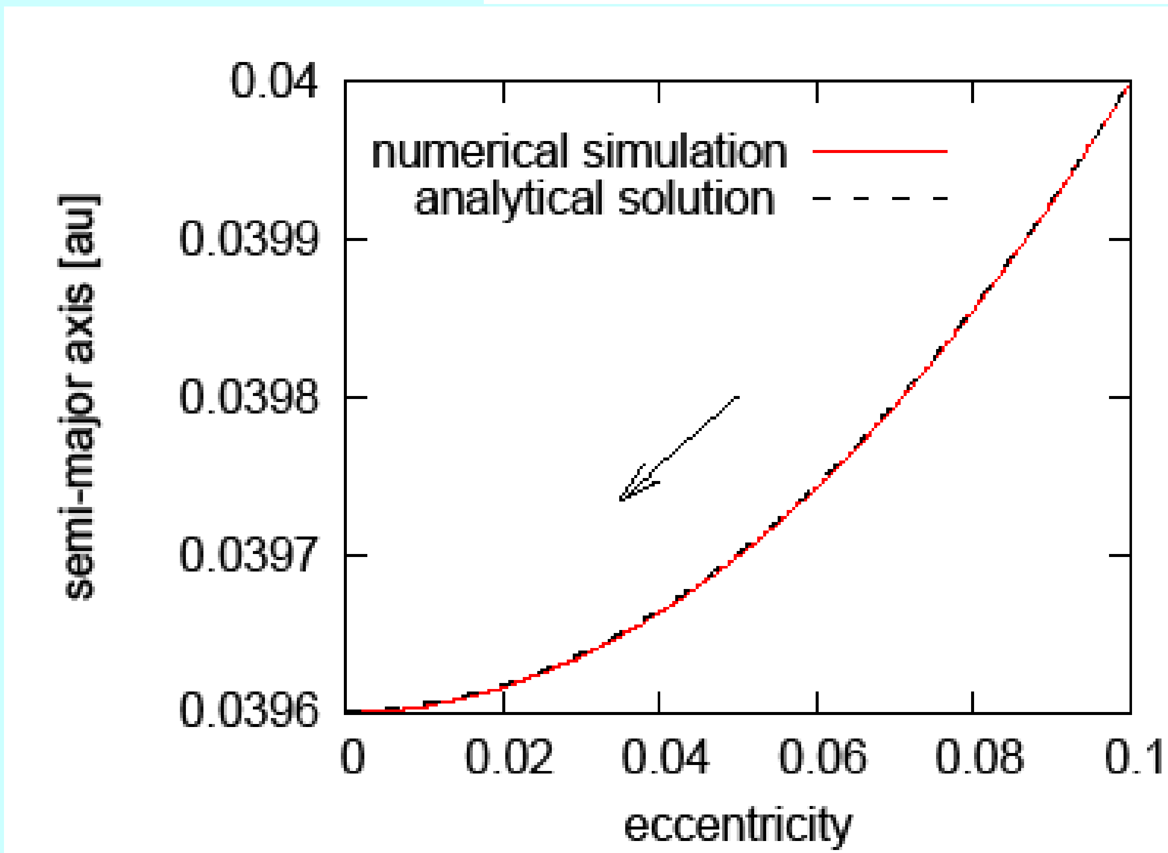
$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{21}{2}\,\frac{k_\mathrm{p}}{Q_\mathrm{p}}\frac{m_0}{m_\mathrm{p}}\left(\frac{R_\mathrm{p}}{a}\right)^5 n e,$$

Hot super-Earth

mass = 5 m_Earth

semi-axis = 0.04 AU

Period ~ 3 days



t~30Myr

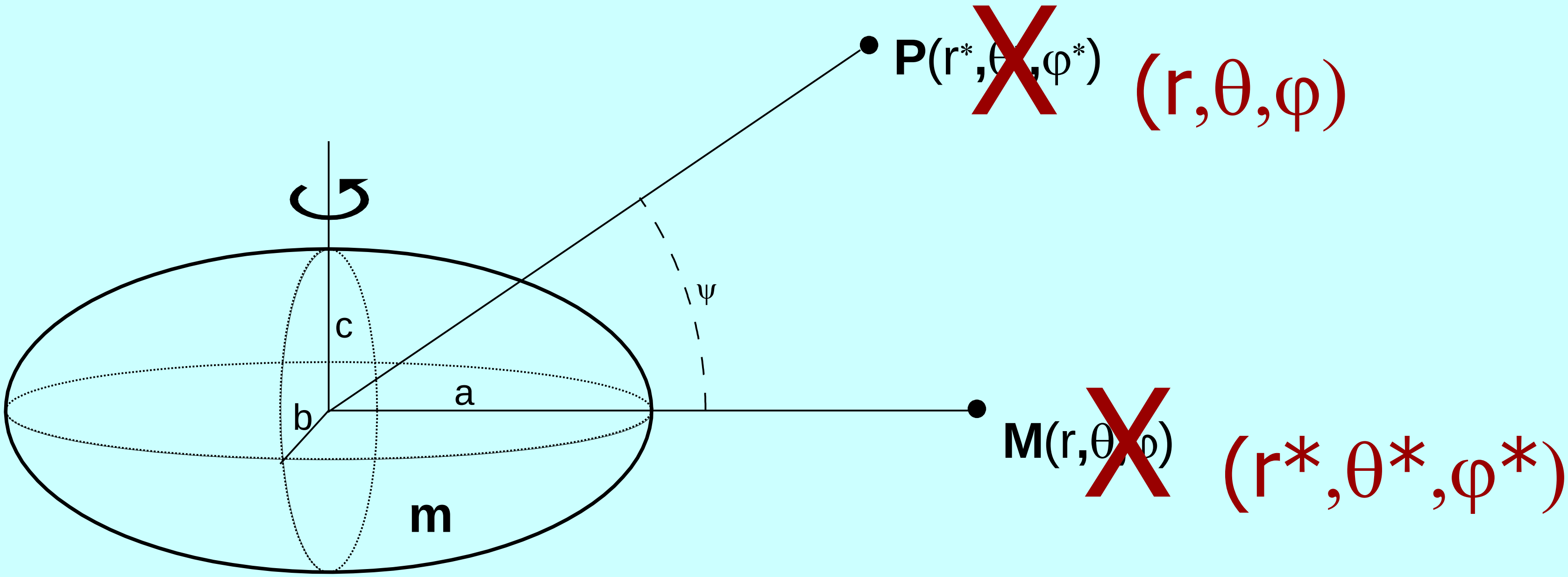
MIGNARD's theory

The Moon and the Planets, **30**, 301 (1979)

Reproduces Darwin (1880)

- constant time lag
- checked to 3rd. order in e,i

!!!



$$U(\mathbf{r}) = k_2 \frac{-G m^* R_{\mathrm{E}}^5}{2 r^{*5} \cdot r^5}\left[3(\mathbf{r}\cdot\mathbf{r}^*)^2 - r^2 \cdot r^{*2}\right]$$

$$\mathbf{r}^* \longrightarrow \mathbf{r}^*(t - \Delta t) + \boldsymbol{\omega}\Delta t \times \mathbf{r}^*;$$

$$V(\mathbf{r},\mathbf{r}^*) = 3\frac{k_2 G m^* R_{\mathrm{E}}^5}{r^5 r^{*5}} \cdot \Delta t \left\{ (\mathbf{r}\cdot\mathbf{r}^*)[\mathbf{r}^* \cdot (\boldsymbol{\omega}\times\mathbf{r}) + \mathbf{r}\cdot\mathbf{v}^*] \right.$$

$$\left. - \frac{(\mathbf{r}^*\cdot\mathbf{v}^*)}{2r^{*2}} [5(\mathbf{r}\cdot\mathbf{r}^*)^2 - r^2 r^{*2}] \right\}.$$

$$\mathbf{F} = -\mathbf{grad}_r V \Rightarrow$$

$$\mathbf{F} = 3\frac{k_2 G m^* R_{\mathrm{E}}^5}{r^5 r^{*5}} \Delta t \left\{ 5\frac{\mathbf{r}}{r^2} \right| (\mathbf{r}\cdot\mathbf{r}^*)[\mathbf{r}^*\cdot(\boldsymbol{\omega}\times\mathbf{r}) + \mathbf{r}\cdot\mathbf{v}^*]$$

$$\left. - \frac{(\mathbf{r}^*\cdot\mathbf{v}^*)}{2r^{*2}} \cdot [5(\mathbf{r}\cdot\mathbf{r}^*)^2 - r^2 r^{*2}] \right] - [\mathbf{r}^*\cdot[\mathbf{r}^*\cdot(\boldsymbol{\omega}\times\mathbf{r}) + \mathbf{r}\cdot\mathbf{v}^*]$$

$$\left. + (\mathbf{r}^*\times\boldsymbol{\omega} + \mathbf{v}^*)(\mathbf{r}\cdot\mathbf{r}^*)] + \frac{(\mathbf{r}^*\cdot\mathbf{v}^*)}{r^{*2}} [5\mathbf{r}^*(\mathbf{r}\cdot\mathbf{r}^*) - \mathbf{r}r^{*2}] \right\},$$

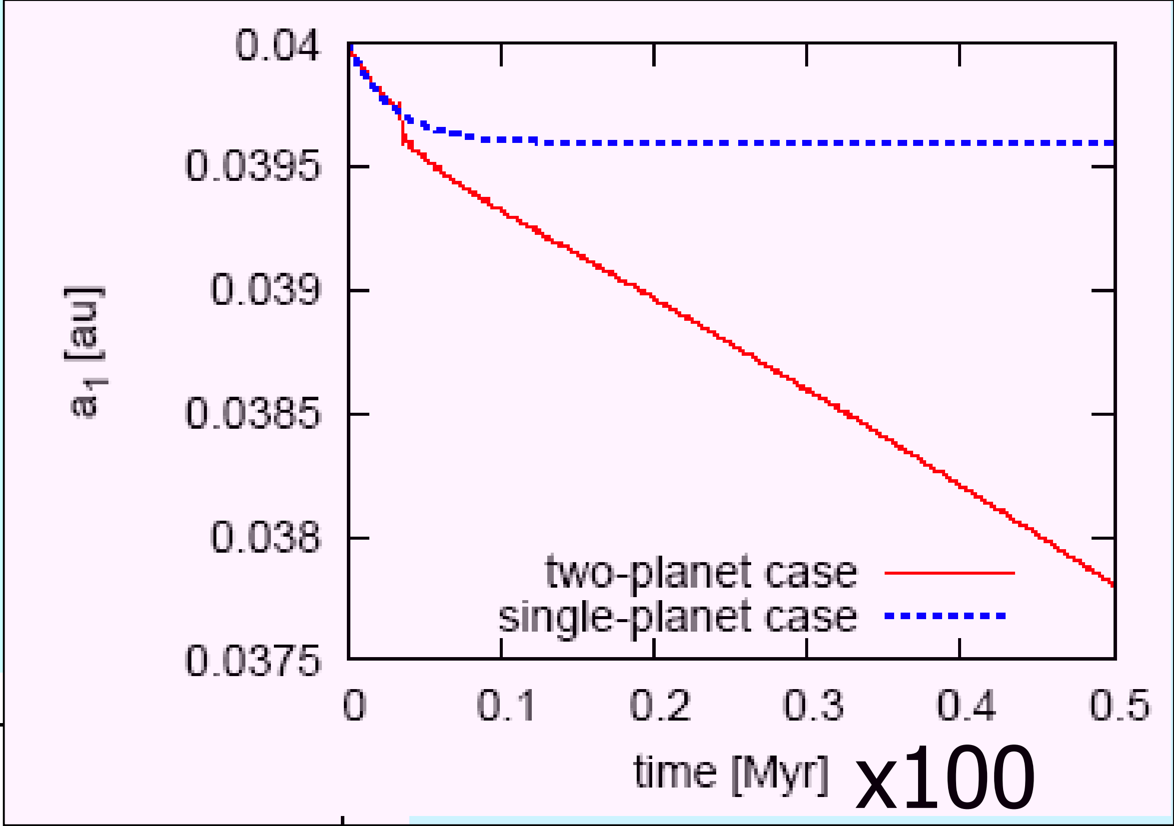
Or, after identification of **r** and **r***:

$$\mathbf{F} = -\frac{3kGM^2R^5}{r^{10}}\left[2\mathbf{r}(\mathbf{r}\cdot\mathbf{v}) + r^2(\mathbf{r}\times\boldsymbol{\Omega}+\mathbf{v})\right]\tau$$

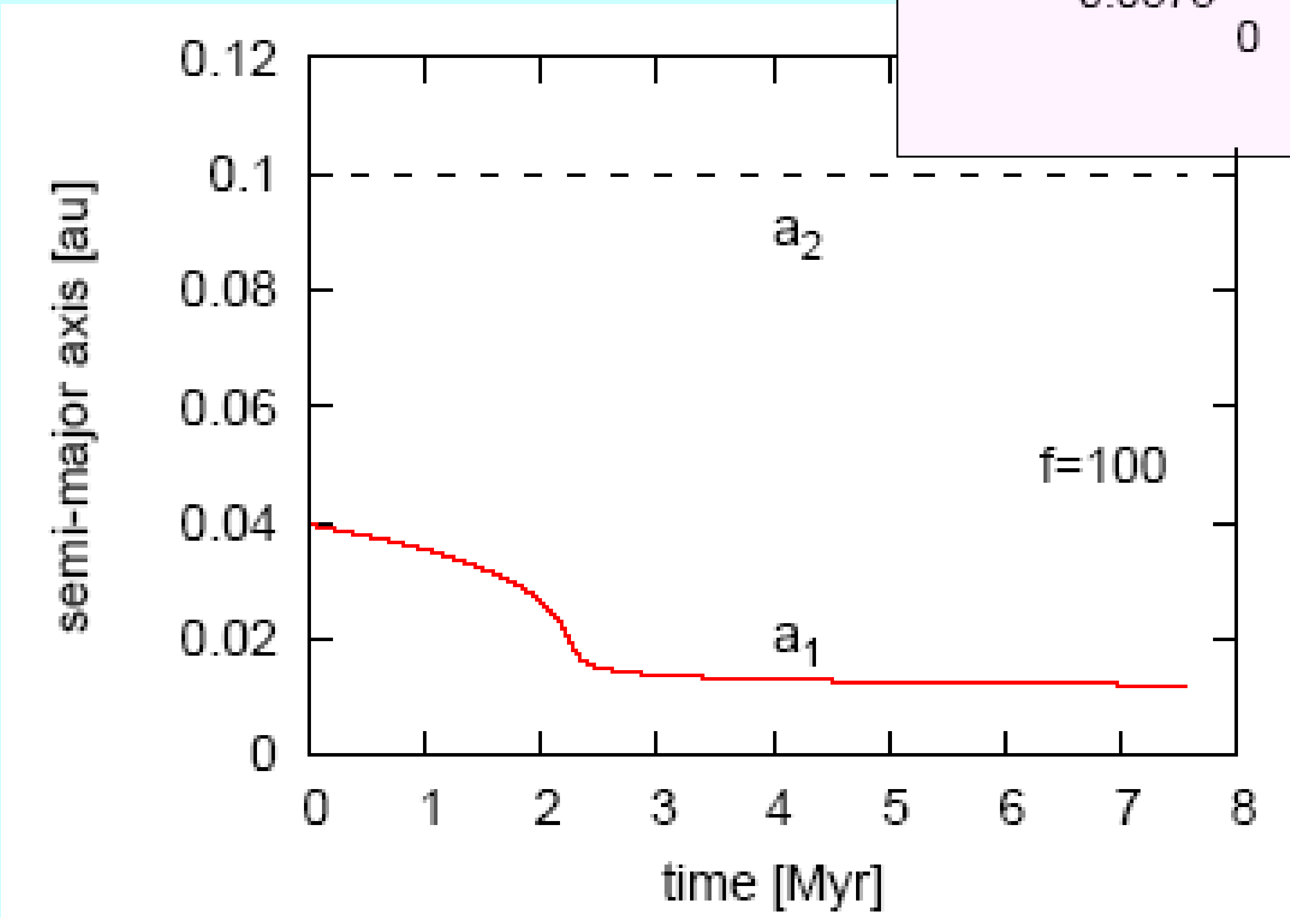
Application:

TWO hot exoplanets
with masses **5 m_Earth** & **1 m_Jupiter**
semi-axes **0.04** and **0.1 AU** resp.

Evolution of the Semi-major axes



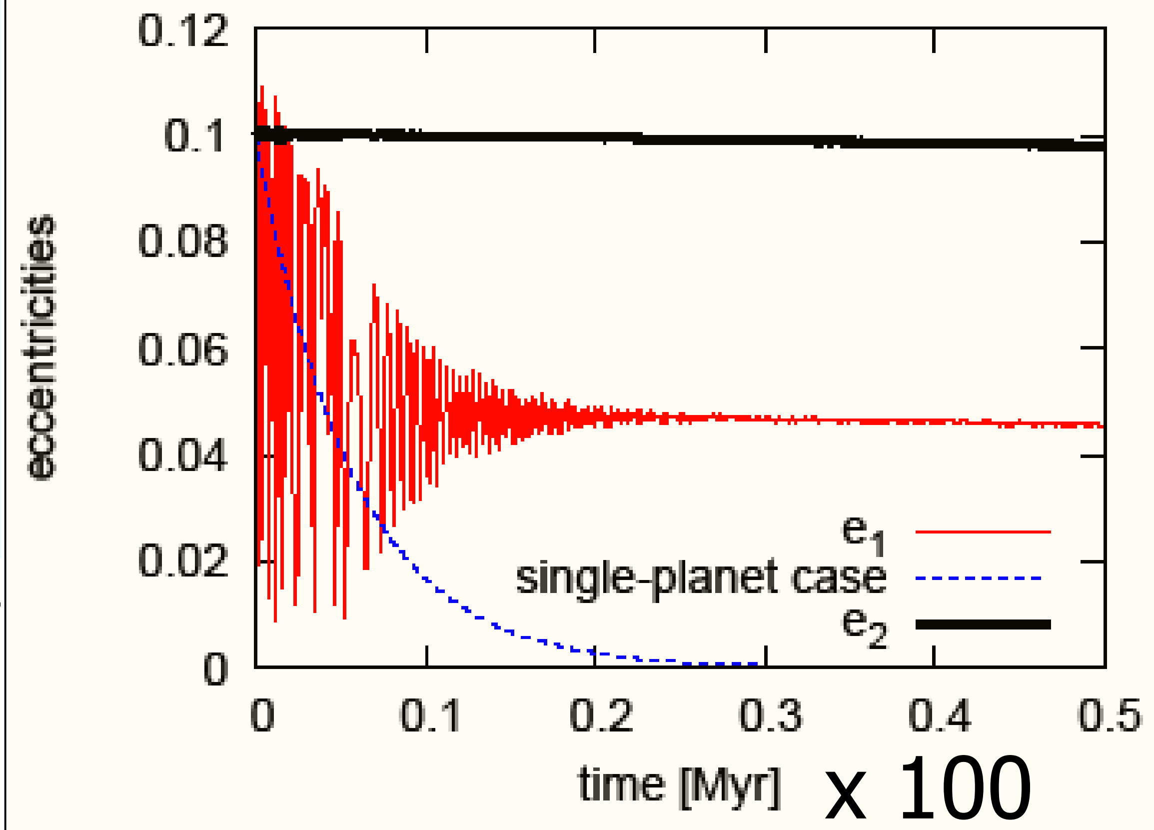
x100



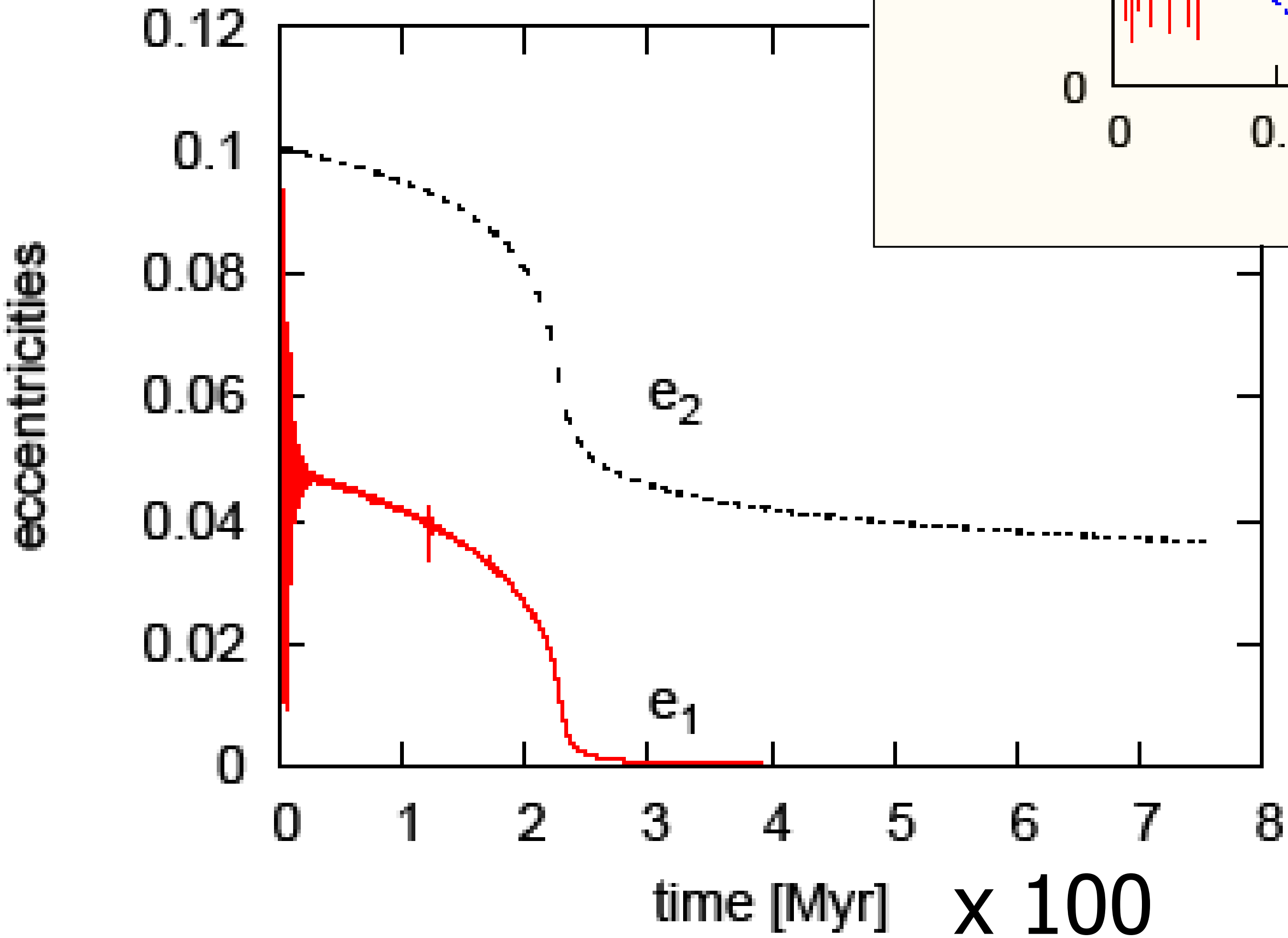
x100

Price to pay:
SCALING !
(to be handled with care)

Evolution of the Eccentricities



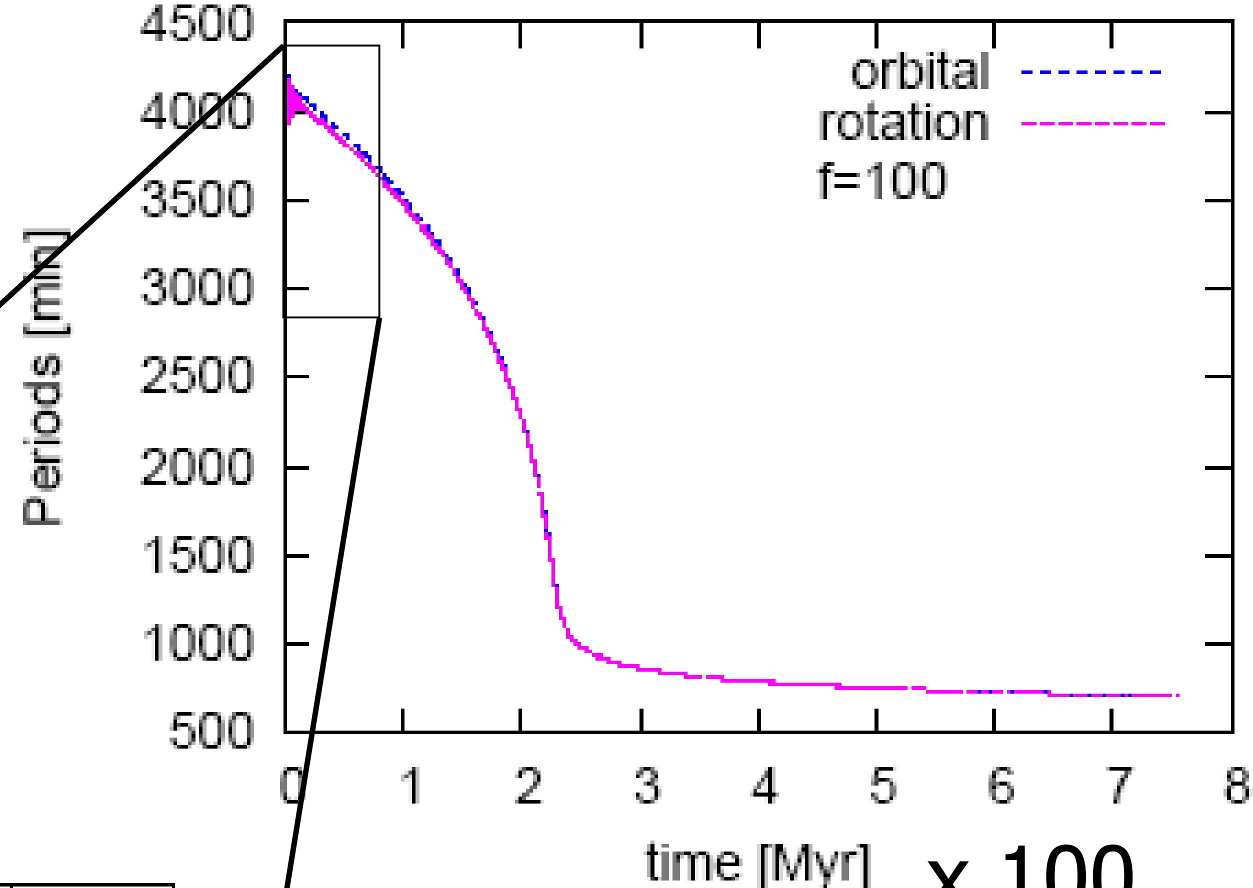
x 100

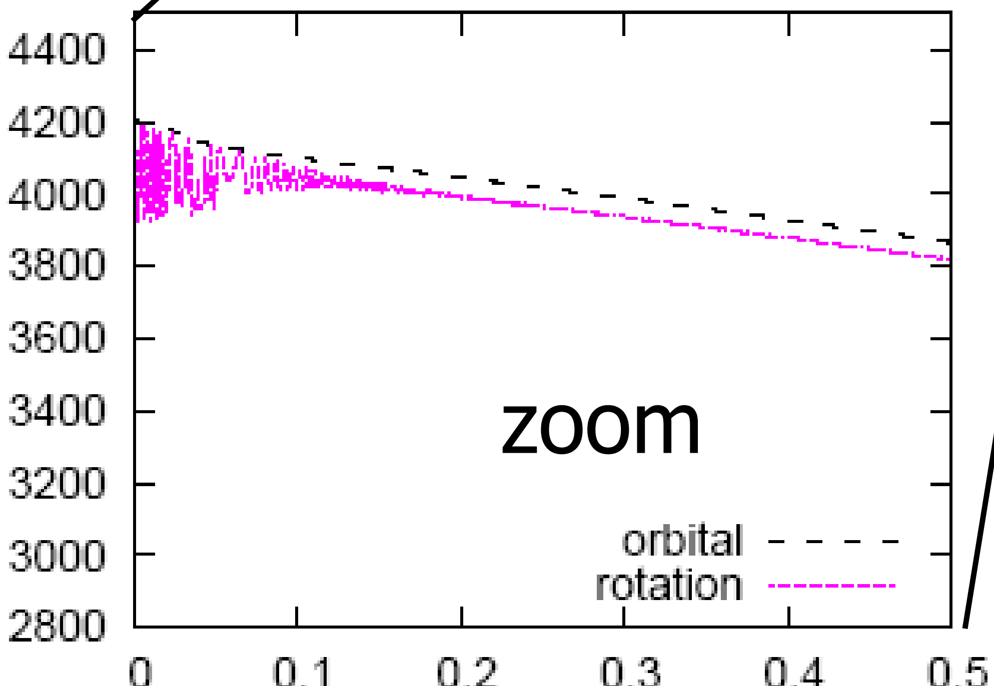


x 100

(Mardling´s
1st. stationary
eccentricity

Evolution of the periods





TIDAL EVOLUTION OF CLOSE-IN SATELLITES AND EXOPLANETS

Lecture 2. Creep tide theory

S.Ferraz-Mello
SDSM 2017

Reference: S.Ferraz-Mello et al.
DDA 2012 (astro-ph 1205.3957)
Cel.Mech.Dyn.Astron.**116**,109(2013)

Recent theories (of the anelastic tide):

Efroimsky, Lainey, Williams (2007-2017)
Remus et al. (2012)
Ogilvie & Lin (2004)
S.Ferraz-Mello (2012-2017)
Correa, Boué et al (2014-16)

- All of them depart in a larger or lesser extent from Darwin's theory.

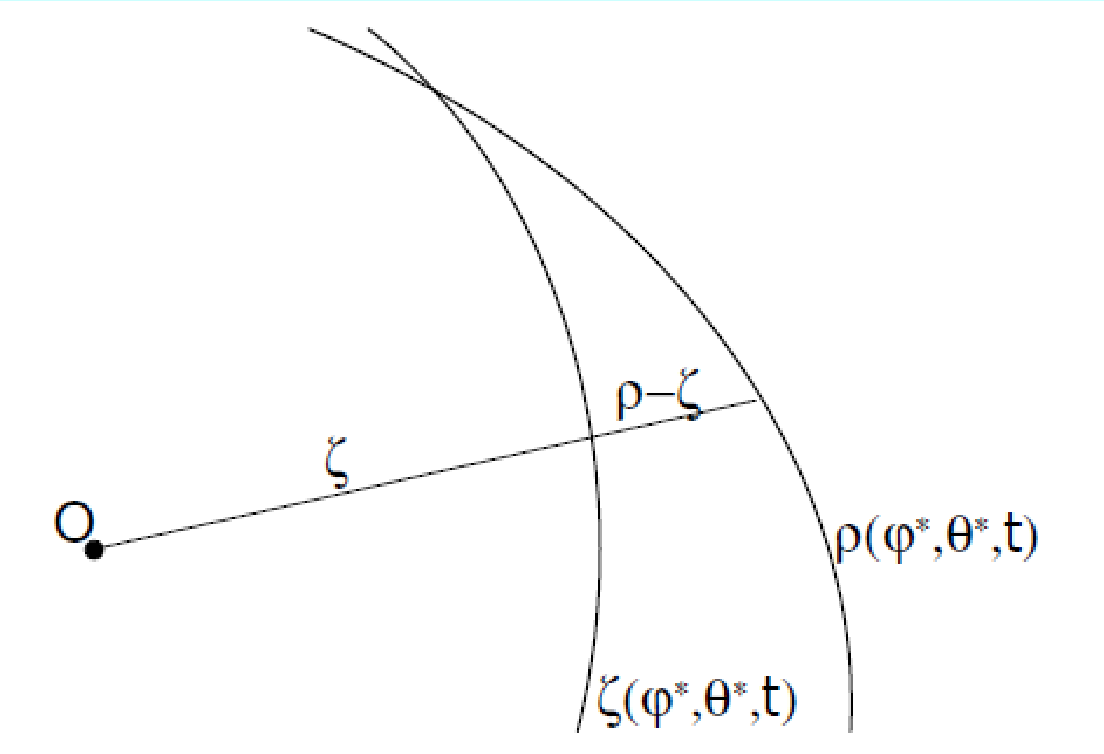
Anelastic Tide

New theory

Aim:

- **Substitution of plugged ad hoc lags by one physical law**

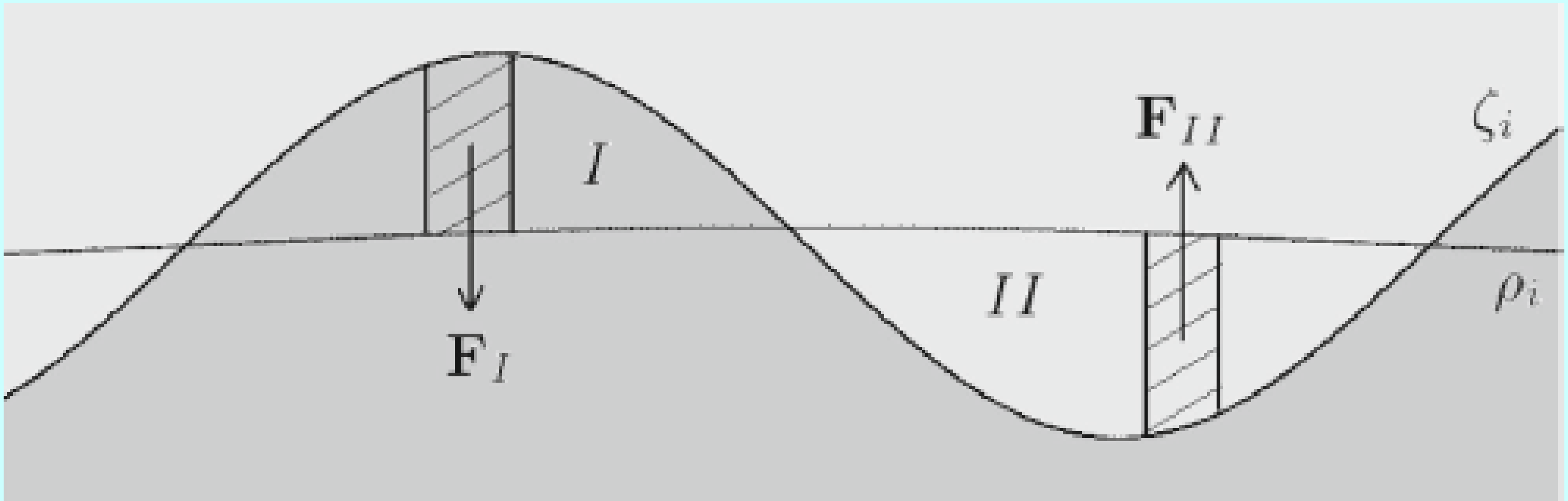
ζ.... Actual Surface of the body
r..... Surface of instantaneous equilibrium (VIRTUAL)



ANSATZ

$$\dot{\zeta} = \gamma(\rho - \zeta).$$

Newtonian
CREEP



Ref: Happel and Brener, Low Reynolds number Hydrodynamics, Kluwer, 1973 + **Darwin, 1879**

(see Folonier & FM, CMDA, in press, 2017)

$$\dot{\zeta} = \gamma(\rho - \zeta).$$

$$\boxed{\gamma = \frac{wR}{2\eta} = \frac{3gm}{8\pi R^2\eta}}$$

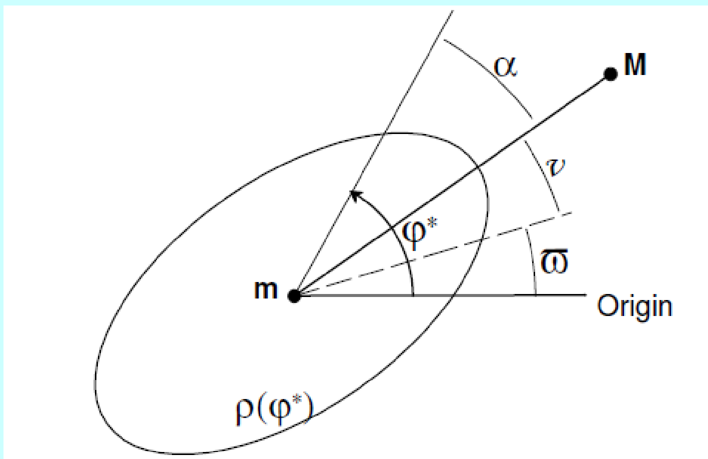
Relaxation factor (critical frequency)

Approx. Solution of the Navier-Stokes equation

Table 1 Typical values of the relaxation factor adopted in applications.

Body	γ (s^{-1})	$2\pi/\gamma$	η (Pa s)
Moon	$2.0 \pm 0.3 \times 10^{-9}$	36,000 d	$2.3 \pm 0.3 \times 10^{18}$
Titan	$2.9 \pm 0.2 \times 10^{-8}$	2500 d	$1.1 \pm 0.1 \times 10^{17}$
Solid Earth	$0.9 - 3.6 \times 10^{-7}$	200-800 d	$4.5 - 18 \times 10^{17}$
Io	$4.9 \pm 1.0 \times 10^{-7}$	730 d	$1.2 \pm 0.3 \times 10^{16}$
Europa	$1.8 - 8.0 \times 10^{-7}$	90–400 d	$4 - 18 \times 10^{15}$
Neptune	2.7–19	< 2 s	$1.2 - 4.8 \times 10^{10}$
Saturn	> 7.2	< 0.9 s	$< 15 \times 10^{10}$
Jupiter	23 ± 4	~ 0.3 s	$4.7 \pm 0.9 \times 10^{10}$
hot Jupiters	8–50	0.1–0.8 s	$5 \times 10^{10} - 10^{12}$
solar-type stars	> 30	< 0.2 s	$< 2 \times 10^{12}$

$$\dot{\zeta} + \gamma\zeta = \gamma R' + \frac{15\gamma R \sin^2\theta^*}{8}\left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^3 \boxed{\left(\frac{a}{r}\right)^3} \cos(2\varphi^* - 2\varpi \boxed{- 2v}).$$



O.D.E. for z(t)

$$\varpi = \varpi_0 - \Omega t.$$

Simplifications **(here!)**:

homogeneous bodies

equator = orbit plane; q* = p/2

solution

$$\zeta = Ce^{-\gamma t} + R' + \frac{15\mu R \sin^2\theta^*}{8}\left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^3 \sum_{k=-N}^{N} \frac{E_k(e)\cos(2\bar{\alpha} + k\ell - \sigma_k)}{\sqrt{\mu^2 + (\nu + kn)^2}}$$

= Superposition of tidal bulges

Prolateness: $$\epsilon_k = \frac{15}{4} E_k(e) \cos\sigma_k \left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^3$$

$$E_k(e) = \frac{1}{2\pi\sqrt{1-e^2}} \int_0^{2\pi} \frac{a}{r} \cos\left(2v + (k-2)\ell\right)\, dv.$$

Cayley functions

$$\sigma_k = \arctan\left(\frac{kn + \nu}{\gamma}\right).$$

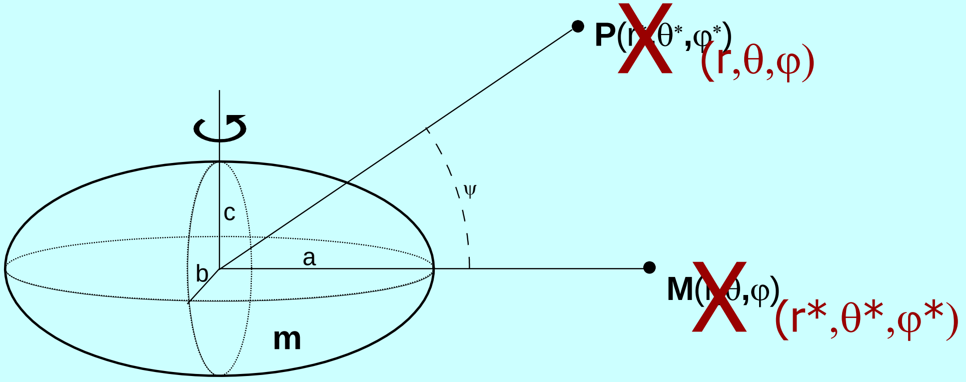
phase of the forced terms

$$U = -GR^2\mu_{\mathrm{m}} \int_0^{\pi} \sin\widehat{\theta}\, d\widehat{\theta} \int_0^{2\pi} \frac{\delta\zeta}{\Delta} d\widehat{\varphi}.$$

$$\delta U = -\frac{GmR^2}{5a^3} \sum_{k\in\mathbb{Z}} \sum_{k+j\in\mathbb{Z}} \Big(3\mathcal{C}_k \cos\overline{\sigma}_k E_{2,j+k} \cos\big((2-k)\ell^* + \overline{\sigma}_k + (j+k-2)\ell\big)$$
$$-\mathcal{C}''_k \cos\overline{\sigma}''_k E_{0,j+k} \cos\big(-k\ell^* + \overline{\sigma}''_k + (j+k)\ell\big)\Big)$$

where

$$\mathcal{C}_k = \frac{1}{2}\overline{\epsilon}_\rho E_{2,k}$$
$$\mathcal{C}''_k = -\frac{1}{2}\overline{\epsilon}_\rho E_{0,k} - \delta_{0,k}\overline{\epsilon}_z$$



Finally,

$$\delta \mathrm{f} = -M.\mathrm{grad}_{\mathrm{r}}\delta U \cdot$$

Torque

$$M_2 = \sum_{k=-N}^{N} -\frac{2k_f G M m R^2 \epsilon_k}{5r^3} \sin(2v + (k-2)\ell - \sigma_k)$$

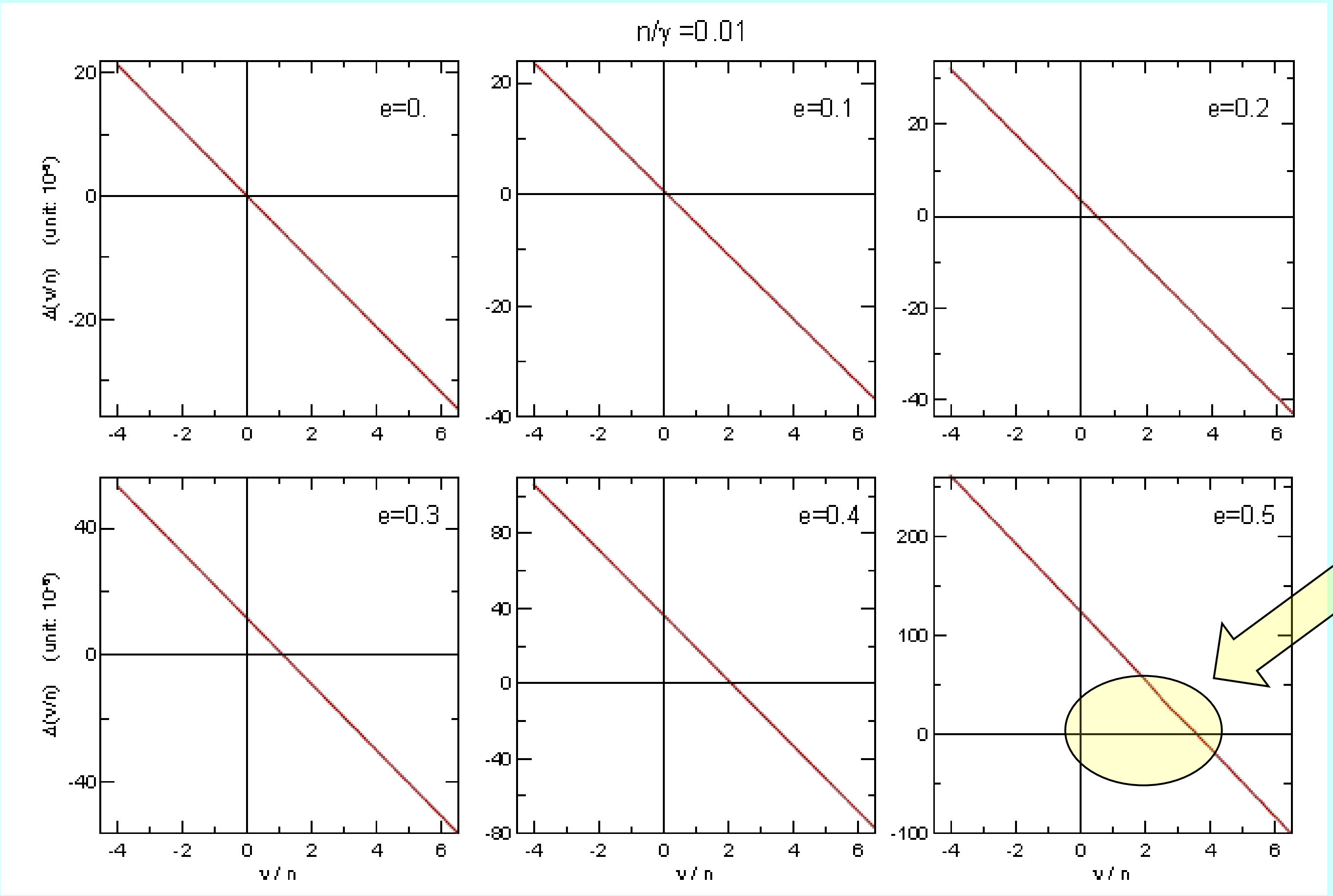
$$\dot{\nu} = -\frac{3GM\overline{\epsilon}_\rho}{2a^3} \sum_{k=2-N}^{2+N} E_{2,2-k} \sum_{j=-N-2+k}^{N-2+k} E_{2,2-k+j} \left(\sin 2\overline{\sigma}_k \cos j\ell + 2\cos^2 \overline{\sigma}_k \sin j\ell\right)$$

Where $\boxed{\nu}$ is the semi-diurnal frequency = $2\Omega - 2\dot{\lambda}$

and $$\sin 2\overline{\sigma}_k = \frac{2\gamma\left(\boxed{\nu} + (2-k)n\right)}{\gamma^2 + \left(\boxed{\nu} + (2-k)n\right)^2}$$

FIRST-order non-linear o.d.e.

Map $y(x) \rightarrow y(x+2p/g)-y(x)$



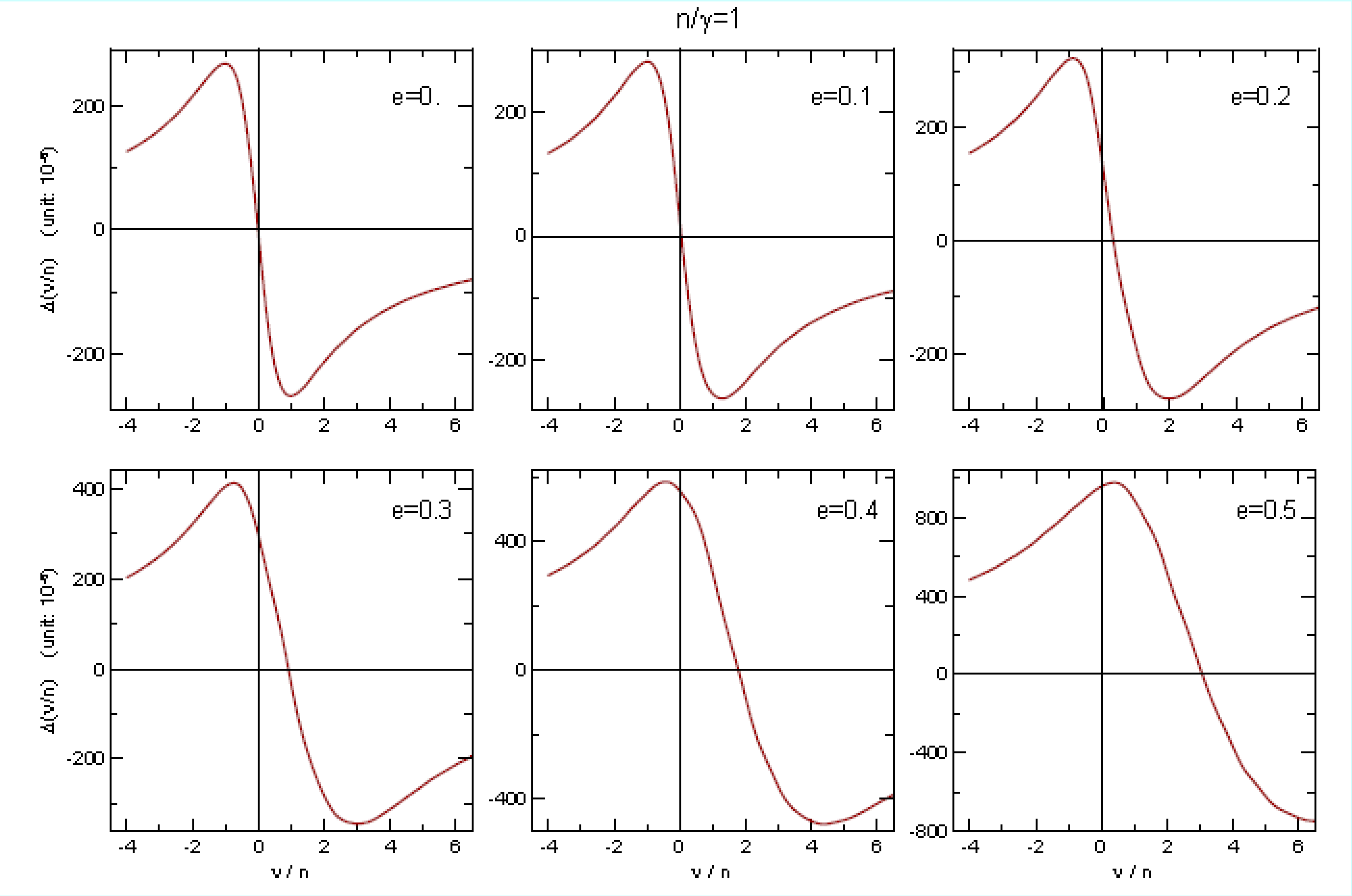
The intersections with the axis y'=0 are attractors

N.B.

These attractors are **supersynchronous**

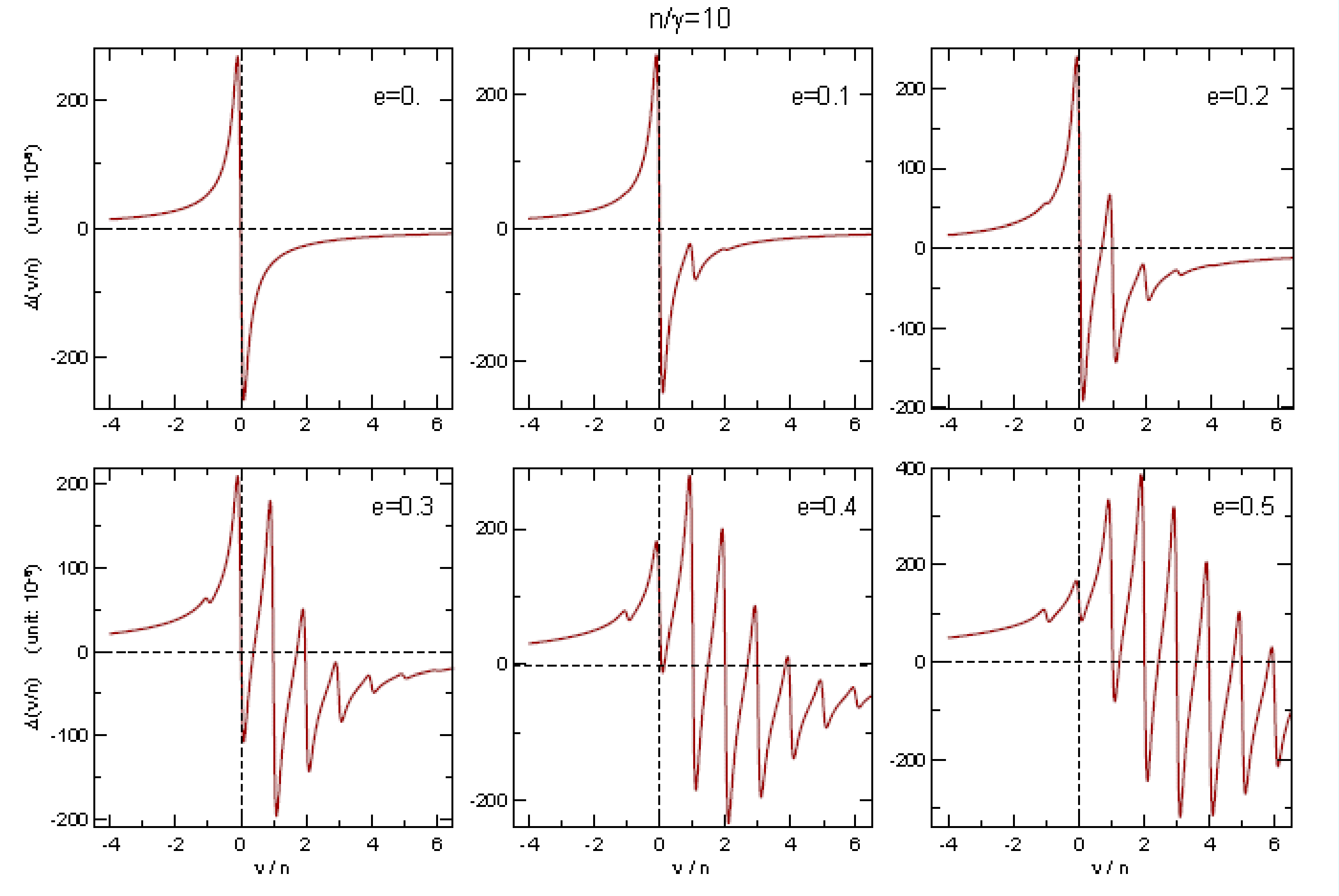
$$\Omega = n + 6ne^2$$

$\gamma >> n$ (ex: **stars**, hot Jupiters)



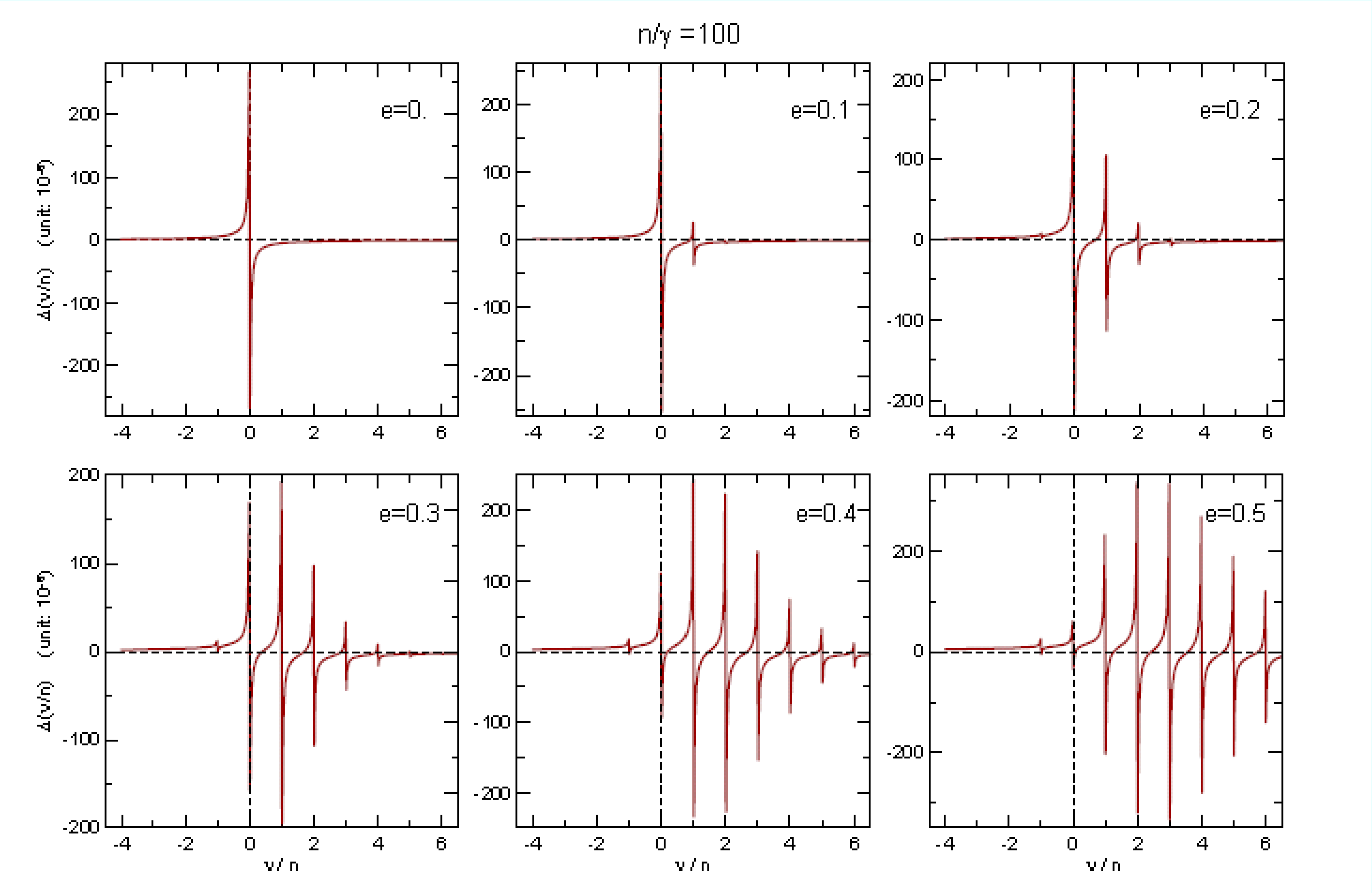
$\gamma \sim n$

Ref: SFM, DDA 2014 & CMDA (2015)
Correia et al. A&A 2013



γ= n/10 (ex: distant satellites, Mercury)

New attractors at n=n,2n,3n,....



$\gamma << n$ (ex: Moon, Titan)

attractors at ν = -n, 0, n, 2n, 3n,....

Ω = n/2, n, 3n/2, 2n, 5n/2, etc.

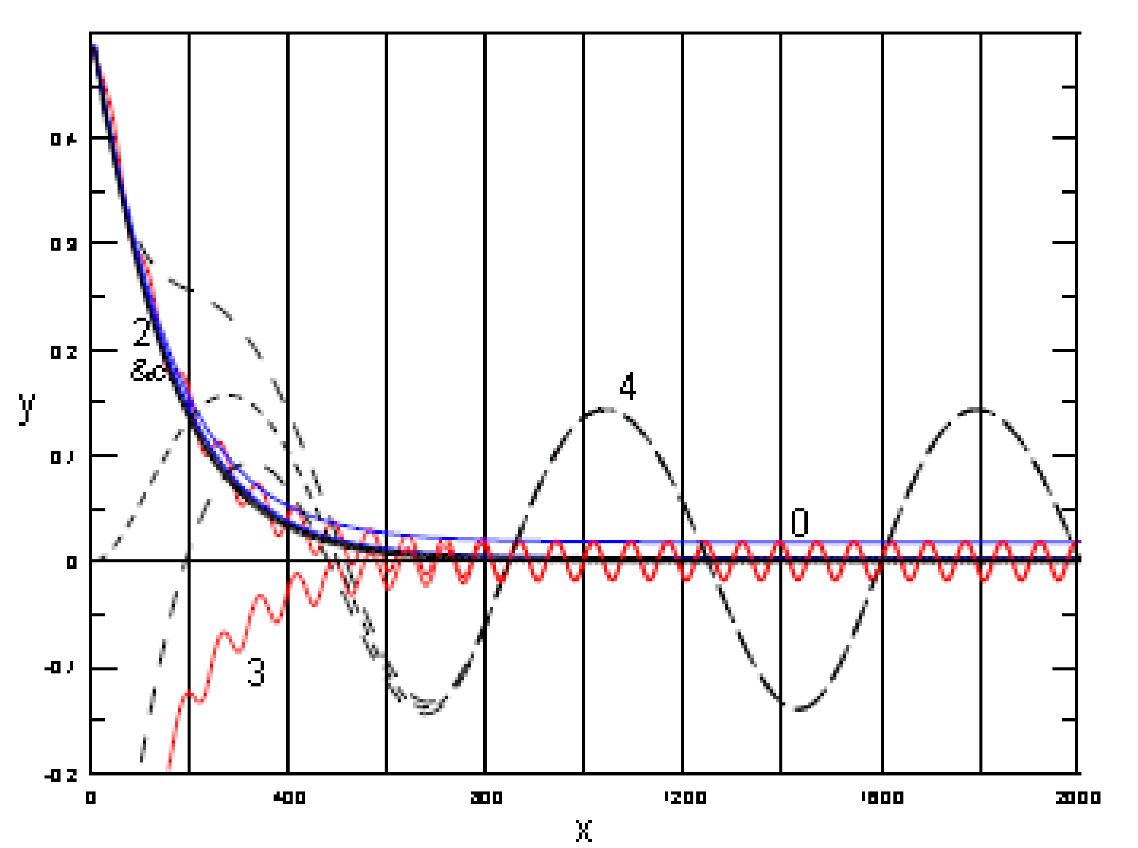
SYNCHRONIZATION

Simulations
near ν=0

(normalized variables)

$$y = \nu/\gamma$$

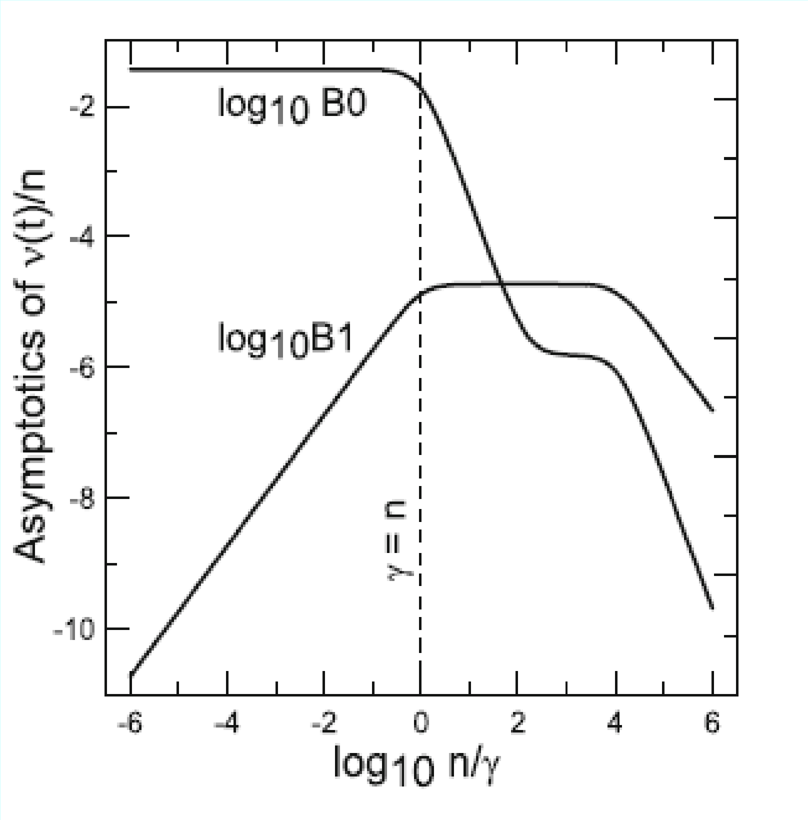
$$x = (n/\gamma)(t - t_0)$$



Parameter $\log_{10} n/\gamma$

Approximated solution

$$\nu/n = B_0 + B_1 \cos(\gamma x + \text{phase})$$

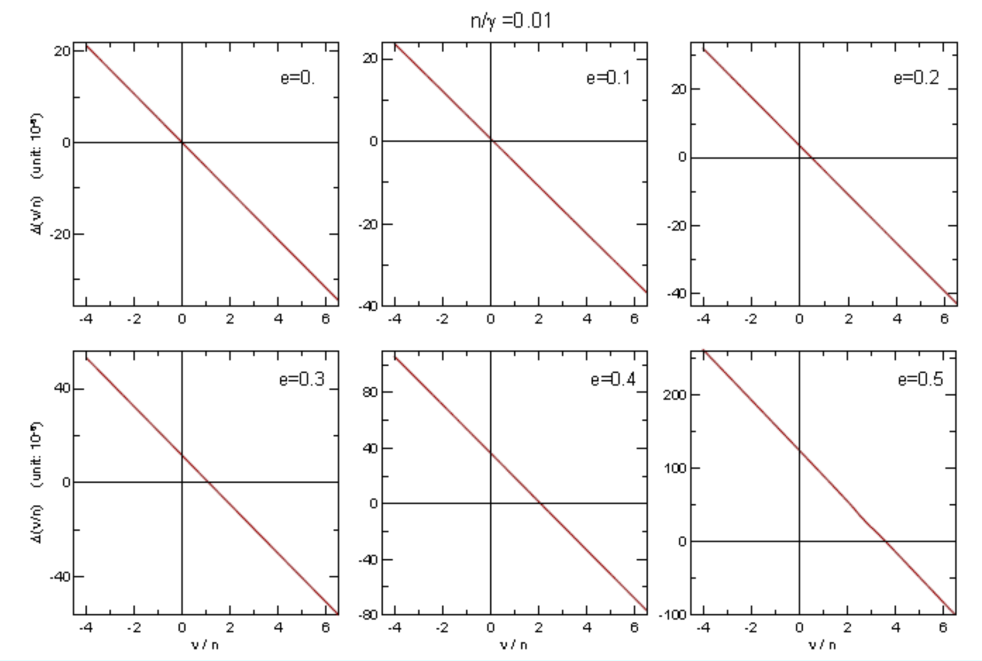


Limits: **Darwin** $\gamma >> n \rightarrow \Omega = n(1+6e^2+..)$

Efroimsky-Lainey $\gamma << n \rightarrow \Omega = n$

(high γ)

STARS



Examples:

CoRoT 15b BD (m=63.3 Jup) around a F7V star
Orbital period: 3.06 d
Star rotation: 2.9 – 3.1 d

KELT 1b BD (=27.4 Jup) around a F5 star
Orbital period: 1.217 d
Star rotation: 1.348 ± 0.4 sin I

But, solar-type stars are affected by wind braking

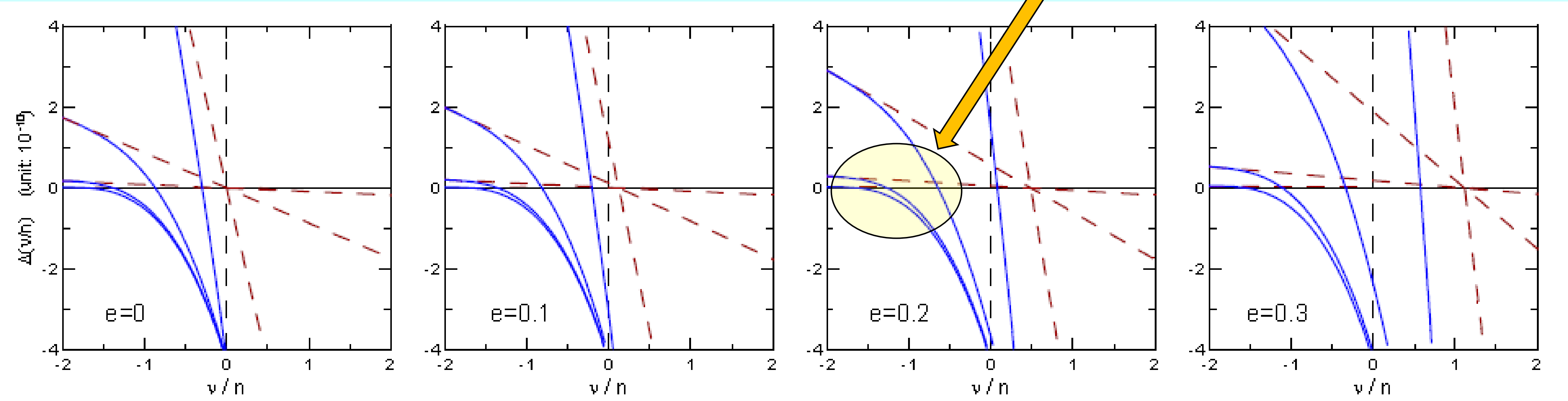
$$\dot{\Omega} = -f_P B_W \Omega^3$$

where

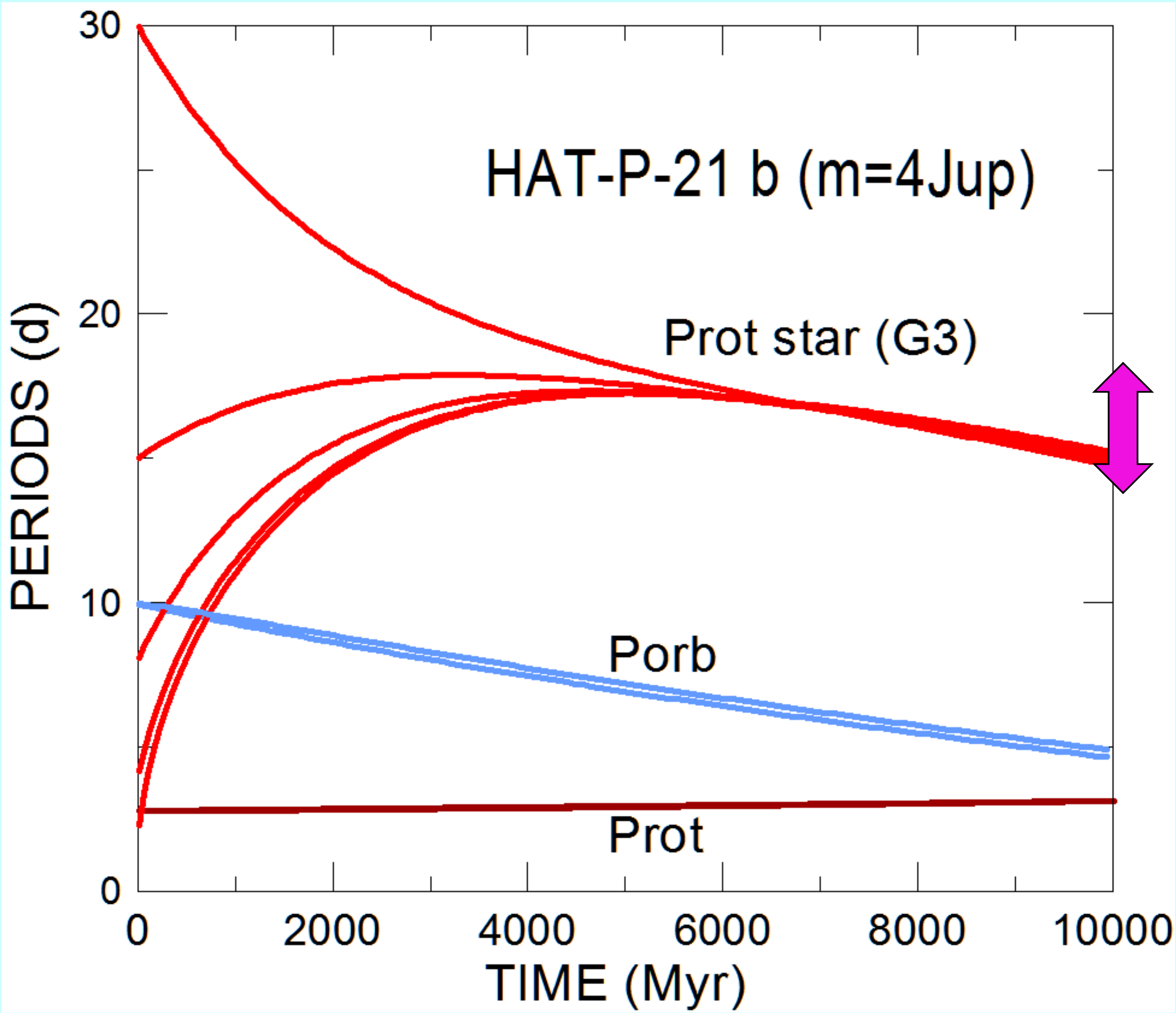
$$B_W = 2.7 \times 10^{47} \frac{1}{C} \sqrt{\left(\frac{R}{R_\odot} \frac{M_\odot}{m} \right)} \qquad \text{(cgs units)}$$

and $0 < f_P < 1$. **THEN**

subsynchronous atractors



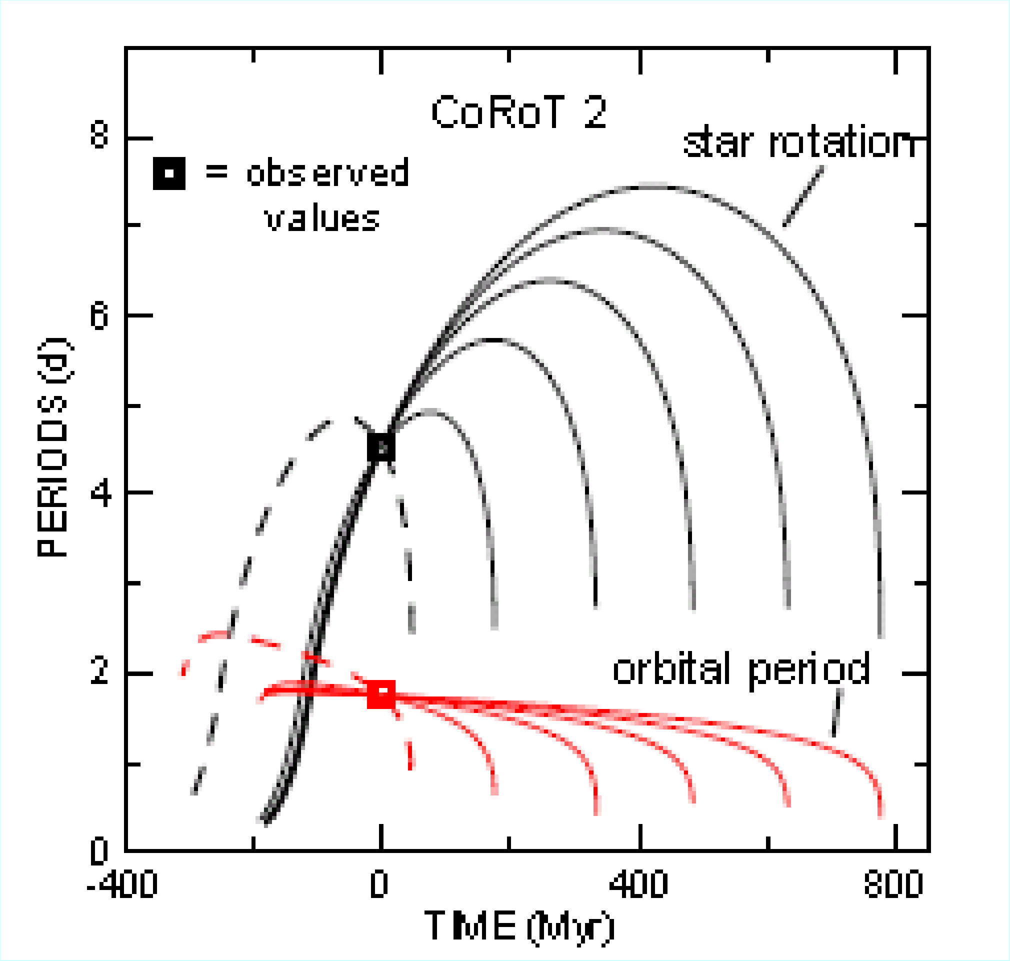
curves: $n/\gamma = 10^{-6}$ to 10^{-3} [brown: f=0; blue: f=1]



$\gamma = 25.5 s^{-1}$

$e = 0.228$

CoRoT 2: A young star

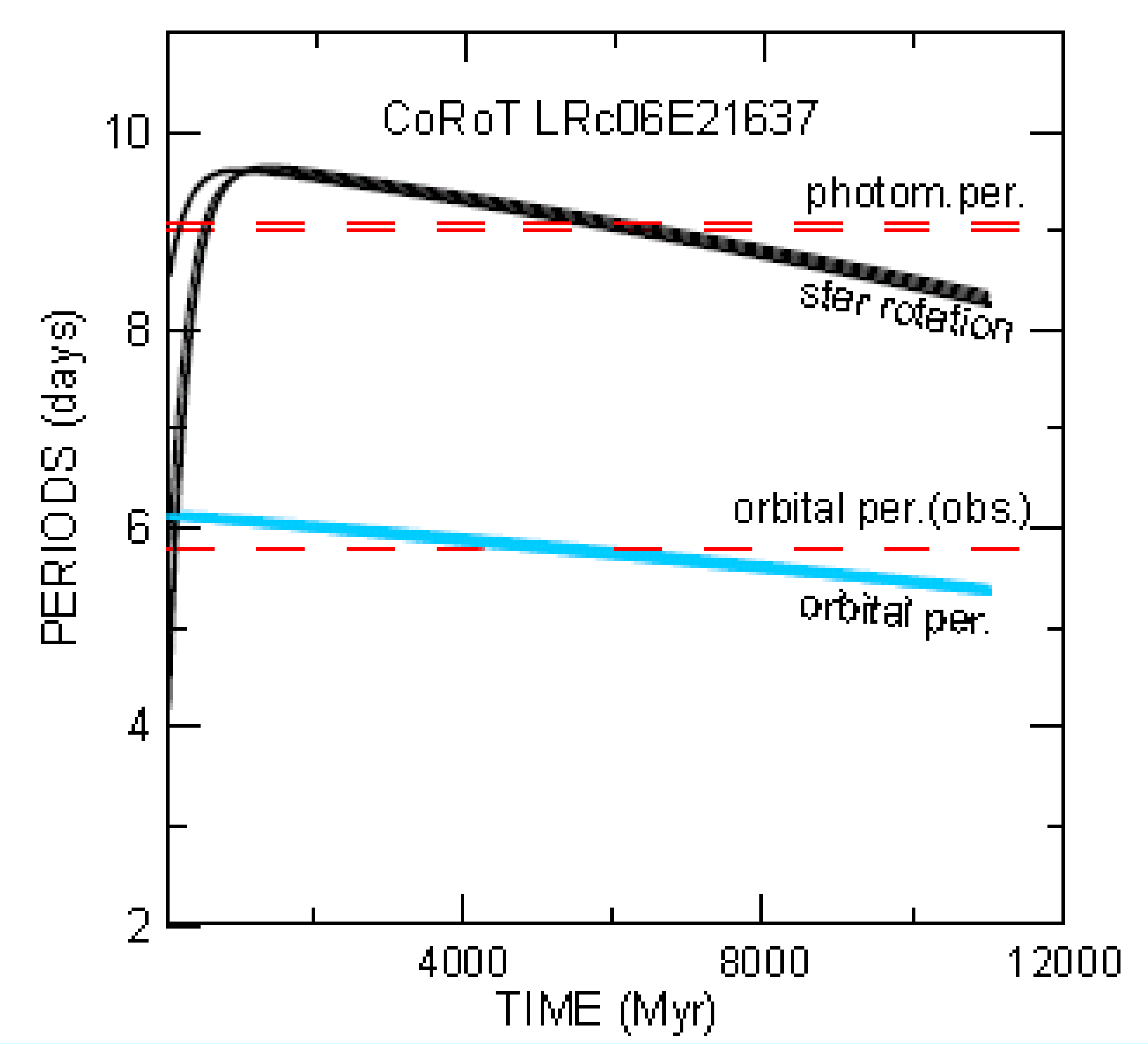


m_{pl}=3.3 jup
γ=20-100 s^{-1}

Result:
Age < 200 Myr

SFM et al. Astrophys. J. (2015)

CoRoT 33: A paradigm



m_{comp}=62 jup

γ = 55 s^{-1}

SFM et al. Astrophys. J. (2015)

VARIATION OF THE ELEMENTS (Lagrange variational equations)

$$\dot{a} = \frac{2}{na}\frac{\partial \mathcal{R}}{\partial \ell}$$

$$\dot{e} = -\frac{\sqrt{1-e^2}}{na^2e}\frac{\partial \mathcal{R}}{\partial \omega} + \frac{1-e^2}{na^2e}\frac{\partial \mathcal{R}}{\partial \ell}$$

(or the equivalent Gauss equations)

$$\mathcal{R} = -(1 + M/m)\delta U$$

$$\frac{da}{dt} = \frac{2nR^2}{5a} \sum_{k\in\mathbb{Z}} \sum_{k+j\in\mathbb{Z}} \Big(3(j+k-2)\mathcal{C}_k \cos\overline{\sigma}_k E_{2,j+k} \sin(j\ell + \overline{\sigma}_k)$$
$$-(j+k)\mathcal{C}''_k \cos\overline{\sigma}''_k E_{0,j+k} \sin(j\ell + \overline{\sigma}''_k)\Big).$$

AVERAGING $$< f > = \frac{1}{2\pi}\int_0^{2\pi} f . \mathrm{d}\boldsymbol{\ell}$$

$$<\frac{da}{dt}> = \frac{nR^2}{5a} \sum_{k\in\mathbb{Z}} \Big(3(k-2)\mathcal{C}_k E_{2,k} \boxed{\sin 2\overline{\sigma}_k} - k\mathcal{C}''_k E_{0,k} \sin 2\overline{\sigma}''_k\Big)$$

BUT… $$\sin 2\overline{\sigma}_k = \frac{2\gamma(\nu + kn)}{\gamma^2 + (\nu + kn)^2}$$ **is not a constant**

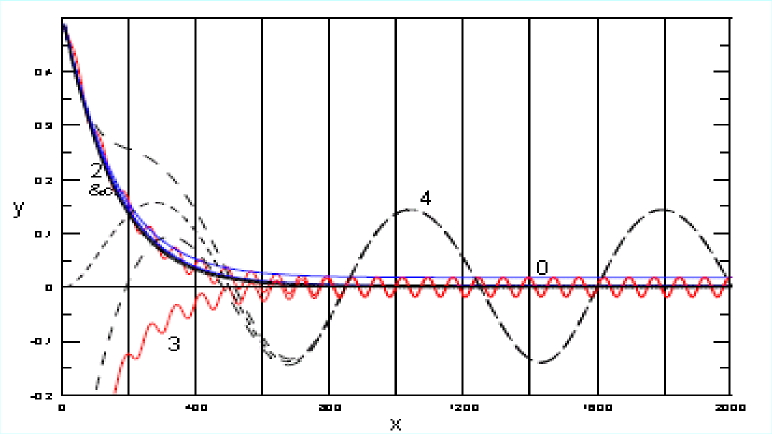
problematic when k=0 and ν~0

Simulations near ν=0

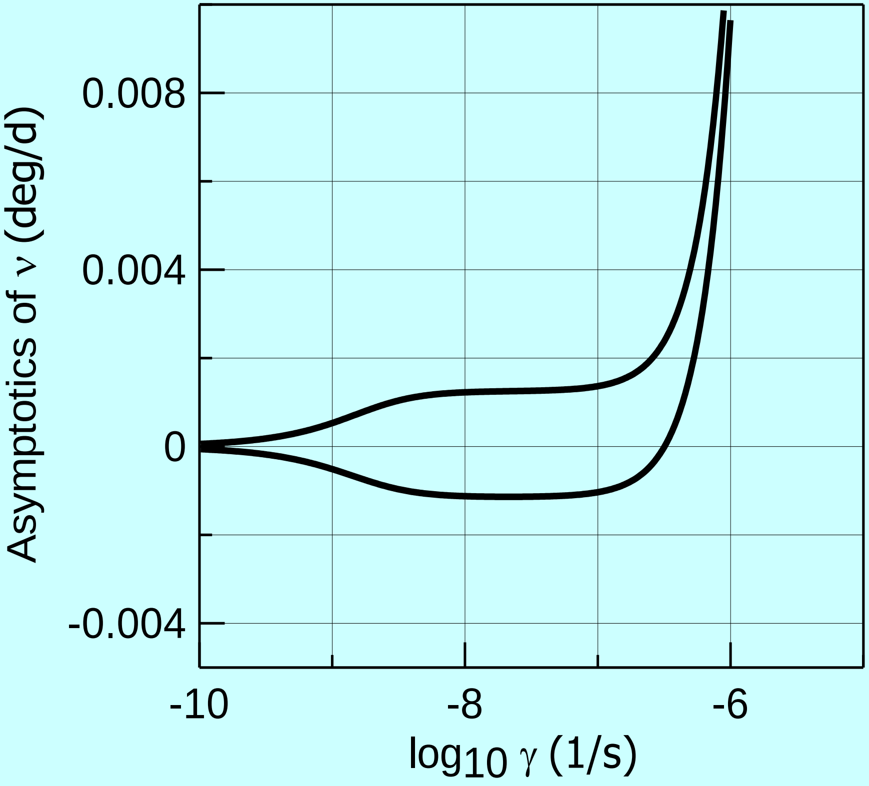
(normalized variables)

$y = \nu/\gamma$

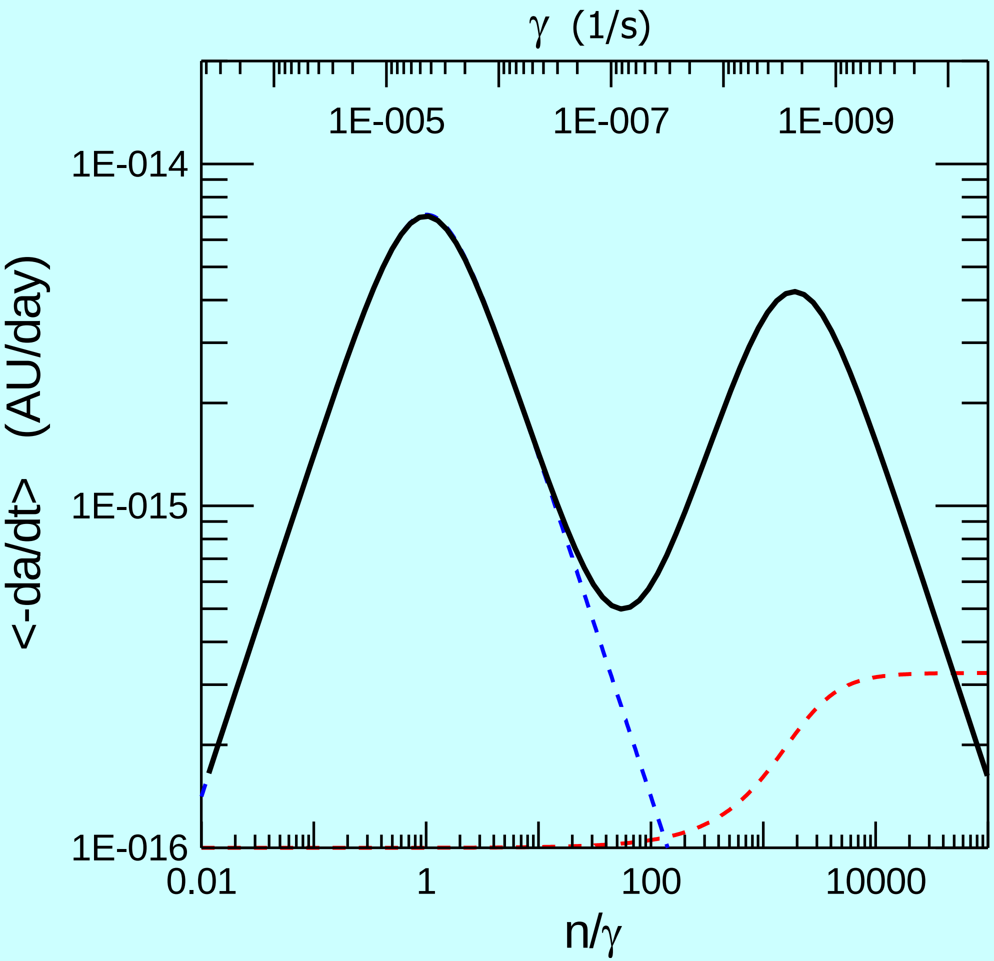
$x = \ell/\gamma$

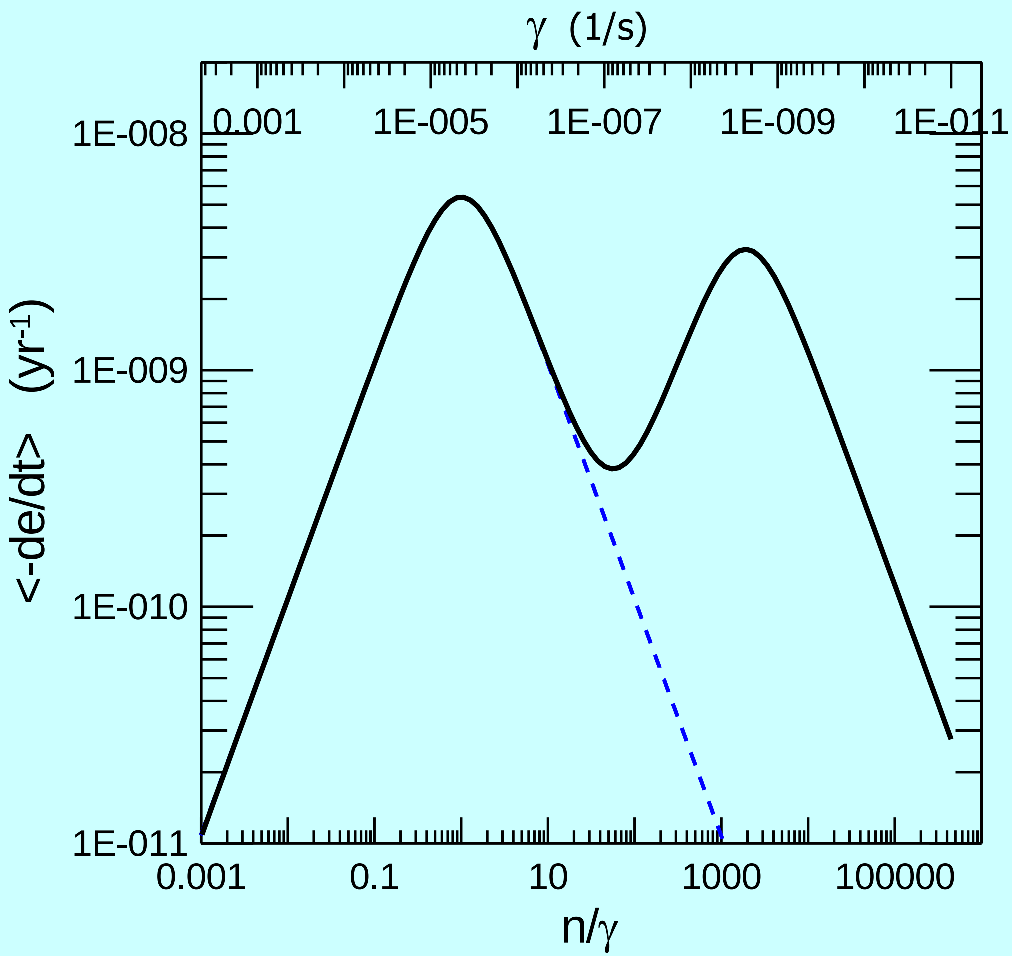


Example TITAN



Titan's da/dt and de/dt





In the saddle: $e'/e=-2\times10^{-8}$ yr^{-1}

Damped to 10% in 10^8 yr

DISSIPATION

Dissipation = “Loss” of orbital energy +
“Loss” of rotational energy

Neglect small variations associated with the moment of inertia and the shape of the body

Averaging (**when $\nu \neq 0$**)
(leading terms)

$$\langle \dot{E}_{\rm tot} \rangle_{(k=0)} = -\frac{3GmM(\Omega - n)R^2\overline{\epsilon}_\rho}{10a^3} E_{2,0}^2 \sin 2\overline{\sigma}_0$$

$$\sin 2\sigma_0 = \frac{2\nu\gamma}{\nu^2 + \gamma^2}$$

Hence, always, $\langle \dot{E}_{\rm tot} \rangle_{(k=0)} < 0$

i.e. mechanical energy **is lost**

Comparison

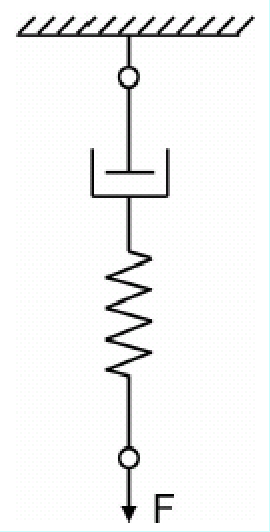
In Darwin's theory:

$$\langle \dot{E}_{\mathrm{tot}} \rangle_{(k=0)} = -\frac{2k_d\, GMm(\Omega-n)R^2\varepsilon_\rho}{5a^3}\,\varepsilon_0$$

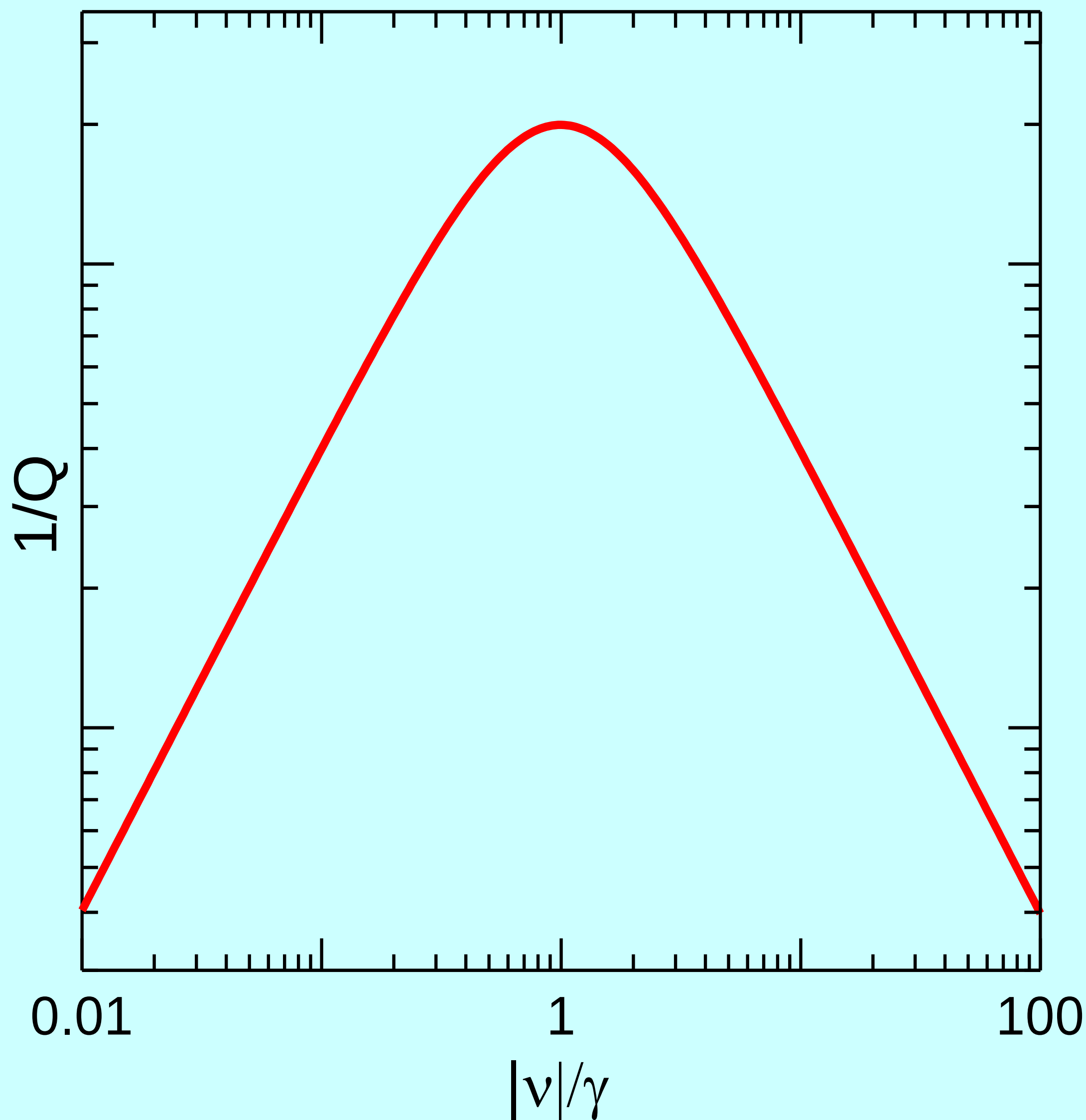
Comparing both results:

$$Q \cong \frac{1}{|\varepsilon_0|} = \frac{2k_d(\nu^2+\gamma^2)}{3|\nu|\gamma}$$

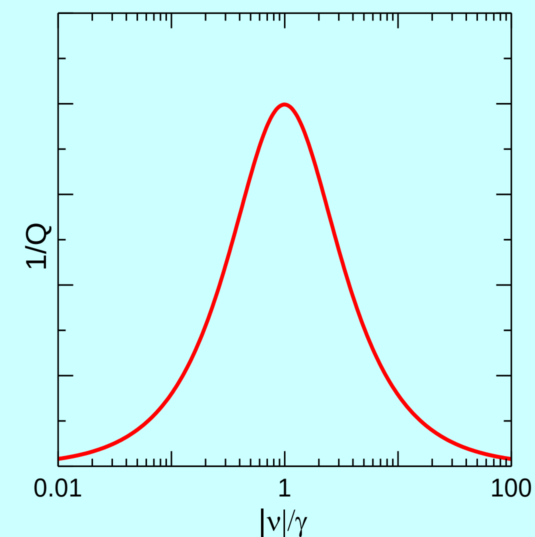
$$Q \cong \frac{1}{|\varepsilon_0|} = \frac{2k_d}{3} \left| \frac{\nu}{\gamma} + \frac{\gamma}{\nu} \right| = \text{dissipation law of a Maxwell body}$$



$\nu=2(\Omega\text{-}n)$

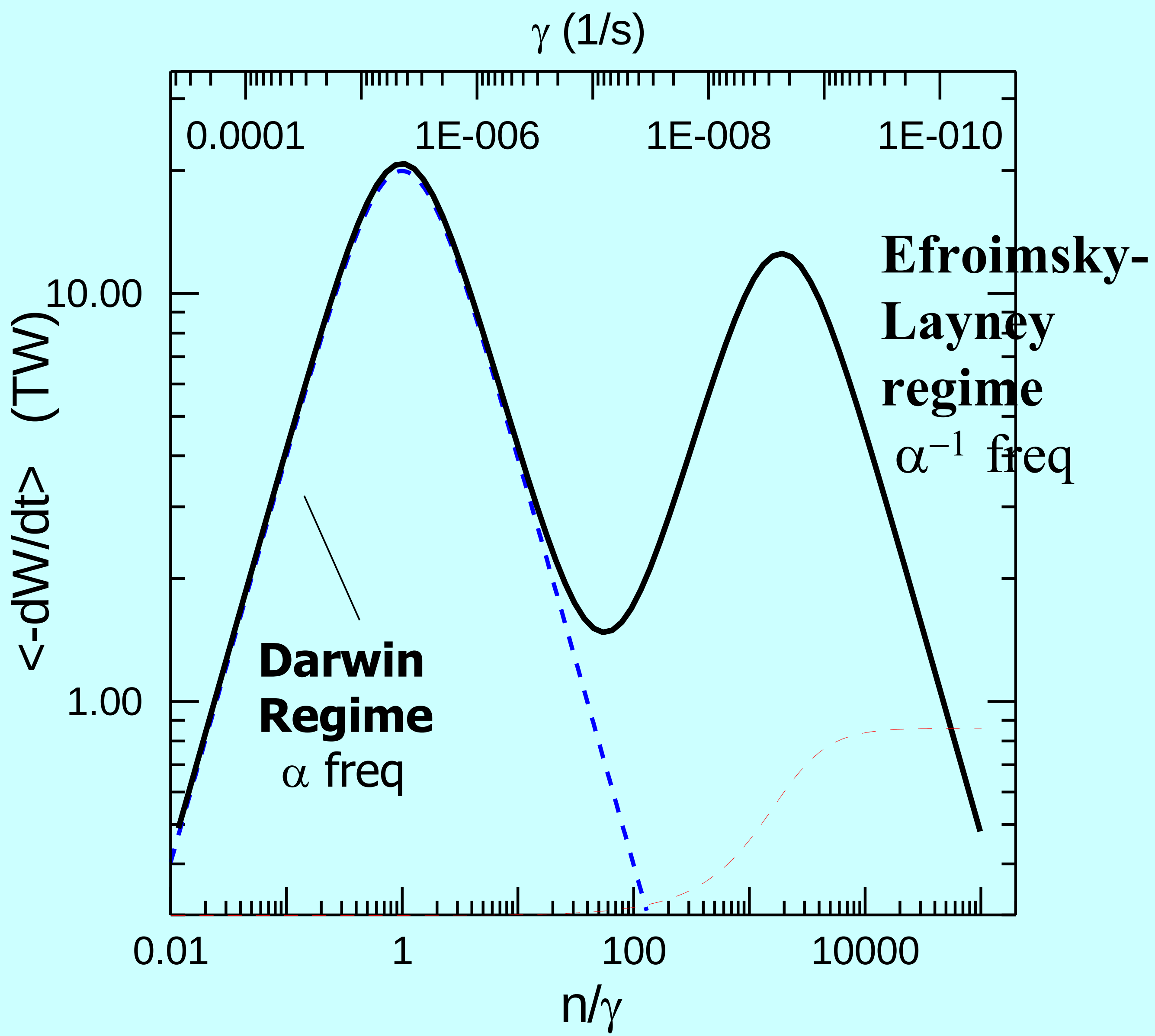


N.B. approximation not valid when ν tends to 0



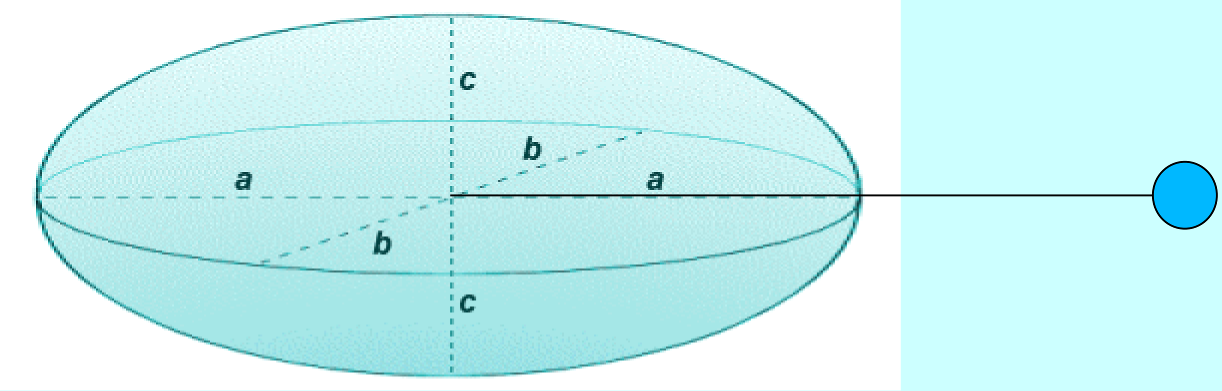
Averaging (**when $\nu \sim 0$**)

Ex: Titan's Dissipation law



Compare to previous figure

putting ELASTIC and ANELASTIC tides together

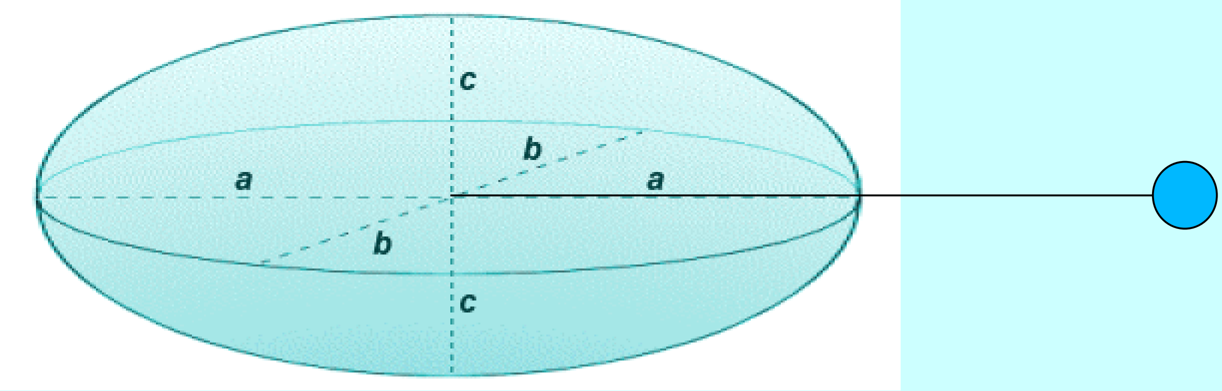


$$\epsilon = \frac{a}{b} - 1 = \frac{15}{4}\left(\frac{M}{m}\right)\left(\frac{R}{r}\right)^3$$

Ex: EARTH (lunar tide)   a-b=134 cm

actual value= 26 cm   (20% only)

putting ELASTIC and ANELASTIC tides together



$$\delta\zeta = \frac{1}{2} R_e \epsilon_\rho \Big(\lambda \cos 2\alpha + \cos\sigma_0 \cos(2\bar{\alpha} - \sigma_0) \Big)$$

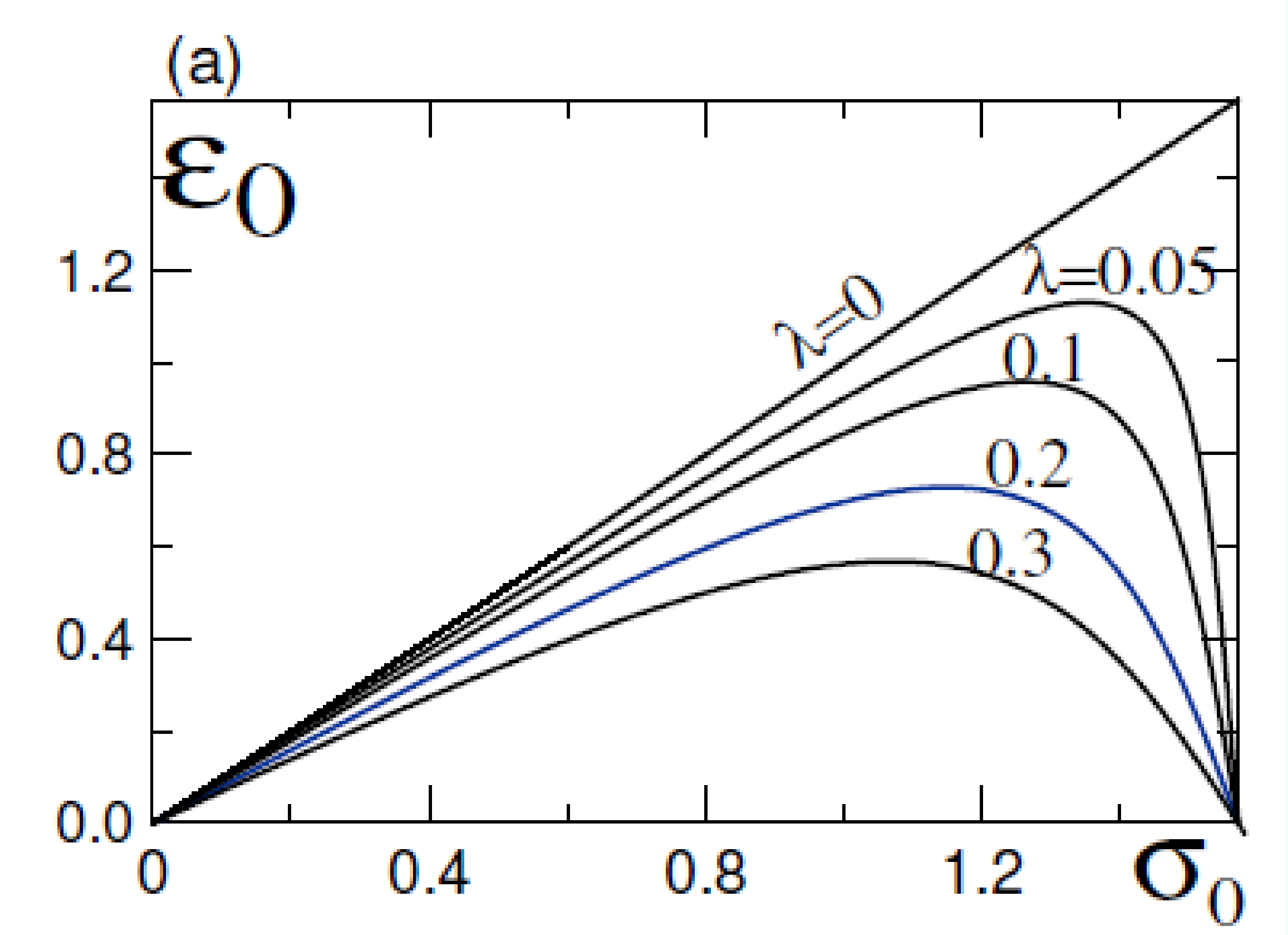
Circular + equatorial approximation.

α = angle between one point at the equator and M.

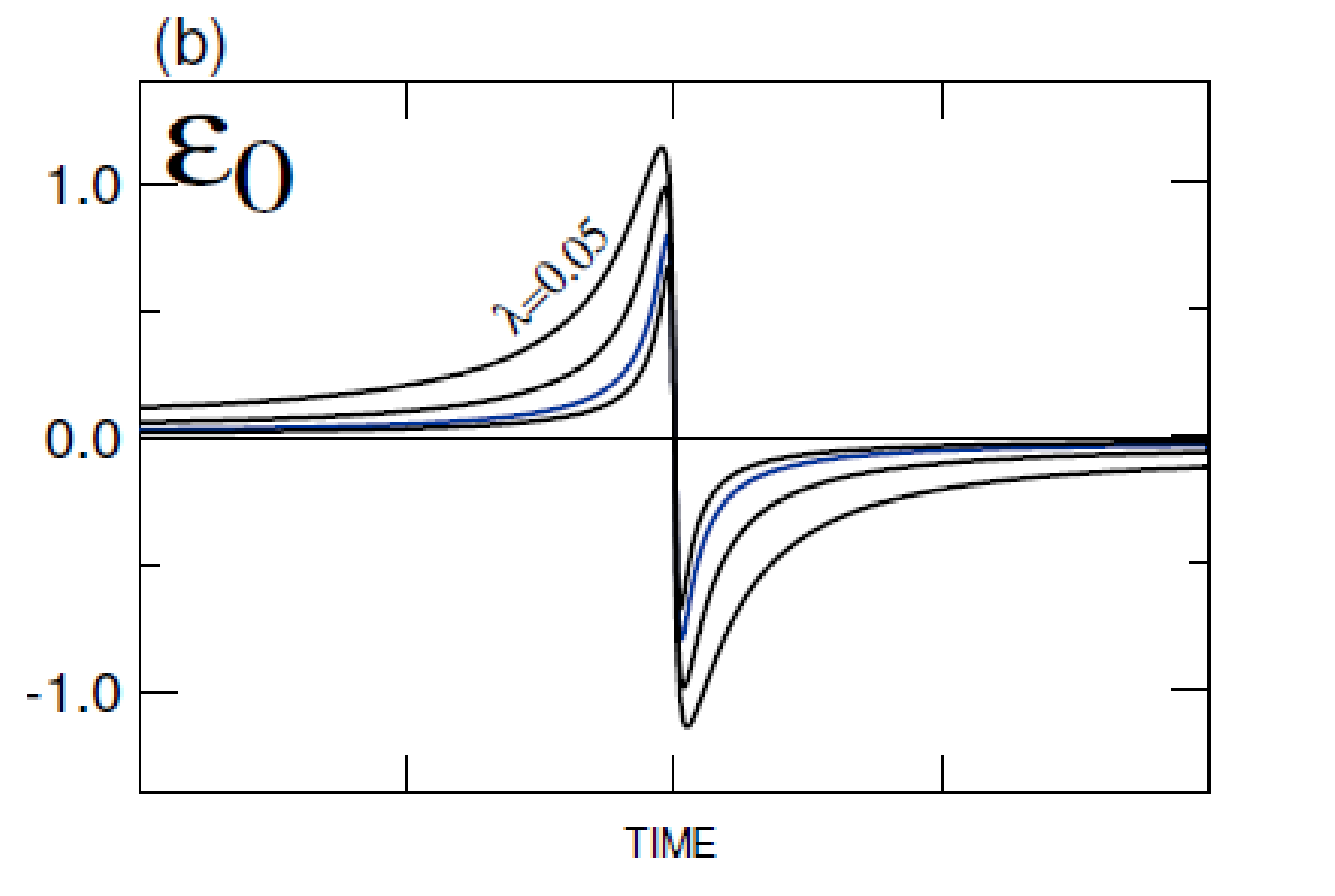
λ = height of the elastic tide bulge w.r.t. max. height of the Jeans ellipsoid

The shape is defined by the composition of the creep tide with a purely elastic tide (without lag)

Geodetic lag (case e=0)



$$\boxed{\sigma_0 = \arctan \frac{\nu}{\gamma},}$$



(b)
ε_0
1.0
0.0
-1.0
λ=0.05
TIME

Setting of the a theory including elastic and anelastic tide.

Anelastic tide (creep eqn.) $\dot{\zeta} = -\gamma(\zeta - \rho)$

Elastic tide: $\lambda\rho$

Introduce new variable: $Z = \boxed{\zeta + \lambda\rho}$

The new equation is:

$$\dot{Z} + \gamma Z = (1 + \lambda)\gamma\rho + \lambda\dot{\rho}$$

Which is the of a Maxwell model

$$\dot{Z} + \gamma Z = \gamma\rho + \lambda\dot{\rho}$$

It may be compred to the Maxwell model introduced by Correia, Boué et al. (A&A 2014) whose basic equation is

$$\dot{Z} + \gamma Z = \gamma \rho + \lambda \dot{\rho}$$

Which is virtually equivalent to the equation resulting from the creep theory:

$$\dot{Z} + \gamma Z = (1 + \lambda)\gamma \rho + \lambda \dot{\rho}$$

but not identical. The two theories give the same results.

THE END

Darwin theory

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(astro-ph 0712-1156)

Cel.Mech.Dyn.Ast.**101**,171,2008

Creep tide theory

S.F.-M.

(astro-ph 1204.3957 and 1505.05384)

Cel.Mech.Dyn.Ast.**116**,109,2013; **122**,359,2015

S.F.-M., H.A.Folonier, E. Andrade-Ines

(astro-ph 1707-09229)