Author: E.M. ALESSI
IFAC-CNR (Italy)
Title: *"Resonant signatures in the LEO region"*
Abstract. In the framework of the space debris problem, in order to mitigate the already critical situation in the Low Earth Orbit (LEO) region, and, more generally, to preserve the circumterrestrial environment, the scientific community is now aware of the need of designing feasible and effective solutions for the satellites' end-of-life. To this end, it is mandatory to obtain a deep understanding of the dynamics at stake. Within the H2020 ReDSHIFT project, we mapped the whole LEO region in terms of initial semi-major axis, eccentricity and inclination, for 16 combinations of longitude of ascending node and argument of pericenter, and 2 initial epochs. The dynamical model considered includes the geopotential of order and degree 5, lunisolar perturbations, solar radiation pressure (SRP) effects and atmospheric drag. The numerical simulations and specific analytical developments revealed the role of resonant inclinations associated with the just mentioned main perturbations. In particular, the resonances corresponding to the gravitational attraction of Moon and Sun and the SRP induce preferential paths which can be exploited to decrease the lifetime of the satellite, either in combination with the effect of the drag, or, if possible, using a SRP enhanced device. Besides the extension of the numerical results, the novelty of our findings consists in the theoretical understanding of the important role of lunisolar and SRP perturbations also in LEO. Concerning the SRP, resonances which were so far considered as secondary can play instead a fundamental role, and for values of area-to-mass ratio which can be achieved with the present technology.

This work is funded through the European Commission Horizon 2020, Framework Programme for Research and Innovation (2014-2020), under the ReDSHIFT project (grant agreement n. 687500).

Joint work with G. Schettino, A. Rossi, G.B. Valsecchi.

Author: K. ANTONIADOU
University of Namur (Belgium)
Title: *"How periodic orbits influence the co-existence of terrestrial planets with giants on eccentric orbits"*

Abstract. Hitherto unprecedented detections of exoplanets have been triggered by missions and ground based telescopes. The quest of "exo-Earths" has become intriguing and the long-term stability of planetary orbits is a crucial factor for the biosphere to evolve. Planets in mean-motion resonances (MMR) prompt the investigation of the dynamics in the framework of the three-body problem (TBP), where the families of stable periodic orbits constitute the backbone of stability domains in phase space. Here, we address the question of the possible co-existence of terrestrial planets with a giant companion on circular or eccentric orbit and explore the extent of the stability regions, when both the eccentricity of the outer giant planet and the semi-major axis of the inner terrestrial one vary, i.e. both non-resonant and resonant configurations. Our study exploits the restricted three-body problem (RTBP). Starting from the circular family and its bifurcation points, the families of periodic orbits in the circular and elliptic RTBP are computed for the 3/2, 2/1, 5/2, 3/1, 4/1 and 5/1 MMRs. We construct maps of dynamical stability to identify the boundaries of the stability domains where such a co-existence is allowed. We also compute the vertical critical periodic orbits (i.e. bifurcation points that can generate spatial families of periodic orbits) and provide hints with respect to vertically stable planetary orbits, as islands in their neighbourhood can host resonant mutually inclined exoplanets. Finally, the maximum mutual inclination of stably evolving planets that can be attained by spatial families of periodic orbits is also discussed.

Joint work with A.-S. Libert.

Author: E. BARRABÉS

Universitat de Girona (Spain)

Title: *"The dynamics of a parabolic restricted three-body problem"*

Abstract. We consider the motion of an infinitesimal mass under the gravitational influence of two masses moving in parabolic orbits and in the same plane. The main features of the problem are the gradient-like character, the Hill's regions, and the invariant manifolds associated to the equilibrium points. From them, we describe the final evolutions of the solutions, forward and backward in time.

This model can be used to understand, at a basic level, the effect of a close encounter of two galaxies. Such a close encounter may cause a significant modification in the mass distribution. Taking into account just one particle within one galaxy, after the close encounter, the particle may jump to the other galaxy or escape. We study in the frame of the planar parabolic problem, the mechanisms that allow to explain the different behaviors. Furthermore, after a close encounter of two galaxies, brigdes and tails can be seen between or around them. A bridge would be a spiral arm between a galaxy and its companion, whereas a tail would correspond to a long and curving set of debris escaping from the galaxy. We use the model to a mechanism that explain the formation of bridges and tails.

Joint work with Josep M. Cors, Mercé Ollé and Laura Garcia.

Author: V. BARUTELLO
Università di Torino (Italy)
Title: *"Recent results in Index Theory in Celestial Mechanics"*

Abstract. Index Theory can be used in Celestial Mechanics both to study linear stability of some classes of periodic orbits and to compute the Morse Index of a huge classes of solutions. In this talk we will focus on some recent results, that give a necessary and sufficient condition for the finiteness of the Morse index of trajectories interacting with the singular set and provide an Index Theorem to link the Morse index to a finite dimensional symplectic invariant, the Maslov Index.

Joint work with Xijun Hu, Riccardo Jadanza, Alessandro Portaluri, Susanna Terracini.

Author: M. BASSETTO
University of Pisa (Italy)
Title: *"Notes on logarithmic spiral trajectories generated by solar sails"*
Abstract. A solar sail represents a very promising option in the set of low-thrust propulsion systems. It exploits the solar radiation pressure that acts on a large reflective surface to generate a propulsive acceleration. The recent successful missions of IKAROS, Nanosail-D2 and LightSail-1 have confirmed the potentialities of such a propulsive concept and laid the foundation for future space missions. The trajectory design for a solar sail-based spacecraft within a heliocentric mission is usually addressed by numerically integrating the spacecraft equations of motion, since in general no closed-form analytical solution exists. In this respect an exception is offered by the special case of logarithmic spiral. The latter is characterized by a constant flight path angle and a thrust vector direction that remains fixed in an orbital reference frame. Therefore, the Sun-spacecraft distance exponentially grows (or reduces) with respect to an angular coordinate measured from a given heliocentric direction. Even though the logarithmic spiral has some intrinsic limitations (such as the inability of generating a circle-to-circle orbit transfer), it may represent an useful tool for the preliminary analysis of some mission scenarios. The aim of this paper is to provide a systematic study about the possibility of inserting a solar sail spacecraft into a heliocentric logarithmic spiral trajectory. The required conditions in terms of solar sail (fixed) attitude, performance, and initial position are discussed. It is shown that the spiral trajectory is required to start from a specific point that depends on the solar sail performance and the parking orbit characteristics. Moreover, the evolution of the osculating orbital parameters are presented, and some potential mission scenarios involving logarithmic spirals are analyzed, including the rotation of the apse line and the phasing trajectories of a spacecraft placed along an elliptic orbit.

Joint work with Lorenzo Niccolai, Alessandro A. Quarta, Giovanni Mengali.

Author: M. BATAILLE
Université de Namur (Belgium)
Title: *"Lidov-Kozai resonance in triple star systems"*
Abstract. About half of the Sun-like stars are part of multiple star systems. Many of them have an orbital period of a few days only. Our work focuses on the Lidov-Kozai tidal migration mechanism and aims to understand which dynamical effects are the most active in the accumulation of stellar companions with short orbital periods in binary star systems. Our framework is the hierarchical three-body problem (octupole), with the effects of tides, stellar oblateness, general relativity and spin down for the host star. Both the orbital evolution and the spin evolution are considered. Using orbital and physical parameters for the stars consistent with current observations, we perform 100 000 numerical simulations of well diversified triple star systems, and compare our results to Fabrycky & Tremaine (2007) and Naoz & Fabrycky (2014). We show that the final distribution of the final systems is very dependent on the initial parameters of the simulations. A similar study is finally realized for Hot Jupiters in binary systems, where the debate about the possible formation mechanisms (disc-planet interactions, planet-planet scattering and Lidov-Kozai migration) of such planets is very intense.

Joint work with A.-S. Libert & A. Correia.

Author: G. BAÙ
Università di Pisa (Italy)
Title: *"New linkage methods for large databases of optical observations"*

Abstract. With the rapid increase of asteroid detections, the identification of asteroids from observations taken in different nights requires more efficient algorithms. For this problem we consider very short arcs (VSAs) of asteroid observations. A VSA is usually not enough to compute an orbit; however, by linear or quadratic interpolation, we can compute an attributable at the mean epoch of the observations, that is a vector whose components are the topocentric angular position and velocity of the asteroid. An orbit is defined by the attributable and the topocentric radial distance and radial velocity of the observed body at that epoch.

The linkage problem consists of computing one or more preliminary orbits by using the information contained in two attributables. One possible way to solve this problem is to impose that the integrals of the motion take the same value at the two epochs of the attributables. An alternative procedure is the classical orbit determination method by Gauss, which uses three angular positions of the asteroid, typically belonging to three different VSAs. In this case the preliminary orbits are computed by solving a polynomial equation of degree 8.

With modern telescopes the number N of observations per night is very large. Using Gauss' method for the identification problem we have to solve $O(N^3)$ polynomial equations. A linkage algorithm, instead, have to be applied $O(N^2)$ times. Therefore, if we could find a polynomial equation for the linkage with low degree, we would significantly decrease the computational complexity of the problem.

We present the results reported in Gronchi et al. (2015) on the use of the two-body integrals to compute preliminary orbits by linking VSAs of observations of celestial bodies. In this work, by combining the algebraic integrals of the two-body problem we find a univariate polynomial of degree 9 in the radial distance of the orbit at the mean epoch of one of the two arcs. Following Gronchi et al. (2017), we show that the degree 9 is optimal in some sense.

Then, we describe a linkage method to join together three VSAs (Gronchi et al. 2017): from the conservation of the angular momentum it is possible to obtain a polynomial equation of degree 8 at the mean epoch of the second VSA.

We conclude with some numerical tests on the performance of the new methods and show that they can also be used when the time separation between the VSAs is large.

Joint work with Giovanni Federico Gronchi, Stefano Marò, Andrea Milani Comparetti.

References

[1] Gronchi, Baù, Marò, CMDA 123(2), 105–122 (2015).
[2] Gronchi, Baù, Milani, CMDA 127(2), 211–232 (2017).

Author: C. BEAUGÉ
Instituto de Astron. Teorica y Experimental, Obs. Astronomico,
Universidad Nacional de Cordoba (Argentina)
Title: *"Multi-body Resonances in Exoplanetary Systems"*
Abstract. In this talk we present a review of some recent results in the study of multi-resonant exoplanetary systems involving 3 or more planetary bodies. While the structure of 2-planet MMRs have been the object of many studies over several decades, multiple commensurabilities among exoplanets is a relatively new field. In particular, little is known about the capture routes and probabilities into resonance, as well as their stability with respect to additional perturbations.

We will discuss several examples of resonant chains in Kepler systems and how the dynamics can be used to constrain planetary masses and eccentricities, and how tidal effects can explain (at least) part of the observed population offset. Finally, we will mention how multi-resonances can be used to define a stability criteria for multi-planet systems and how different types of commensurabilties contribute to this limit.

Author: L. BENET
UNAM (Mexico)
Title: *"A simple model for the location of Saturn's F ring"*
Abstract. In this paper, we introduce a simplified model to understand the location of Saturn's F ring. The model is a planar restricted five-body problem defined by the gravitational field of Saturn, including its second zonal harmonic J2, the shepherd moons Prometheus and Pandora, and Titan. We compute accurate long-time numerical integrations of non-interacting test-particles initially located in the region between the orbits of Prometheus and Pandora, and address whether they escape or remain trapped in this region. We obtain a wide region of initial conditions of the test particles that remain confined. We consider a dynamical stability indicator for the test particles' motion defined by computing the ratio of the standard deviation over the average value of relevant dynamical quantities, in particular, for the mean-motion and the semi-major axis. This indicator separates clearly a subset of trapped initial conditions that appear as very localised stripes in the initial semi-major axis and eccentricity space for the most stable orbits. Retaining only these test particles, we obtain a narrow eccentric ring which displays sharp edges and collective alignment. We relate the accumulation stripes of stable ring-particles to resonances, mostly involving Prometheus' outer Lindblad and co-rotation resonances, but not exclusively. Comparison of our results, including the angular precession of the ring, with the nominal data for the F ring shows some correspondence.

Joint work with Á. Jorba.

Author: A. BOSCAGGIN
Università di Torino (Italy)
Title: *"Scattering parabolic solutions for the N-centre problem in the three dimensional space"*

Abstract. For the N-centre problem in the three dimensional space, we prove the existence of entire parabolic trajectories having prescribed asymptotic directions. The proof relies on a variational argument of min-max type; Morse index estimates and regularization techniques are used in order to rule out the possible occurrence of collisions.

Joint work with W. Dambrosio and S. Terracini (Torino).

Author: M. CAPINSKI
AGH University of Science and Technology (Poland)
Title: *"A topological mechanism for diffusion in a priori chaotic dynamical systems, with application to the Neptune-Triton elliptic restricted three-body problem"*

Abstract. We present a topological mechanism of diffusion in a priori chaotic systems. The method leads to a proof of diffusion for an explicit range of perturbation parameters. The assumptions of our theorem can be verified using interval arithmetic numerics, leading to computer assisted proofs. As an example of application we prove diffusion in the Neptune-Triton planar elliptic restricted three body problem.

Joint work with Marian Gidea.

Author: J. CARDOSO DOS SANTOS
UNESP - São Paulo State University (Brazil)
Title: *"Study of the roto-translational dynamics using intermediaries: numerical experiments"*

Abstract. The present work deals with the roto-translational motion of an axisymmetric rigid body, considering this body under the influence of a central gravitational field. Following Ferrándiz and Sansaturio, a Hamiltonian formalism is considered based on the total angular momentum and the canonical variables associated, and the model is shaped as a perturbation of the Keplerian motion plus the free-rotation of a rigid body, where the elimination of the nodes is used to reduce two degrees of freedom of the problem. Despite this, the system of equations of motion is non-integrable. The concept of intermediary Hamiltonian is then used to propose a simplification of the system which is integrable in terms of elliptical integrals. An alternative procedure proposed by one of the authors may be followed identifying a second model, where now the canonical transformation known as elimination of the parallax is used before the process of building of a second intermediary Hamiltonian. This second intermediary becomes integrable in terms of elementary functions. As part of the full two-body dynamics, different values of the parameters: relative masses of the bodies and the shape of the rigid body, are considered to visualize possibilities of applications to problems such as binary asteroids and artificial satellite attitude propagation. Numerical experiments are made to compare the order of approximation of the two intermediary Hamiltonians with respect to the full model under the MacCullagh approximation. In all the cases considered in the numerical simulations, both models present good precision after hundreds of orbits for a system with a rigid body in an eccentric relative orbit, with some advantage for the model with elimination of the parallax.

Joint work with Sebastián Ferrer (Universidad de Murcia – Spain) and Daniel J. Scheeres (University of Colorado – USA).

Author: V. CARRUBA
UNESP (Brazil)

Title: *"Detection of the Yarkovsky and YORP effect for young asteroid families"*

Abstract. Young families are families that formed in timescales of 20 Myr or less. Because of their very young age, chaotic dynamics did not have enough time to erase traces of the event that formed the family. The longitudes of nodes (and, in some cases, of pericenters) of the family members with respect to the parent body converge to within a very limited range when integrated backward over the estimated age of the family. This allows to (i) identify family members and (ii) estimate the family age with a precision not available for other, older asteroid groups. While this behavior was studied and understood in previous works, discoveries of new asteroids over the last ten years dramatically improved the number of young asteroid families such as Karin, a subfamily of the Koronis asteroid family, Veritas, Iannini, and others. Here we took advantage of this new larger sample to (i) refine the age estimate of these families, and (ii) detect and study the impact that the Yarkovsky and YORP effects had on the group dynamical evolution. The Karin family should be 5.764 ± 0.011 Myr old, and the YORP effect was most likely responsible for affecting the spin obliquity distribution of the $D < 2$ km sized Karin population, producing a bimodal distribution. The Veritas family is found to be 8.23 ± 0.37 Myr old, and (490) Veritas is most likely an interloper. Despite the very chaotic dynamical environment of this family, we were able to infer Yarkovsky induced drift rates for 274 of the most regular family members. Preliminary results for other young families may also briefly discussed in this work.

Joint work with Dr. David Nesvorný and Dr. David Vokrouhlický.

Author: C. COLOMBO
Politecnico di Milano (Italy)
Title: *"Orbit manoeuvring enhancing natural perturbations"*
Abstract. Natural orbit perturbations are responsible for the trajectory divergence from the nominal two-body problem, increasing the requirements for orbit control; whereas, in space situation awareness, they influence the orbit evolution of space debris that could cause hazard to operational spacecraft. However, the dynamics of natural orbit perturbations can be leveraged to significantly reduce extreme high mission cost and create new opportunities for space exploration. Alternatives to high fidelity models of the dynamics to predict the actual orbit evolution are semi-analytical techniques, based on averaging of the disturbing potential function (Brouwer 1959, Deprit 1981), which separate the constant, short-period and long-period effects, thus reducing the computational time for long-term analysis. In an era of unlimited computational resources, when the burden of high-fidelity numerical propagation is not anymore a problem, a recent trend emerged in resorting to semi-analytical propagation techniques for Earth-centred orbits and to apply them to new space engineering problems. In this work semi analytical techniques and Hamiltonian dynamics are used first as a tool for understanding the underlying dynamics of orbit perturbations. Then, an optimiser is proposed that progressively explores the phase space and, through spacecraft propulsion manoeuvres, governs the effect of perturbations to reach the desired orbit. Within the optimisation, the dynamics model is progressively improved so that the final optimal result reflect the actual orbit evolution. Two mission applications are presented: the end-of-life design of the ESA INTEGRAL mission, enhancing the effect of luni-solar perturbation through delta-v manoeuvres and the use of variable geometry solar sail for the end-of-life of nano-satellites in medium earth orbit. In the first case the manoeuvre is computed in the eccentricity-inclination-anomaly-of-perigee map, first introduced by Kozai (1962), in the second case the effect of solar radiation pressure is modulated to achieve a long term grow of the orbit eccentricity.

Author: B. CONWAY
University of Illinois Urbana (USA)

Title: *"Asteroid Deflection with Safe Harbors Found via Numerical Optimization"*

Abstract. Deflection is perhaps the best option for mitigation of the danger presented by a hazardous asteroid. Deflection experiments (e.g. AIDA) are also being considered in advance of an actual hazard. If it were feasible to "safely park" the asteroid in a region of phase space that does not yield a significant future impact risk (i.e. a "safe harbor") with the Earth, this should certainly be done. In this work a metaheuristic method is used to optimize all of the relevant mission parameters for a spacecraft to deflect an asteroid via impact. The objective of the numerical optimizer is to maximize the deflection obtained while assuring that subsequent close approach distances of the asteroid to the Earth are only increased by the initial deflection.

Joint work with Siegfried Eggl and Daniel Hestroffer, IMCCE, Observatoire de Paris.

Author: A. CORREIA
University of Aveiro (Portugal)
Title: *"Tidal evolution of circumbinary systems"*
Abstract. We investigate the secular dynamics of three-body circumbinary systems under the effect of tides. We use the octupolar non-restricted approximation for the orbital interactions, general relativity corrections, the quadrupolar approximation for the spins, and the viscous linear model for tides. We derive the averaged equations of motion in a simplified vectorial formalism, which is suitable to model the long-term evolution of a wide variety of circumbinary systems in very eccentric and inclined orbits. We show that circumbinary planets with initial arbitrary orbital inclination can become coplanar through a secular resonance between the precession of the orbit and the precession of the spin of one of the stars. We also show that circumbinary systems for which the pericenter of the inner orbit is initially in libration present chaotic motion for the spins and for the eccentricity of the outer orbit.

Joint work with Gwenael Boué (IMCCE) and Jacques Laskar (IMCCE).

Author: J.M. CORS

`Universitat Politécnica de Catalunya (Spain)`

Title: *"Coorbital quasi-periodic motion in the three-body problem"*

Abstract. We consider the dynamics of coorbital motion of two small moons about a large planet which have nearly circular orbits with almost equal radii. Within the framework of the planar three-body problem we establish the existence of quasi-periodic motions and KAM 3-tori. The study is based on a combination of normal form and symplectic reduction theories and the application of a KAM theorem for high-order degenerate systems. We approach the problem as a perturbation of decoupled Kepler. This approximation is valid in the region of phase space where coorbital solutions occur.

A joint work with J.F. Palacián and P. Yanguas.

References

J.M. Cors and G.R. Hall. Coorbital periodic orbits in the three body problem. SIAM J. Appl. Dyn. Syst. (2003), no. 2, 219–237.

J. M. Cors, J.F. Palacián and P. Yanguas. Asymptotic stability estimates near an equilibrium point. In preparation.

Author: A. CRIDA
Université Côte d'Azur / Observatoire de la Côte d'Azur (France)
Title: *"Formation of Janus and Epimetheus from Saturn's rings as coorbitals, thanks to Mimas' 2:3 inner Mean Motion Resonances"*

Abstract. Janus and Epimetheus orbit Saturn at 151461 km on average, on mutual horseshoe orbits with orbital separation 50 km, exchanging position every 4 years. This configuration is unique and intriguing: Lissauer et al. (1985) have shown that their orbital separation should converge to zero in about 20 Myrs only, and no satisfactory model for the origin of this co-orbital resonance exists yet. Charnoz et al. (2010) have shown that Janus and Epimetheus probably formed from the spreading of the rings beyond the Roche radius. Here, we study this phenomenon in the frame of the elliptical restricted 3-body problem, where ring particles are perturbed by mean motion resonances with the outer satellite Mimas.

Two types of resonances play different roles. The Lindblad resonance (LR) confines the rings radially, and prevents their spreading (like the B-ring into the Cassini division). The Corotation resonance (CR) confines the rings azimuthally in two capture sites (akin Neptune's arcs). Because of Saturn's J_2, the CR is 130 km closer to Saturn than the LR. A few hundred million years ago, the 2:3 mean motion resonances with Mimas were just inside the Roche radius; hence the rings could not spread and the two capture sites were full of ring material. When Mimas migrated outwards so that its 2:3 mean motion resonances receded past the Roche radius, the captured material agglomerated into two bodies of $\sim 10^{15}$ kg on the exact same orbit. These bodies then migrated outwards together due to their interaction with the rings, in mutual horseshoe orbits. The rings then spawn new small satellites, eventually accreted by the proto-Janus and the proto-Epimetheus following the pyramidal regime of the ring spreading model (Crida & Charnoz 2012). The two bodies then grow in mass following a Fibonacci sequence, and this excites their orbital separation, leading to a configuration close to the present one.

Joint work with Maryame El Moutamid.

Author: A. DEL VIGNA
Università di Pisa / SpaceDyS (Italy)
Title: *"Short arc orbit determination and imminent impactors in the Gaia era"*

Abstract. The need of an automated system for the prediction of short-term impacts became evident after two important events: the Chelyabinsk meteor and the impact of the asteroid 2014AA on the Earth, only 21 hours after its discovery. In this field, short-arc orbit determination techniques are crucial. Even if the observations are too few to obtain an orbit, the attributable can be computed. It contains information about the sky position and motion, but it leaves unknown the topocentric distance and the radial velocity. The systematic ranging explores a raster in the topocentric range and range-rate space, and allows us to identify the regions filled with impact solutions. If a reliable nominal solutions does not exist, we make use of a two-step systematic ranging to scan the Admissible Region (AR), selecting the density and the kind of sampling (uniform in the range or in its logarithm) on the basis of the number of connected components of the AR and of the extension of the first component (if it is the only one). On the other hand, if a reliable nominal orbit exists, the use of the grid is no longer necessary: we compute a spider web sampling in a neighborhood of the nominal solution, following the level curves of the target function.

In both cases, we then compute a sampling of the Manifold of Variations, a 2-dimensional manifold parametrized on a subset of the AR, using an iterative procedure, namely the doubly constrained differential corrections. Moreover, we start from the hypothesis of a Guassian density on the residuals space, and we pull it back to the sampling space. We do not consider any other a priori assumption, such as a population model. In this way, we are able to compute the impact probability for possibly impacting objects, and to assign to each one of them a score, representing the probability for it to be a Near-Earth, a Main Belt or a distant object.

We set up an automated system for the scanning of the NEOCP and the computation of the risk assessment of recently detected objects. Such a fast-response system (i.e. within few minutes from the release of new observations) is important not to loose interesting objects before their uncertainty grows too much, and hence it will help in the follow-up activities.

Joint work with F. Spoto, A. Milani, G. Tommei, P. Tanga, F. Mignard.

Author: J.-B. DELISLE
University of Geneva (Switzerland)
Title: *"Spin dynamics of close-in planets exhibiting large TTVs"*

Abstract. We study the spin evolution of close-in planets in compact multi-planetary systems. The rotation period of these planets is often assumed to be synchronous with the orbital period due to tidal dissipation. We show that planet-planet perturbations can drive the spin of these planets into non-synchronous or even chaotic states. In particular, we show that transit timing variations (TTVs) are a very good probe to study the spin dynamics. We apply our model to KOI-227 b and Kepler-88 b, which are both observed undergoing strong TTVs. We also perform numerical simulations of the spin evolution of these two planets. We show that for KOI-227 b non-synchronous rotation is possible, while for Kepler-88 b the rotation can be chaotic.

Ref.: Delisle, J.-B., Correia, A.C.M., Leleu, A., Robutel, P. submitted to A&A.

Author: L. DELL'ELCE
INRIA (France)
Title: *"Optimal Low-Thrust Orbital Transfer by Averaging Multiple Frequencies"*

Abstract. When the state of an Hamiltonian system can be decomposed into slow and fast oscillatory components, averaging the equations of motion over the instantaneous period of the fast variables is a valuable practice to simplify the dynamics of the system and gain understanding of the long-term evolution of the flow. The perturbed Kepler problem belongs to this category. Hence, averaging techniques were exploited in astrodynamics since the early space age to develop efficient analytic or semi-analytic orbital propagators. More recent contributions in optimal control were addressed to the study of extremal flow of low-thrust orbital transfers in energy and time minimization.

Averaging the controlled system facilitates the challenging task of providing a reliable initial guess to shooting algorithms. The quality of this guess can be possibly enhanced if multiple fast-oscillating perturbations are accounted for, e.g., sectorial gravitational harmonics or third-body perturbations. Nonetheless, available results in control consider a single fast variable, namely the angular position of the satellite on a slow-varying orbit.

In this talk, we discuss the orbital transfer problem in the presence of multiple fast-oscillating perturbations. Although the enhancement of the realism of the model is appealing, averaging multiple frequencies is only mathematically sound when the instantaneous frequencies are non commensurable. However, important applications exist where near-resonant regions are crossed or even targeted. For example, geosynchronous orbits exhibit a one-to-one resonance between the orbital and the Earth rotation periods. In this case, averaging over the two variables independently lacks a rigorous justification. A preliminary step is to use frequency map analysis to gain insight into this problem.

Joint work with Jean-Baptiste CAILLAU and Jean-Baptiste POMET.

Author: M. DI CARLO
University of Strathclyde (United Kingdom)
Title: *"Surrogate-based optimisation of low-thrust trajectories"*

Abstract. In this work the use of a surrogate model to estimate the cost of low-thrust transfers and to globally optimise low-thrust trajectories is presented. The paper will show how surrogate models can be used to obtain a quick estimation of the cost and time of flight required for any low-thrust trajectory rendezvous. The data points for the construction of the surrogate model are generated by solving a two-point boundary value problem (TPBVP) defined by a set of initial and final non-singular orbital elements. For the solution of the TPBVP, a direct transcription method based on Finite Orbital Elements (FOE) is used. Finite Orbital Elements are based on an asymptotic analytical solution of the motion under a low-thrust acceleration. The resulting surrogate model has a wide range of possible applications and overcomes some limitations of analytical formulas, currently available in the literature, for the estimation of the cost of low-thrust transfers. In fact these formulas often consider the variation of a single or a small group of orbital elements at a time and do not consider the angular position of the spacecraft on its orbit.

It will be shown how the proposed surrogate model can be used to globally optimise low-thrust trajectories in conjunction with higher fidelity, expensive models. In this case, the surrogate model is adaptively updated during the optimisation process and the higher fidelity model is evaluated only when required. This form of adaptive surrogate-based global optimisation is applied to the deisgn of GTO-GEO low-thrust transfers using an expensive high fidelity model including perturbations from the Earth's aspherical potential, drag and third body. The results are compared to those obtained using a traditional nonlinear programming method for low-thrust optimisation, that provides only locally optimal solutions, strongly dependent on the initial guess.

Joint work with Massimiliano Vasile.

Author: L. DIMARE
SpaceDys (Italy)

Title: *"Use of a semi-linear method to predict the impact corridor on ground"*

Abstract. We propose an adaptation of the semi-linear algorithm [1,2] for the prediction of the impact corridor on ground of an Earth impacting asteroid. The algorithm starts from a least squares orbit, for which an impact on Earth is possible at some epoch in the future, with a positive impact probability IP>0, as provided by the main impact monitoring systems CLOMON2 and Sentry, respectively at the university of Pisa and at NASA-JPL [3]. Since the IP is greater than zero, there exist sets of orbital parameters compatible with the least squares solution and leading to an impact. Starting from one of them and using the covariance of the least squares solution, the semi-linear method provides the boundary of the impact corridor on ground, corresponding to the piece of the initial uncertainty region that leads to the impact.

References

[1] Milani A. (1999) Icarus, 137, 269–292.

[2] Milani A. and Gronchi G. F. (2010), Theory of Orbit Determination, Cambridge University Press.

[3] Milani A. et al. (2005) Icarus, 173, 362–384.

Author: S. EFIMOV
MIPT (Russia)

Title: *"Non-integrable model describing celestial bodies' dynamics in first order mean motion resonance"*

Abstract. Mean motion resonance (MMR) is the dynamical situation characterized by commensurability between the orbital periods of two celestial bodies. Effects of bodies' interaction in the resonance are very different from non-resonant case. Study of MMR is very important, in particular, for problems of formation and stability of the solar system. The commensurability $(p+q):p$ (where p is a natural number and q a non-zero integer) is referred to as resonance of order $|q|$. Systems with the first order MMR are usually considered to be integrable [1]. However, there is evidence, that dynamics of Pluto, which is in $2:3$ orbital resonance with Neptune, is chaotic [2]. In this work, it is shown that integrability of the problem vanishes when second-order perturbation terms are accounted for. The non-planar circular restricted three-body problem is considered and the model Hamiltonian describing evolution of the system in the first order MMR more accurately is introduced. Following the technique originated from [3, 4], canonical variables are divided into slow and fast and the averaging method is applied to study long-term evolution of the orbit. It is shown that the phase space of the system contains a region with adiabatic chaos. Some characteristics of emerging chaos are calculated, classification of all possible long-term evolution scenarios is presented and bifurcations in phase portraits of the system are analyzed.

Joint work with Vladislav Sidorenko.

References

[1] Sessin W. and Ferraz-Mello S. Motion of two planets with periods commensurable in the ratio 2:1 solutions of the Hori auxiliary system // Celestial Mechanics. 1984. 32. 307-332.

[2] Wisdom J. and Sussman G.J. Numerical evidence that the motion of Pluto is chaotic // Bulletin of the American Astronomical Society. 1988. 20. 910.

[3] Wisdom J. A perturbative treatment of motion near the 3/1 commensurability // Icarus. 1985. 63. 272-289.

[4] Neishtadt A.I. Jumps in the adiabatic invariant on crossing the separatrix and the origin of the 3:1 Kirkwood gap // Sov. Phys. Dokl. 1987. 32. 571-573.

Author: C. EFTHYMIOPOULOS
Research Center for Astronomy and Applied Mathematics (Greece)
Title: *"Secular dynamics in the Earth's Geostationary domain"*

Abstract. The talk will provide a review of our recent work, in collaboration with F. Gachet, A. Celletti and G. Puccaco, on the problem of the secular dynamics in the Earth's geostationary orbital domain. We implemented normal form analytical approximations to an original system of 8 degrees of freedom, including the Earth's J2 and J22, lunisolar gravitational perturbations and the solar radiation pressure. The main outcome of our analytical approach is a precise determination of the forced equilibrium (i.e. forced eccentricity and invariant Laplace plane position), which appears as an equilibrium point of a suitably defined secular Hamiltonian. The role of resonances and corresponding small divisors is analyzed. Finally, we discuss the applicability of proper elements for test particles in the GEO domain, as well as the latter's utility in the characterization of various populations of GEO space debris.

Joint work with F. Gachet, A. Celletti and G. Pucacco.

Author: L. FAGGIOLI
SpaceDyS for ESA - NEOCC (Italy)
Title: *"A new list of Near Earth Asteroids with detected Yarkovsky effect"*
Abstract. When an asteroid orbit is affected by non-gravitation perturbations, non-gravitational forces need to be taken into account since they are as important as collisions and gravitational perturbations for the overall understanding of the asteroid orbital evolution.

The Yarkovsky effect is a non-gravitational phenomenon related to the anisotropic thermal emission of solar system objects. This perturbation can be modeled knowing the physical properties of asteroids, and its consequences on the motions can be measured from accurate astrometry.

The knowledge of the physical properties of asteroids is usually not sufficient to produce the thermophysical models needed for the computation of the Yarkovsky acceleration. Nevertheless, it can often be measured as a semimajor axis drift if the astrometric dataset contains extremely accurate observations (e.g. radar data), or if the observations span a sufficiently long time interval.

Farnocchia et al. 2013 list 21 NEAs with a measurable semimajor-axis drift. Since 2013, the number of asteroids for which it is possible to detect the Yarkovsky effect has grown. This is due to the increased quality and time span of the observations, and to new radar measurements that have since become available. We analyze about 400 objects chosen by a suitable criteria involving the semi-major axisaccuracy, among which we are able to detect the Yarkovsky effect for about 50 NEAs, employing a high precision dynamical model, including the Newtonian attraction of 16 massive asteroids and the planetary relativistic terms, and a suitable astrometric data treatment. We present a list of objects with a significant detection of Yarkovksy effect and a value compatible with the Yarkovsky mechanism. Moreover, we perform an in depth investigation about 10 cases with a dubious detected Yarkovsky effect.

The computed non-gravitational perturbations have been added to the web portal of the ESA SSA-NEO Coordination Centre, highlighting the fact that the orbit has been computed taking the Yarkovsky effect into account. The inclusion of non-gravitational perturbations can also affect the results of the impact monitoring, as in the case of (410777) 2009 FD, (29075) 1950 DA, (99942) Apophis and (101555) Bennu.
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Joint work with Alessio Del Vigna (3)(1), Andrea Milani (3)(1), Federica Spoto (4), Giovanni B. Valsecchi (5)

Author: E. FANTINO
`Space Studies Institute of Catalunya (Spain)`
Title: *"Low-energy tour of the Galilean moons"*

Abstract. Motivated by current interest in the exploration of Jupiter's system, we designed a tour of Europa, Ganymede and Callisto based on the circular restricted three-body problem and Keplerian orbits. The former model is used to generate the initial conditions in the neighborhood of Lyapunov orbits of the collinear libration points of the several Jupiter-moon-spacecraft systems: these initial conditions may belong either to the invariant manifolds or to transit trajectories of the Lyapunov orbits. The latter type guarantees lower propellant cost, shorter flight times and an easier generation of the observation orbits around the moons. Out of the sphere of influence of the secondary, the three-body trajectories can be approximated by arcs of Keplerian ellipses with one focus at Jupiter. The intersection between elliptical arcs originating at two consecutive moons can be computed analytically. The difference in velocity at the intersection constitutes the cost of an individual moon-to-moon transfer. Exploring the intersections over a wide range of energies and at all the relative orientations would still be time consuming were it not for the important finding that the cheapest intersection between two elliptical orbits occurs when they are mutually tangent. This corresponds to a specific planetary configuration and remarkably reduces the search space. The link between consecutive moon-to-moon transfers in a tour that starts at Europa, visits Ganymede, continues to Callisto, returns to Ganymede and ends up at Europa is a problem of time matching or orbit phasing. The issue has been faced by designing the observation orbits around the intermediate moons (Ganymede and Callisto) in such a way as to add time flexibility to the trajectory thus ensuring the required phasing. These so-called re-phasing orbits increase the propellant consumption of the mission in the form of impulsive maneuvers of 100 m/s each. The complete tour takes 75 days and requires 4.5km/s of Δv. The replacement of the impulsive maneuvers with low-thrust arcs makes the tour affordable, although the time required for its execution increases considerably. We present these findings and we compare them with the open literature.

Joint work with A. Viale (Padua Univ., Italy), R. Castelli (Vrije Univ. Amsterdam, The Netherlands), K.C. Howell (Purdue Univ., USA).

Author: M. FENUCCI
Università di Pisa (Italy)
Title: *"On the linear stability of some periodic orbits of the N-body problem with the symmetry of platonic polyhedra"*

Abstract. In the last few years many interesting periodic orbits of the classical Newtonian N-body problem have been discovered as minimizers of the Lagrangian action functional, on a particular subset of the T-periodic loops in H^1. The interest in this classical problem was revived by the numerical discovery of the now famous figure-eight solution of the three body problem, by C. Moore in 1993. In 2000 A. Chenciner and R. Montgomery rediscovered this particular orbit, giving a formal proof of its existence using the direct method of calculus of variations. The figure-eight is a first example of a N-body choreography, that is a solution in which N equal masses chase each other around a fixed closed curve, equally spaced in phase. Moreover, T. Kapela and C. Simó (2007) and G. Roberts (2007) independently proved the linear stability of such orbit, using numerical methods.

In this presentation we want to study the linear stability of periodic orbits with the symmetry of the Platonic polyhedra, in which N coincides with the order of the rotation group of the polyhedra. The particles have all the same mass and the orbits found are invariant under the rotations of the group. The existence of such orbits is given by G. Fusco, G. F. Gronchi and P. Negrini (2011), still using variational methods. In the first part we see how to set up an algorithm in order to find all the different sets on which we are able to state the existence of a minimizer of the action functional. This algorithm gives us a list of periodic orbits that we have to examine. In the second part we describe a numerical method (proposed by C. Moore and M. Nauenberg) that permits us to obtain an approximation of the motion. In the last step, we describe how we can correct such approximation using a shooting method, and how we can state the linear stability or instability of our orbits. However, we will see that these orbits are all unstable.

Joint work G.F. Gronchi.

Author: S. FERRAZ-MELLO
University of São Paulo (Brazil)
Title: *"Short-period librations due to tides in planetary satellites "*
Abstract. The application of the creep tide theory to the rotation of close-in satellites and exoplanets shows a variety of behaviors with two extreme cases: gaseous planets with fast relaxation (low viscosity) and satellites and Earth-like planets with slow relaxation (high viscosity). In the simplest Darwinian regime of gaseous bodies, the rotation tends to a stationary rotation a bit faster than the orbital motion. The excess of angular velocity is $\sim 6ne^2$. In the case of close-in planetary satellites and Earth-like planets, the rotation is damped to attractors with periods nearly commensurable with the orbital period, but the final solutions are not stationary. They are forced oscillations (the so-called physical librations) around one attractor. The use of averaged models in the study of the spin-orbit dynamics of planetary satellites and other stiff bodies is not appropriate because these motions are strongly dominated by a short-periodic oscillation. We discuss the consequences of these librations in the evolution of the systems and apply the new theory to the study of the rotation of some Saturnian satellites whose oscillations were determined from Cassini's observations.

Joint work with H.A. Folonier, Universidade de São Paulo.

References

Ferraz-Mello, S., Tidal synchronization of close-in satellites and exoplanets. A rheophysical approach, Cel. Mech. Dyn. Astr. 116:109 (2013).

Ferraz-Mello, S., Tidal synchronization of close-in satellites and exoplanets: II. Spin dynamics and extension to Mercury and exoplanet host stars, Cel. Mech. Dyn. Astr. 122:359 (2015).

Folonier, H.A., Tide on differentiated planetary satellites. Application to Titan. PhD Thesis, Univ. Sao Paulo (2016).

Author: H.A. FOLONIER
University of São Paulo (Brazil)
Title: *"Titan's Synchronous Rotation"*

Abstract. The Cassini radar observation of Titan over several years show that the rotation is slightly faster than the synchronous motion (Lorenz et al. 2008; Stiles et al. 2008 and 2010; Meriggiola 2016). The seasonal variation in the mean and zonal wind speed and direction in Titan's lower troposphere causes the exchange of a substantial amount of angular momentum between the surface and the atmosphere (Tokano and Neubauer, 2005; Richard et al. 2014). The rotation variation is affected by the influence of the atmosphere when we assume that Titan is a differentiated body and the atmosphere interacts only with the outer layer.

In this work, we calculate "variations of Titan's rotation" when the body is formed by two independent rotating parts and assuming that friction occurs at the interface of them. The tides are considered using the extension of the Ferraz-Mello's creep tide theory (Ferraz-Mello, 2013 and Ferraz-Mello, 2015) to the case of one body formed by two homogeneous parts.

Joint work with S.Ferraz-Mello, Universidade de São Paulo.

Author: A. FORTUNATI
University of Bristol (United Kingdom)
Title: *"Normal forms with negligible small divisors for a class of non-autonomous Hamiltonians"*

Abstract. The aim of the talk is to illustrate the ideas behind the theory of normal forms for nearly-integrable non-autonomous Hamiltonian systems with a real-analytic perturbation which is summable in time. It will be shown how, in the paradigmatic case of time decaying perturbations, the small divisors do not affect the well-posedness of the (ad-hoc) perturbative scheme and its "quantitative" arguments. The perpetual stability for the class of systems at hand, is obtained as a corollary of the normal form result.

From a joint work with S. Wiggins.

Author: J. GALAN VIOQUE
Universidad de Sevilla (Spain)

Title: *"Global continuation of symmetric orbits in the Sitnikov problem"*

Abstract. In [1] Llibre and Ortega studied analytically making use of the global continuation theorem the families of symmetric periodic orbits of the elliptic Sitnikov problem for non necessarily small values of the eccentricity e, and showed that some periodic orbits for e = 0 can be continued to all values of e in [0, 1). In [2] Ortega and Rivera analyzed the bifurcation of solution from the center of mass which is an equilibrium of the problem. There are also numerical studies by Belbruno et al [3] and Jiménez-Lara and Escalona Buendía [4] describing the families of periodic orbits for almost all values of the eccentricity.

In this work we concentrate on the **stability** and **bifurcation** behavior of the families of symmetric periodic orbits that are born at the circular problem or emanate form the equilibrium solution and provide complementary information to the existing results. We present a combination of analytical estimates of the eccentricity intervals of ellipticity and numerical results based on a continuation technique [5,6].

References

[1] J. Llibre and R. Ortega, On the families of periodic orbits of the Sitnikov problem, SIAM J. Applied Dynamical Systems., 7 (2008), 561-576.

[2] R. Ortega and A. Rivera, Global bifurcations from the center of mass in the Sitnikov problem. DCDS, Series B 14, 719-732 (2010).

[3] E. Belbruno, J. Llibre, and M. Ollé, On the families of periodic orbits which bifurcate from the circular Sitnikov motions, Celestial Mech., 60 (1994), 99-129.

[4] L. Jiménez-Lara and A. Escalona-Buendía, Symmetries and bifurcations in the Sitnikov problem. Celestial Mech. Dynam. Astronom., 79 (2001), 97-117.

[5] Muñoz-Almaraz, F.-J., E. Freire, J. Galán, E. Doedel and A. Vanderbauwhede (2003) Continuation of periodic orbits in conservative and Hamiltonian systems. Physica D 181, 138.

[6] Galán, J., Muñoz-Almaraz, F. J., Freire, E., Doedel, E. and Vanderbauwhede, A.: 2002, Stability and bifurcations of the figure-8 solution of the three-body problem, Physical Review Letters 88, 241101-4.

Author: C. GALES

`Al. I. Cuza University of Iasi (Romania)`

Title: *"Resonance effects within LEO, MEO and GEO regions"*

Abstract. Due to the large number of debris, produced during the human space activity, the circumterrestrial space became unsafe. The impact of operative spacecraft or satellites with large enough space debris could result in a disastrous situation; the accumulation of debris in specific regions of the sky - where most of operative satellites are positioned - cannot be neglected anymore.

Space agencies have seriously considered the extent of the danger and started space situational awareness programs to investigate all sources of hazard both for Earth and its orbiting environments. Among these programs, monitoring and cataloguing the space debris population is now an ongoing activity with an ever enriching database.

On the theoretical side, it is of seminal importance to understand the global dynamics of this population in each specific region, LEO, MEO and GEO. The study of dynamics might provide practical solutions in the assessment of space debris mitigation measures and in the development of maintenance and control strategies.

In this talk, we describe several recent results describing the dynamics of resonances within LEO, MEO and GEO regions. Two types of resonance affect the motion of space debris, namely tesseral resonances, which occur when there is a commensurability between the Earth's rotation period and the orbital period of the space debris, and lunisolar resonances, which involve commensurabilities among the slow frequencies of orbital precession of the debris and the perturbing body. Tesseral resonances provoke variations of the semi-major axis on a time scale of the order of hundreds of days, while lunisolar resonances influence the evolution of the eccentricity and inclination on a much longer time scale, of the order of tens (or hundreds) of years.

By using both analytical and numerical tools, we provide a description of the main dynamical features of the two types of resonance, highlighting the phenomena occurring in each region of the sky. In particular, in the LEO region we discuss the interplay between the conservative and dissipative effects and we show that the orbital decay, caused by the air drag, can be balanced by resonant effects. In MEO and GEO, we show the existence of various dynamical phenomena such as splitting and overlapping of resonances, chaotic variations of the orbital elements, bifurcations, variations of the amplitude of resonances with respect to various parameters etc. We discuss the importance of these phenomena in the evaluation of the long-term evolution of the orbital elements, in particular the semi-major axis, eccentricity and inclination.

This talk refers to several works in collaboration with Alessandra Celletti.

Author: M. GIDEA
Yeshiva University, New York (USA)
Title: *"Arnold Diffusion in the Three- and Four-Body Problem"*

Abstract. One possible formulation of the Arnold diffusion problem is that arbitrarily small, time-dependent, generic perturbations of autonomous Hamiltonian systems yield trajectories along which the energy grows by some quantity that is independent of the size of the perturbation, as well as trajectories for which the energy performs chaotic excursions. Verifying diffusion in concrete systems, where the perturbation is explicit, is a challenging problem. We present several models from celestial mechanics in which we prove diffusion rigorously, and construct algorithmically diffusing trajectories. Moreover, we obtain trajectories with optimal diffusion speed, and provide estimates on the Hausdorff dimension of the set of initial conditions that undergo diffusion. The results are obtained via analytical arguments and rigorous numerical computations. The motivation resides with the dynamics of Jupiter's comet Oterma, of the Neptune-Triton system, and of Jupiter's Trojan asteroids.

This is based on joint works with Jaime Burgos, Maciej Capinski and Rafael de la Llave.

Author: I. GKOLIAS
Politecnico di Milano (Italy)
Title: *"Exploiting the dynamics near GEO for optimal end-of-life disposal manoeuvres"*

Abstract. The services provided by satellites in geosyncrhonous orbits are valuable in everyday life. However, the human space activity has made the geostationary ring densely populated. Apart from the operational satellites, the pieces of defunct spacecraft, upper stages and other smaller space debris add up to approximately 1200 catalogued objects. The future exploitation of the region heavily depends on the measures we will take to keep it clean. Moreover, action needs to be taken since there is no natural cleansing mechanism at this altitude. To this end, ESA requirements suggest that space systems operating in the Geostationary (GEO) protected zone should be disposed into graveyard orbits with less than 0.005 eccentricity and a minimum perigee altitude above the geostationary altitude.

An important role in applying these guidelines plays the understanding of the natural dynamics that govern the motion of satellites, not only at the exact geostationary altitude but also a few hundred kilometres above and below. In this region, the perturbations from the Earth's gravity field, the third body interactions with the Sun and the Moon and the solar radiation pressure (SRP) all become relevant to a higher degree. In this work, both high-fidelity, semi-analytical propagation tools and linearised methods are used to study the long-term evolution of the orbits near the GEO ring. In order to identify possible regions of the phase space that can serve as graveyard orbits, we compute dynamical stability maps, frequency maps and study their properties. We particularly focus on understanding the interplay between the tesseral, lunisolar and solar radiation pressure induced resonances in order to rigorously justify our findings. Finally, we exploit the computed maps to design fuel efficient manoeuvres to stable graveyard orbits that have minimum intersection with the GEO protected region.

Joint work with Camilla Colombo.

Author: M. GUZZO
Università degli Studi di Padova (Italy)
Title: *"Computation of the hypertube manifolds in the spatial circular restricted three-body problem with chaos indicators"*

Abstract. The stable and unstable manifolds of the center manifolds originating at the Lagrangian equilibrium points L1 and L2 in the spatial circular restricted three-body problem are hypertubes that can be computed with chaos indicators suitably designed for this purpose. The definition of the chaos indicators is not trivial, since the mandatory use of the regularizing Kustaanheimo-Stiefel variables introduces discontinuities in the well known Fast Lyapunov Indicators. We illustrate the method for the Sun-Jupiter mass ratio and represent the topology of the asymptotic manifolds using sections and three-dimensional representations.

Author: A. HARO
University of Barcelona (Spain)
Title: *"A posteriori KAM theorems for systems with first integrals in involution"*
Abstract. Some relevant Hamiltonian systems in Celestial Mechanics have first integrals in involution. A classic technique to study such systems, known as symplectic reduction, is based in reducing the number of degrees of freedom by using the first integrals. In this talk we present two a posteriori KAM theorems for Hamiltonian systems with first integrals in involution, including the isoenergetic case, without using symplectic reduction. The approach leads to efficient numerical methods and validating techniques.

This is a joint work with Alejandro Luque.

Author: K. HOWELL
School of Aeronautics and Astronautics, Purdue University (USA)
Title: *"Orbital Infrastructures to Support Space Exploration"*
Abstract. Innovative trajectory concepts have enabled and enhanced a significant number of recent space missions, including robotic science missions, both Earth-focused and interplanetary, as well as the new directions for the human space flight program. The next step is the evolution of a sustainable space-based infrastructure in cis-lunar space to support all such activities. The orbital network is a significant component in creating an environment for a permanent human presence that also enables missions to various translunar destinations. Yet, significant challenges remain to deliver wide-ranging orbit options and to leverage the latest computational capabilities.

Author: Á. JORBA
Universitat de Barcelona (Spain)
Title: *"On the computation of high order power expansions of Poincaré maps"*
Abstract. In this talk we will explain a computational technique (called jet transport) to compute high order derivatives of Poincaré maps with respect to initial data and/or parameters. The method is based on the use of automatic differentiation techniques on the propagation of the flow of the ODE. If the flow propagation is done by means of a Taylor method, the resulting procedure is efficient enough to be carried out even with extended precision arithmetic. As applications, we mention the effective computation of normal forms, centre manifolds and stable/unstable manifolds of periodic orbits. In the talk we will discuss the effective computation of 1D and 2D stable/unstable manifolds of periodic orbits on concrete examples.

This presentation summarises collaborations with A. Farrés, G. Gimeno, M. Jorba-Cuscó, N. Miguel and M. Zou.

Author: M. JORBA-CUSCÓ
Universitat de Barcelona (Spain)
Title: *"Computation of the Centre Manifold of a Solar Sail Periodic Orbit"*
Abstract. A Solar sail is a spacecraft endowed with a large and highly reflecting surface to take advantage of the solar radiation pressure to propel the spacecraft.

In this work we focus on the motion of a solar sail in the Earth-Moon system. The model used is a coherent version of the Bicircular Problem extended to include the effect of the Solar Radiation Pressure on the sail. This model can be regarded as a periodic time-dependent perturbation of the well-known Restricted Three Body Problem. This system can be written in Hamiltonian form as a three and a half degrees of freedom. Thus, the classical Lagrangian points are no longer equilibria but they are replaced by periodic orbits with the same period as the time-dependence. The model has three parameters: one of them describes the performance of the sail and the other two describe its orientation. This leads to a three-parametric family of periodic orbits that raise from each Lagrangian point.

We focus in a periodic orbit near the point L_2. Then we perform a partial normal form process to the Hamiltonian function to obtain a new autonomous Hamiltonian that has uncoupled elliptic and hyperbolic parts. Therefore, we can reduce the study of the motion near the periodic orbit to the study of a two degrees of freedom autonomous Hamiltonian system and, by fixing some section and energy level, to the study of a family of Area Preserving Maps.

Joint work with Ariadna Farrés and Ángel Jorba.

Author: T. KAPELA
Jagiellonian University (Poland)
Title: *"Computer assisted proof of the KAM stability of the Eight"*
Abstract. We set up a methodology for computer assisted proofs of the existence and the KAM stability of an arbitrary periodic orbit for Hamiltonian systems. As an example of application in the 3-body problem we prove the KAM stability of the well-known figure eight orbit and two selected orbits of the so called family of rotating Eights.

Author: K.V. KHOLSHEVNIKOV
St. Petersburg State University (Russia)
Title: *"Metric Spaces of Keplerian Orbits"*
Abstract. Different questions in Celestial Mechanics lead us to study the structure of the space H of Keplerian orbits and several its subspaces H_s. First attempt to bring metrics in one of H_s , say H_1 (H without rectilinear, circular, and coplanar orbits passing in the opposite directions), was made by Southworth and Hawkins in 1963. Results were rather useful for the astronomical practice, but incorrect from the theoretical point of view. The corresponding "distance" D (named "criterion" by the authors) did not satisfy the triangle axiom. Hence the pair (H_1, D) represents a pseudo-metric space only. In 1970 Moser (and almost simultaneously Stiefel and Scheifele) proved a brilliant theorem showing that the existence of singularities in any system of elements for elliptic orbits is a direct consequence of the topology of the space H_2 of elliptic Keplerian orbits with fixed negative energy. This space has a topological structure of the product of 2-dimensional spheres. Later different authors made several modifications of D, but they all turned out to be pseudo-metric. In 2004 Kholshevnikov and Vassiliev introduced an irreproachable metric in the space H_3 of elliptic orbits. But it had a serious drawback: parabolic and hyperbolic orbits were excluded. So it was impossible to use it when comparing orbits of comets and meteoroid streams. In 2010 Maruskin proposed a Riemannian metric in H_3 regarding it as a 5-dimensional surface embedded in R^6. It suffered the same drawback. In 2008 Kholshevnikov proposed a metric in the space H_4 of all Keplerian orbits, and another one in the space H_5 of non-rectilinear Keplerian orbits. In 2015 Kholshevnikov introduced a more friendly metric in H_5, and pseudo-metrics in three factor-spaces of H_5 , where we neglect nodes, pericenters, or both nodes and pericenters. In 2017 Milanov proved that these pseudo-metrics satisfy all axioms of metric spaces. Here we describe properties of these metrics.

Author: Z. KNEŽEVIĆ
Serbian Academy of Sciences and Arts (Serbia)
Title: *"Errors of analytical asteroid proper elements"*

Abstract. The asteroid proper elements computed from an analytical secular perturbation theory suffer from two important drawbacks: the availability only for objects with low to moderate eccentricity/inclination, and lack of the estimate of their uncertainty. The former problem may not be very important because asteroids in general have orbits of fairly low eccentricity and inclination, and because other theories have been developed which can handle the high eccentricity/inclination cases.

Regarding the latter problem, accuracy estimates exist for only a very small set of asteroids, representative of the main families. The typical uncertainty of analytical proper elements was found to be on the order of 0.002-0.003 in proper eccentricity, and 0.001-0.002 in proper inclination, while the corresponding instabilities of the proper semi-major axis were typically one or two orders of magnitude smaller. These uncertainties are by a factor of 3 to 4 larger than the corresponding values for the so-called synthetic proper elements, computed by means of a set of purely numerical procedures.

The computation of analytical proper elements has been carried out for more than 25 years as part of the regular maintenance of the catalog available via the AstDyS service. However, if indeed they are of so much poorer accuracy with respect to synthetic proper elements, as indicated on the basis of the above mentioned limited tests, and possibly insufficient for the more and more demanding family classification applications, then their maintenance can safely be discontinued in the future.

For the purpose of determination of the accuracy of analytical proper elements we carried out two different tests, one that exploits the plain comparison of the analytical and synthetic values, with the aim to determine whether they differ more then the corresponding error bars of synthetic elements permit, and the other consisting in repeating the direct determination of instabilities of analytical elements themselves, but on a much larger sample of asteroids.

Here we present some results of the comparison of analytical and synthetic proper elements, obtained for a sample of 10 000 asteroids, but indicative of the method and of the results to be expected for the entire proper elements catalog with about 400 000 entries, as well as some preliminary results of the direct determination of instabilities with the larger asteroid sample.

Joint work with A. Milani.

Author: I. KOSENKO

National Research University (Russia)

Title: *"Motion of a satellite with a variable mass distribution in a central field of Newtonian attraction"*

Abstract. Dynamics of a spacecraft with a variable mass distribution in a central field of Newtonian attraction is considered. Within the so-called "satellite approximation" the equations of spatial attitude motion are derived, in particular, from the Hamilton variational principle. The satellite multibody system mass center performs elliptic motion of arbitrary fixed eccentricity. Special rules of the mass redistribution providing prescribed in advance attitude motions are indicated. For detected classes of relative equilibria, existing under appropriate rules of the mass redistribution, necessary conditions of stability are investigated. Special attention is paid to investigation of stability for spatial motions of the attitude satellite dynamics. The obtained results are illustrated within Magnus' approach, presuming utilisation of moments of inertia as barycentric coordinates. There exist at least three observations related to dynamics of spacecrafts with a variable mass distribution. (a) First of all, there are effects related to "natural" mass redistribution due to crew motions, liquid sloshing, etc. These effects, and in particular, methods of compensation of appearing objectionable attitude motions were studied in numerous publications. (b) Another point relates to possibilities of using spacecraft's mass redistribution to provide propulsive motion of its center of mass. This idea belonging to Beletsky and Giverts was developed by Donov and was a subject of different discussions. (c) Later related problems were intensively studied, in particular, in frame of the satellite multibody system dynamics attitude motion.

Joint work with Prof. Alexander Burov and Prof. Anna Guerman.

Author: E. KUZNETSOV
Ural Federal University (Russia)
Title: *"Lidov-Kozai effect: flips and collisions"*

Abstract. Several variants of Lidov-Kozai effect are considered for the Sun–Earth–Satellite problem. Some of them cannot be described by the standard integrable model. Different cases of dynamical evolution (including flips and collisions with the Earth) occur depending on initial positions of the perturbing body. So, the method of double averaging is incorrect for these cases.

Joint work with Leonid Sokolov.

Author: G. LARI
Università di Pisa (Italy)
Title: *"The contribution of JUICE data to the determination of the Jupiter system's energy dissipation"*

Abstract. The Galilean satellites of Jupiter are some of the most fascinating celestial bodies of the Solar System, not only for their composition and internal structure, but also for their dynamics. It is well known that the three inner Galilean satellites are locked in a three-body resonance, called Laplace resonance. Moreover, the tides between Jupiter and Io dissipate a huge quantity of energy, resulting in an acceleration along the satellite's orbit. Because of the Laplace resonance, also Europa and Ganymede suffer the dissipation effects. The amount of this energy dissipation is yet a matter of discussion in the scientific community and it can result in different evolutions of the satellites' resonant state. JUICE is an ESA space mission that will perform several flybys of the Galilean moons and it will end with a nine months orbiting phase around Ganymede. The mission will provide different kinds of data (range, range-rate, vlbi and camera observations) that will allow to obtain informations about satellites' geophysical properties and to improve the knowledge of their dynamics. With ORBIT14 software, an orbit determination code developed by the Celestial Mechanics Group of the University of Pisa, we simulated these observables and we performed a least squares fit in order to solve for all the dynamical parameters we chose to determine. In particular, in this presentation, we will discuss the results we obtained for the formal uncertainties of the dissipative coefficients and possible strategies to improve them. Moreover, we will compare them with the values present in the literature.

Joint work with Andrea Milani.

Author: J. LASKAR
IMCCE, Observatoire de Paris (France)
Title: *"AMD-Stability of planetary systems"*

Abstract. In a planetary system, the AMD (Angular Momentum Deficit) is the difference between the planar circular angular momentum and the total angular momentum. This quantity is conserved between collisions in the average system, and decreases during collisions. It was introduced to understand the long-term evolution of the solar system (Laskar, 1997), and shows how the process of accretion of planetesimals can give rise to distributions of the planets that are functions of the initial distribution of matter in the disk (Laskar, 2000).

This leads to the concept of AMD-stability. A planetary system is AMD-stable if the AMD in the system is not sufficient to allow collisions. The advantage of this notion is that it becomes possible to verify very quickly whether a newly discovered planetary system is stable or potentially unstable, without any numerical integration of the equations of motion. These principles have been applied to the 131 multiple planetary systems of the exoplanet.eu database whose orbital elements are sufficiently well determined (Laskar and Petit, 2017a).

AMD-stability, based on the secular evolution, addresses to long time stability, in absence of mean motion resonances. On the other hand, criterions for short term stability have been established on the basis of Hill radius (Marchal & Bozis 1982; Gladman 1993; Chambers et al. 1996; Smith & Lissauer 2009; Pu & Wu 2015) or on the overlap of mean motion resonances (Wisdom 1980; Duncan et al. 1989; Mustill & Wyatt 2012; Deck et al. 2013). Both long and short time scales can be in fact combined owing some modification of the AMD-stability criterion (Petit, Laskar & Boué, 2017).

References

Laskar, J. and Petit, A.C., 2017, AMD-stability and the classification of planetary systems, A & A, in press.

Petit, A.C., Laskar, J. and Boué, G., 2017, AMD-stability in presence of first order mean motion resonances, A & A, submitted; https://arxiv.org/abs/1705.06756

Author: C. LHOTKA
Space Research Institute (Austria)
Title: *"Stable Cassini states of a rigid body in the $p:q$ spin-orbit resonant problem. Application to Mercury"*

Abstract. We investigate generalized stable Cassini states in the spin-orbit resonant problem. Our study includes the study of the effect of the gravitational potential up to degree and order 4, and $p:q$ spin-orbit resonances with p, $q \leq 8$ and $p \geq q$. We derive new formulae that link the gravitational field coefficients with its secular orbital elements, and its rotational parameters. The formulae can be used to predict the orientation of the spin-axis and necessary angular momentum at exact resonance. We also develop a simple pendulum model to approximate the dynamics close to resonance and make use of it to predict the libration periods and widths of the oscillatory regime of motions in phase space. Our analytical results are based on averaging theory that we also confirm by means of numerical simulations of the exact dynamical equations. Our results are applied to the case of planet Mercury.

Author: A.-S. LIBERT
University of Namur (Belgium)
Title: *"Dynamical mechanisms during the formation of non-coplanar planetary systems"*

Abstract. The orbits of extrasolar planets are more various than the circular and coplanar ones of the Solar system. Evidence of mutually inclined orbits has been reported for detected giant planets these last years. Several dynamical mechanisms leading to an excitation of the orbital inclinations during the late-stage formation of planetary systems are studied in detail here, in particular the inclination-type resonance, the nodal resonance and the Kozai resonance, for two different stages of the formation process.

Firstly, by use of extensive n-body simulations, we follow the evolution of three giant planets in the late stage of the gas disc, investigating the gravitational interactions among the planets during the migration phase. Our simulations, which take into account the Type-II migration, the damping of planetary eccentricity and inclination, and an exponential decrease of the disc mass, reproduce the semi-major axis and eccentricity distributions of the detected giant planets. We show that, starting from quasi-circular and quasi-coplanar orbits, highly mutually inclined systems can form, despite the strong eccentricity and inclination damping, due to planet-planet scattering and/or resonant phenomena. Particular attention is given on the different inclination-growth mechanisms at small to moderate eccentricities, guided by the computation of vertical critical orbits and the bifurcation of families of spatial periodic orbits.

Secondly, we study the impact of inclined massive giant planets on the terrestrial planet formation process. In particular, we follow the gravitational interactions of mutually inclined giant planetary systems with an inner disc of planetesimals and embryos, studying the physical and orbital parameters of the formed terrestrial planets, as well as their water content. We show that terrestrial planets can form on stable inclined orbits through the classical accretion theory, even in coplanar giant planet systems emerging from the disc phase.

Joint work with S. Sotiriadis (University of Namur).

Author: A. LUQUE
Instituto de Ciencias Matemáticas - Madrid (Spain)
Title: *"A-posteriori rigorous validation of KAM tori"*

Abstract. In this talk we present a methodology to rigorously validate a given approximation of a quasi-periodic Lagrangia torus of an exact symplectic map. That is, we check the hypothesis of an a-posteriori KAM theorem and we prove the existence of a true invariant torus nearby. Our method is sustained in the a-posteriori KAM formulation developed in the last decade by R. de la Llave and collaborators.

To check the hypotheses of the theorem, we use rigorous fast Fourier transform in combination with a sharp control of the discretization error. An important consequence is that the rigorous computations are performed in a very fast way. Indeed, with the same asymptotic cost of using the parameterization method to obtain numerical approximations of invariant tori.

We will discuss the application of the method to the standard map and the Froeschlé map.

Author: S. MARÒ
Instituto de Ciencias Matemáticas - Madrid (Spain)

Title: *"Long term dynamics in mean motion resonances with crossing singularities"*

Abstract. We consider the long term dynamics of the restricted N body problem, modeling the dynamics of an asteroid moving in the gravitational field of the Solar system. In particular we deal with the case of a mean motion resonance with a planet. The asteroid evolution is computed by averaging over the fast angles using standard techniques of perturbation theory. We focus on the critical case where the trajectory of the asteroid crosses the one of a planet during the evolution. This produces a singularity in the averaged equations of motion. We prove that the averaged vector field can be extended to two Lipschitz continuous vector fields in a neighborhood of crossing configurations and we define generalized solutions, going beyond this singularity. Moreover, we prove that the orbit distance between the asteroid and a planet is differentiable also in case of crossings. Some applications will be given considering mean motion resonances with Jupiter and crossing with the Earth. This work extends the results in (Gronchi-Tardioli 2013) to the resonant case.

Joint work with G. Gronchi.

Author: A. MILANI
Università di Pisa (Italy)
Title: *"Towards a chronology of the asteroid main belt collisional history"*

Abstract. A recent development in the field of proper elements and asteroid families has been the computation of very large datasets of proper elements, for more than 500 000 asteroids, and the derivation of correspondingly large asteroid families classifications. E.g., we have proposed 125 families with more than 123 000 members, and this classification is semiautomatically updated when new data are available.

An achievement made possible by these bigdata classifications has been the computation of ages for more than 50 families, with a consistent method based on the Yarkovsky effect signature in the family distribution.

Because the new families contain much smaller asteroids than the previous ones, many of the families turned out to be of the cratering type, that is leaving a substantial parent body and with many small fragments. The question is what characterizes the families of this type, for which we propose a quantitative definition. Among our findings, there are surprisingly many cratering families with two separate ages, thus resulting from two collisional events. We have also found many new cases of cratering families, previously ignored, and we list a number of dubious cases. We include results on cratering families obtained by using resonant proper elements, including the confirmation of the Astraea family and peculiar properties of some families in the Trojan swarms.

The ages have been computed with a rigorous procedure, allowing to give a formal uncertainty to the age estimations. However, these uncertainties are still quite large. We propose in this paper the procedures which can be used to improve these estimates. We also discuss the possibility of estimating the original dispersion of velocities immediately after the creation of the fragments and their escape from the gravitational well of the parent body.

Joint work with Federica Spoto and Zoran Knežević.

Author: I. MILIC ZITNIK
`Astronomical Observatory - Belgrade (Serbia)`
Title: *"Impact of Yarkovsky Effect and Mean-Motion Resonances on Main Belt Asteroid's Transport"*

Abstract. The long-term dynamics of main belt asteroids is governed by complex interaction among the gravitational and non-gravitational phenomena. A detailed study about this interaction has not been performed yet. The most important gravitational mechanisms are orbital resonances and the most important non-gravitational effects are the Yarkovsky and the Yarkovsky-O'Keefe-Radzievskii-Paddack forces. This motivated us to study the effect of different mean-motion resonances (MMRs) on the mobility of an asteroid's semimajor axis due to the Yarkovsky effect. We established our findings about the effect of 11 two-body MMRs with Jupiter, on the mobility of an asteroid's semimajor axis caused by the Yarkovsky effect. This study was accomplished using numerical integrations of test particles. The obtained results revealed that MMRs could either speed up or slow down the drift in the semimajor axis. Moreover, this allowed us to determine the distribution that represents the best data obtained for time delays, dtr, caused by the resonances on the mobility of an asteroid. We found a certain functional relationship that describes dependence of the average time lead/lag <dtr> on the strength of the resonance, SR, and the semimajor axis drift speed, da/dt. As the Yarkovsky effect scales as 1/D, an important consequence of this relationship was that average time lead/lag <dtr> is directly proportional to the diameter D of an asteroid. Also, we analyzed how the time spent inside the resonance depends on orbital eccentricity, and proposed the relation that taking this parameter into account as well.

Joint work with Bojan Novakovic.

Author: A. MORBIDELLI
Observatoire de la Côte d'Azur (France)
Title: *"Dynamical evolution of distant TNOs induced by Planet Nine"*

Abstract. The observational census of trans-Neptunian objects with semi-major axes greater than ~ 250AU exhibits unexpected orbital structure that is most readily attributed to gravitational perturbations induced by a yet-undetected, massive planet. Although the capacity of this planet to reproduce the observed clustering of distant orbits in physical space, a coherent theoretical description of the dynamical mechanisms responsible for these effects remains elusive. In this work, we characterize the dynamical processes at play, from semi-analytic grounds. We begin by considering a purely secular model of orbital evolution induced by Planet Nine, and show that it is at odds with the ensuing stability of distant objects. Instead, the long-term survival of the clustered population of long-period KBOs is enabled by a web of mean-motion resonances driven by Planet Nine. Then, by taking a compact-form approach to perturbation theory, we show that it is the secular dynamics embedded within these resonances that regulates the orbital confinement and perihelion detachment of distant Kuiper belt objects. We also consider the effects of the orbital inclination of PL9 and show a simple mechanism confining in libration the longitude of the node of distant TNOs. In light of the developed qualitative understanding of the governing dynamics,we offer an updated interpretation of the current observational dataset within the broader theoretical framework of the Planet Nine hypothesis.

Work done in collaboration with K. Batygin (GPS, Caltech).

Author: M. MURAWIECKA
University of Namur (Belgium)
Title: *"Space debris dynamics: resonances in the MEO"*

Abstract. The MEO region is the one hosting a large number of important operational satellites along with a significant population of space debris. From the dynamical point of view, the region is intersected by the tesseral $2:1$ resonance as well as the lunisolar ones, and some secondary resonances emerge. The picture becomes even more complicated if the orbits of high eccentricity are considered. It is essential to understand thoroughly the dynamics of these resonances, for the sake of development of mitigation strategies. In this work, we aim to expand the analysis of the MEO region dynamics already performed by other authors (Celletti and Gales 2014; Daquin et al. 2015, 2016) onto those secondary resonances. We study the structures appearing in the stability maps of the region by the use of the MEGNO chaos indicator, taking into account various force models (the geopotential, the Sun and Moon attraction, and the solar radiation pressure). By means of the frequency analysis, we determine the resonant periods and amplitudes and identify the secondary resonances. We formulate an analytical model of local perturbed pendulum that would explain these structures.

Joint work with A. Lemaitre.

Author: Z. OLIKARA
University of Colorado - Boulder (USA)
Title: *"Generating heteroclinic chains in a planar restricted 4-body problem"*
Abstract. Heteroclinic connections allow a spacecraft to naturally travel between libration point orbits in the Sun-Earth and Earth-Moon systems. Connections exist not only within a system but also between systems. The current work considers motion in a Sun-Earth-Moon planar restricted 4-body problem, which allows both cases to be studied using a single model. Since the transfers are not isolated, this work is founded on a 2-parameter continuation algorithm to automatically map out the full connection space within each heteroclinic family.

By expressing each computed family's boundary conditions in an appropriate coordinate set, linking planar connections is reduced to locating intersections between regions on a plane. An approximate trajectory can be constructed that follows a prescribed itinerary between the Sun-Earth and Earth-Moon libration points. Since the spacecraft is shadowing a heteroclinic connection chain (i.e., it departs each libration point orbit after a specified number of revolutions), we include a corrections procedure to refine the complete trajectory within the 4-body model.

The resulting trajectory allows a spacecraft to tour the libration points using very little fuel. Since the model is non-autonomous (the influences of the Sun, Earth, and Moon are included simultaneously), libration point orbits at different energy levels can be naturally connected. We intend to demonstrate that transfer chains including multiple switches between the Sun-Earth and Earth-Moon systems are possible.

Joint work with Daniel J. Scheeres, University of Colorado Boulder.

Author: R. ORTEGA
`Universidad de Granada (Spain)`
Title: *"A dissipative Kepler problem"*
Abstract. We consider the Kepler problem with a linear dissipative force. This system has a vector valued first integral that can be interpreted as an asymptotic Runge-Lenz vector. From this fact it is possible to describe the geometry and dynamics of the orbits. In contrast to other dissipative forces there is no circularization in the linear case.

Joint work with A. Margheri and C. Rebelo

Author: R.I. PÁEZ
Research Center for Astronomy and Applied Mathematics (Greece)
Title: *"Unveiling Nekhoroshev instability and chaotic diffusion along resonances"*
Abstract. Several types of dynamical systems appearing in Celestial Mechanics can be represented by simple models of chaotic diffusion along resonances. The clarification of the diffusion mechanisms becomes then an important question. In the present work, we will focus on the use of a technique based on normal form computations, that allows to unveil the fastest diffusing chaotic orbits, i.e. orbits undergoing a "Nekhoroshev instability". In particular, we implement a normalization process that completely erases the deformation effects over the orbits' evolution in action space. This, in turn, makes possible to visualize the chaotic diffusion in the separatrix domain of the resonant chaotic layers. Furthermore, we demonstrate that only a few terms in the remainder of the optimal normal form drive the chaotic jumps in action space. Finally, we obtain precise quantitative estimates of the diffusion rate, by implementing a Melnikov-type analysis of the dynamics induced by the above remainder terms.

Joint work with M. Guzzo and C. Efthymiopoulos.

Author: J. PALACIÁN
Universidad Pública de Navarra (Spain)
Title: *"Quasi-periodic Motions of the Spatial Three-body Problem Related to Inner Rectilinear Motions"*

Abstract. In the context of KAM theory and the spatial three-body problem, specifically in the regime where the Hamiltonian can be split as the sum of two Keplerian systems plus a small perturbation, we deal with the quasi-periodic motions of the three bodies such that the two inner particles describe near-collision orbits. More precisely the inner particles never collide, but they follow orbits that are close to bounded straight lines. These solutions fill in invariant 5-tori and the motions occur either near the axis that is perpendicular to the invariable plane or near the invariable plane. The outer particle's orbit has an eccentricity varying between zero and a value that is upper bounded by $e_2^{\max} < 1$ and lies near the invariable plane. Our approach consists in a combination of a regularisation process with the construction of the reduced spaces at different levels and the explicit determination of sets of symplectic coordinates. Moreover we use an isoenergetic theorem by Han, Li and Yi on the existence of quasi-periodic solutions for Hamiltonian systems with high-order proper degeneracy. All these elements allow us to calculate explicitly the torsions for most possible combinations that the three particles' orbits can achieve. Extensions to the N-body problem are briefly discussed.

This is a joint work with F. Sayas and P. Yanguas.

Author: A. PERMINOV
`Ural Federal University (Russia)`
Title: *"The orbital evolution of the four-planetary system Sun–Jupiter–Saturn–Uranus–Neptune on long-time scales"*

Abstract. The study of planetary systems orbital evolution is one of important problems of celestial mechanics. We present averaged semi-analytical motion theory for a planetary system with four planets. The Hamiltonian of the four-planetary problem is written in Jacobi coordinates and it is expanded into the Poisson series in orbital elements of the Poincaré second system. This system has only one angular element, that is mean longitude. It allows simplifying angular part of the series expansion. The expansion is constructed up to the second degree of the small parameter, which is the ratio of the sum of planetary masses to the mass of the star.

The averaged Hamiltonian of the four-planetary problem is constructed by means of the Hori-Deprit method. We obtain the generating function for the transformation between osculating and averaged elements, the functions for the change of variables and right-hand sides of the motion equations in averaged elements. Analytical transformations are implemented by means of the Piranha echeloned Poisson series processor.

We have applied our averaged motion theory to the investigation of orbital evolution of Solar system's giant planets. The results of numerical integration of the averaged motion equations for Sun–Jupiter–Saturn -Uranus–Neptune's system on a time interval of 10 billion years is considered. The integration is performed by Everhart method of the 15th order. The planetary motion has an almost periodic character. Eccentricities and inclinations of the planetary orbits save small values. Also, resonant properties of planetary motion are considered. The accuracy of numerical integration is given.

Joint work with Eduard Kuznetsov.

Author: A. PETIT
`IMCCE - Observatoire de Paris (France)`
Title: *"AMD-stability in presence of mean-motion resonances"*
Abstract. The AMD-stability criterion allows to discriminate between a-priori stable planetary systems and systems for which the stability is not granted and needs further investigations (Laskar 2000, Laskar & Petit 2017). AMD-stability is based on the conservation of the Angular Momentum Deficit (AMD) in the averaged system at all orders of averaging. While the AMD criterion is rigorous, the conservation of the AMD is only granted in absence of mean-motion resonances.

Here we extend the AMD-stability criterion to take into account mean-motion resonances, and more specifically the overlap of first order mean-motion resonances (MMR). If the MMR islands overlap, the system system will experience generalized chaos leading to unstability. Following the work of (Chirikov 1979; Wisdom 1980; Deck et al. 2013), we compute the phase space areas where MMR overlap. We then improve the definition of AMD-stability to take into account the short term chaos generated by MMR overlap. We analyse the outcome of this improved definition of AMD-stability on the classification proposed in (Laskar & Petit 2017) on selected multi-planet systems from the Extrasolar Planets Encyclopeadia (www.exoplanet.eu).

Joint work with Laskar, J., Boué, G.

Author: G. PICHIERRI
Observatoire de la Côte d'Azur (France)
Title: *"On the stability of resonant planetary systems of Super-Earths"*
Abstract. Super-Earths are estimated to be the most abundant exoplanets in the Galaxy. They are often present in multi-planet systems. Although current models predict that, due to their migration through the protoplanetary disk, Super-Earths should reside in a chain of mutual mean motion resonances, many observed systems are not even close to such an orbital configuration. Numerical simulations suggest that many resonant systems should eventually become unstable, but a definitive explanation for these instabilities has not been provided.

We study the stability of multi-planetary systems in chains of mutual mean motion resonances and its dependence on the mass of the planets. To obtain the initial resonant configuration, we perform numerical simulations where the planets' interaction with the protoplanetary disk results in slow convergent migration. Once the resonance configuration is obtained, we slowly deplete the gas. Finally we slowly increase the masses of the planets and obtain a critical mass above which the planetary system becomes dynamically unstable. We perform this study for different populations of planets and various first order resonances.

In the case of two planets, we have available a quasi-integrable approximation of the resonant dynamics. This allows us to calculate analytically the equilibrium eccentricities arising from the resonant interaction and from the damping caused by the disk. Subsequently we can follow the resonant dynamics as the masses increase, thus obtaining a complete (albeit approximate) analytic description of the evolution of the system.

By looking at the fundamental frequencies of the system, we obtain an explanation for the onset of the instability that destroys the resonant configuration.

Joint work with A. Morbidelli and A. Crida.

Author: E. PILAT-LOHINGER
University of Vienna (Austria)
Title: *"Habitable planets in binary stars"*

Abstract. The formation and evolution of habitable planets is of particular interest in astrophysics nowadays. Since a large fraction of low-mass stars exist in binary and multiple star systems, an important question is the possibility of planetary habitability in such stellar systems. It is well known that planets in binary stars can have two types of motion, i.e. S-types and P-types. In both cases it is important that the regions of stable motion coincide with the habitable zone, i.e. the area around a star where a planet has appropriate conditions for habitability. In this presentation we discuss how dynamical perturbations and stellar activity might influence the formation of habitable environments in tight binary stars.

(Joint work with A. Bazso, D. Bancelin and C. Johnstone)

Author: G. PINZARI
University of Padova (Italy)
Title: *"Regarding Sun-Earth-Asteroid systems as perturbed two-centre systems"*
Abstract. The Law of Universal Gravitation, according to which, any two masses in the Universe attract each other with a law going as the inverse squared distance, was stated in 1687 by Isaac Newton. Newton was aimed to find a theoretical explanation to the Kepler's Laws (1609–1619). At the same time, he provided the *exact solution* of the simplest gravitational system: the two–body problem, or: the problem of Sun and Earth. He also tried to attack the analogous problem with three masses: Sun, Earth and Moon, but then gave up, calling it a "head ache problem". At the end of the XX century, Henri Poincaré proved the *non–integrability* of the three-body problem, and this led him to formulate the concept of *chaos* in dynamics. A major breakthrough came from Kolmogorov–Arnold–Moser theory (1954, -62, -63), the main object of which was that of providing an estimate of "stable motions", from the probabilistic point of view.

Much less known as further example of *exactly solved* gravitational system is the so–called two–centre problem, solved by Euler in XVIII century. Maybe due to the difficulty of handling, at practical level, Euler's solutions, the two–centre problem has been not frequently used in the study of the three–body problem. An attempt in this direction is often attributed to Charlier. The aim of this talk is to discuss a canonical setting that allows to do that in a seemingly not too complicate way, at least in the case that the three masses are much different one from the other (Sun, Earth and an Asteroid). As a by-product, we shall show that, with our method, it is possible to characterize exactly collisions Earth-Asteroids in terms of the levels of a quasi-integral associated to it, and to prove the global existence of a plenty KAM tori even when the orbits of the asteroid encircles the Earth, a situation at risk of collisions. Moreover, the analysis highlights the possibility of Arnold diffusion.

Author: E. PITJEVA

Institute of Applied Astronomy of Russian Academy of Science (Russia)

Title: *"Kuiper Belt: its mass and influence on planetary motion"*

Abstract. The Kuiper belt consists of a large number of distant bodies beyond Neptune's orbit. The main part is located between the resonances of 3:2 and 2:1 of the mean motion with Neptune, inside a ring area with radial distance between 39.4 and 47.8 AU and inclinations $i < 5°$. Perturbation from the Kuiper belt affects the orbits of planets and should be taken into account when high-accuracy planetary ephemerides are constructed. Also, a more reliable influence of this belt on the motion of the planets will permit to decide how to search a distant major planet and how to take into account its additional gravitational influence. For this purpose, high-precision Saturn data obtained by the Cassini spacecraft are used mainly. However, the total gravitational acceleration created by the Kuiper belt for Saturn is comparable to or even exceeds the perturbations of a hypothetical remote (about 300-400 au) planet with the mass of 10 Earth mass. One-dimensional ($R = 43$ AU) and two-dimensional models ($R_1 = 39.4$, $R_2 = 47.8$ AU) of the Kuiper belt have been considered, and the difference of gravitational effect it was analyzed from them simulating the effect of Kuiper belt on motion of Saturn, Uranus, Neptune. Estimations of masses of 30 large transneptunian bodies have been obtained from motion of their satellites or from their sizes and densities. These objects were included into the simultaneous numerical integration of planets and the Moon while constructing planetary ephemerides. Estimation of the mass of the other bodies belonging to the Kuiper belt was based on the new version of the ephemerides of planet and the Moon at IAA RAS - EPM2017, where more 800 thousand positional observations of planets and their satellites, mostly radar data from spacecraft were fitted. The comparison of estimations of the total mass of the Kuiper belt obtained for two dynamical models and statistical estimates of other authors are given.

Joint work with Pitjev Nikolay.

Author: M. PONTANI
Università di Roma "La Sapienza" (Italy)
Title: *"Hyperbolic rendezvous with Earth-Mars cycling spacecraft"*
Abstract. In recent years, the scientific community has shown a strong interest toward both robotic and human interplanetary missions, and some space agencies are planning to carry out a human mission to Mars within the next three decades. Several criticalities accompany the design of a similar mission, which will require a spacecraft considerably more massive than those already employed for robotic exploration. Cycler mission architectures consider the use of a large space vehicle that cycles continuously between the Earth and Mars, describing a near-ballistic path that includes flybys at the two planets. While this large "cycler" spacecraft can be equipped with the life support system appropriate for a long interplanetary flight with a crew, taxi vehicles of reduced size are sufficient to ensure the connection between the interplanetary vehicle and each planet. This research addresses the determination of the optimal paths that lead the taxi to rendezvous with the cycler while the latter travels in the proximity of the Earth, where its trajectory is represented by a Keplerian hyperbola. In general, orbit rendezvous between two space vehicles can be achieved through one or more maneuvers, using either high thrust or continuous low thrust. Minimization of the propellant required by the taxi implies maximizing its payload mass. Regardless of the payload nature, propellant minimization can be regarded as the objective of the rendezvous trajectory optimization problem. Two classes of rendezvous problems are considered: (a) multiple-impulse hyperbolic rendezvous, if high thrust is employed by the taxi for short durations, or (b) low-thrust hyperbolic rendezvous, if the taxi is equipped with a continuous, low-thrust propulsion system. In both cases, propellant consumption is minimized through selection of the optimal thrust direction. For impulsive rendezvous (a), this work proposes several solutions, including an option that considers an abort strategy and is therefore particularly suitable for human transportation toward the cycler. Conversely, low-thrust hyperbolic rendezvous (b) requires a longer transfer time, and appears more appropriate for cargo taxis. This research has the ultimate purpose of describing several different optimal options for accomplishing the orbital rendezvous between a taxi vehicle and a cycling shuttle that connects the Earth and Mars, pointing out their respective possible uses, advantages, and disadvantages.

Joint work with Bruce Conway.

Author: A. PORTALURI
Università degli Studi di Torino (Italy)
Title: *"Stability dreams (with a symplectic friend) in Celestial Mechanics"*
Abstract. Is the solar system stable? This is one of the oldest open question in dynamical systems. It is still a lively and very active research field, even if many famous scientists like Newton, Lagrange, Maxwell, Poincaré and Birkhoff proved many astonishing results in this direction.

A lot of useful techniques are developed so far to tackle this problem: KAM theory, symplectic and contact methods, blow-up techniques, computer-assisted proofs, etc. One more (variationally oriented piece) we add to this arsenal: the index theory!

In this talk we try to shed some lights on the ideas behind some very recent results on this topic and we discuss some new perspectives and challenges in Celestial Mechanics. We show some amazing classes of equivariant periodic orbits and we prove very recent (in)stability results for a plethora of periodic motions via symplectic techniques.

Joint work with: V. Barutello, X. Hu and S. Terracini.

Author: A. POUSSE
Università degli studi di Napoli "Federico II" (Italy)
Title: *"On the stability of co-orbital motion in the three-body problem"*
Abstract. The co-orbital motion, that corresponds to a particular domain of trajectories where two bodies (two exo-planets or a planet and an asteroid) gravitate around a star with the same period, possesses a very rich dynamics connected to the Lagrange's equilateral configurations L_4 and L_5 as well as to the Eulerian's aligned configurations L_1, L_2 and L_3. A major example in the solar system is given by the Trojan asteroids harboured by Jupiter in the neighbourhood of L_4 and L_5. A second astonishing configuration is given by the satellites of Saturn, Janus and Epimetheus that exchange their orbits every four years and whose dynamics is known as "horseshoe". Eventually, the "quasi-satellite" asteroids, recently observed harbouring several planets in the solar system, are associated with a new type of co-orbital dynamics whose motion, in a rotating frame with a planet, describe the trajectory of a remote retrograde satellite. The main goal of our work is to prove long time stability results for the above co-orbital dynamics. Therefore we shall sketch a rigorous proof (and up to our knowledge, the first one) of existence of the "horseshoe" dynamics over infinite times in the three-body problem thanks to KAM theory.

Joint work with P. Robutel (1) and L. Niederman (1,2)
(1) IMCCE - Observatoire de Paris
(2) Université Paris XI

Author: M. SAILLENFEST
Università di Pisa (Italy) - Observatoire de Paris (France)
Title: *"Secular representations for the long-term dynamics beyond Neptune"*
Abstract. I will present the development and the application of semi-analytical secular models, designed to describe generically the long-term dynamics of transneptunian objects. One-degree-of-freedom systems are obtained in both the non-resonant and resonant cases, allowing to represent every possible trajectory by the level curves of the Hamiltonian. The application to known objects reveals pathways to high perihelion distances. In particular, distant resonant objects can be tracked back to their resonance capture. The effect of a potential external perturber will be also discussed.

Author: M. SANSOTTERA
Università degli Studi di Milano (Italy)
Title: *"Analytical treatment of long-term evolution of extrasolar systems: an extension of the classical Laplace-Lagrange secular theory"*

Abstract. One of the most remarkable properties of extrasolar planets is their possibly high orbital eccentricities, in contrast to the quasi-circular planetary orbits of the Solar System. The classical Laplace-Lagrange secular theory uses the circular approximation as a reference, thus its applicability to extrasolar systems can be doubtful.

We aim to show here that perturbation theory reveals very efficient for describing the long-term evolution of extrasolar systems. More precisely, we study the long-term evolution of two-planet extrasolar systems by extending the Laplace-Lagrange theory. We identify three categories of systems: (i) secular systems, whose long-term evolution is accurately described by an extension of the classical Laplace-Lagrange theory to a high order in eccentricities; (ii) systems that are near a mean-motion resonance, for which an extension of the Laplace-Lagrange secular theory to order two in the masses is required; (iii) systems that are really close to or in a mean-motion resonance, for which a resonant model has to be used.

In the first two cases, we determine the fundamental frequencies of the motion and compute precisely the long-term evolution of the Keplerian elements with a totally analytical method, based on Lie series. Coming to the resonant systems, we show how the long-term evolution can be accurately reproduced by including appropriate resonant combinations of the fast angles via a resonant normal form. This result extends the Laplace-Lagrange secular approximation to resonant systems.

Joint work with A.-S. Libert.

Author: D. SCHEERES
University of Colorado (USA)
Title: *"Minimum energy configurations in Celestial Mechanics and new relative equilibria for the Full 3-Body Problem"*

Abstract. Celestial Mechanics systems have two fundamental conservation principles that enable their deeper analysis: conservation of momentum and conservation of (mechanical) energy. Of the two, conservation of momentum provides the most constraints on a general system, with three translational symmetries (which can be trivially removed) and three rotational symmetries. If no external force acts on the system, these quantities are always conserved independent of the internal interactions of the system. Conservation of energy instead involves assumptions on both the lack of exogenous forces and on the nature of internal interactions within the system. For this reason energy is often not conserved for "real" systems that involve internal interactions, such as tidal deformations or impacts, even though they may conserve their total momentum. Thus mechanical energy generally decays through dissipation until the system has found a local or global minimum energy configuration that corresponds to its constant level of angular momentum.

Author: G. SCHETTINO
IFAC-CNR (Italy)
Title: *"Testing General Relativity with the BepiColombo mission to Mercury"*
Abstract. BepiColombo is a joint ESA/JAXA mission for a comprehensive exploration of the planet Mercury. The mission is now scheduled for launch in October 2018 and for orbit insertion at the end of 2025. The Mercury Orbiter Radio science Experiment (MORE) is one of the on-board experiments, devised to enable a better understanding of both Mercury geophysics and fundamental physics. One of the main scientific goals of MORE is to perform a very precise test of General Relativity (relativity experiment). Thanks to the state of the art on-board and on-ground instrumentation, the extremely accurate tracking from the Earth will allow to precisely reconstruct the heliocentric orbit of Mercury enabling to constrain the value of several post-Newtonian and related parameters with an unprecedented accuracy. The Celestial Mechanics Group of the University of Pisa developed a novel dedicated software, ORBIT14, to perform the simulations of the experiment and to determine simultaneously all the parameters of interest within a global least squares fit. We present the results of a full set of numerical simulations, carried out in a mission up-to-date realistic scenario. The results are highly encouraging: in the framework of metric theories of gravitation, an accuracy of some parts in 10^{-6} for the Eddington parameter beta and at least of 10^{-5} for the Nordtvedt parameter η can be attained, while accuracies at the level of 5×10^{-7} and 10^{-7} can be achieved for the preferred frames parameters α_1 and α_2, respectively. Finally, we will present a comprehensive discussion on the issue of curing rank deficiency in the simultaneous determination of the orbits of Mercury and the Earth with MORE.

Joint work with Giacomo Tommei, Daniele Serra and Andrea Milani.

Author: T. M.-SEARA

Universitat Politecnica de Catalunya (Spain)

Title: *"Oscillatory orbits in the restricted planar three-body problem"*

Abstract. The restricted planar three-body problem models the motion of a massless body under the Newtonian gravitational force of two bodies evolving in Keplerian ellipses.

Since Chazy (1922), it is known that the possible states the body $q(t)$ can approach as time tends to infinity are four:

- Hyperbolic: $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to c > 0$ as $t \to \pm\infty$.
- Parabolic: $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to 0$ as $t \to \pm\infty$.
- Bounded: $\limsup_{t\to\pm\infty} \|q\| < +\infty$.
- Oscillatory: $\limsup_{t\to\pm\infty} \|q\| = +\infty$ and $\liminf_{t\to\pm\infty} \|q\| < +\infty$.

Examples of all these types of motion, except the oscillatory ones, were already known by Chazy.

In this talk, we prove the existence of oscillatory motions for any value of the masses of the primaries assuming they move in ellipses whose eccentricity is small enough, as a consequence of the transversal intersection of the stable and unstable manifolds of periodic orbits at "infinity", and using techniques of Arnold diffusion. We plan to extend these results to the non restricted case.

This is a joint work with M. Guardia, P. Martin, L. Sabbagh.

Author: D. SERRA
Università di Pisa (Italy)
Title: *"New advances in Chaotic Orbit Determination"*

Abstract. Chaotic orbit determination intervenes in many practical problems, such as the chaotic rotation state of a celestial body, the chaotic orbit of a planet-crossing asteroid experiencing many close approaches, or the chaotic orbit of a spacecraft orbiting a giant planet system undergoing many close encounters with its satellites. It is known that chaotic dynamical systems are characterized by the existence of a predictability horizon, connected to its Lyapounov time, beyond which the prediction of the state starting from the initial state of the system is meaningless. In (Spoto and Milani, 2016), the authors applied the differential corrections algorithm for the determination of an orbit and of a dynamical parameter of a simple discrete system, the standard map, with observations distributed beyond the predictability horizon. They defined a computability horizon, a time limit beyond which numerical calculations are affected by numerical instability, and gave a formula for approximating its value. Aiming to push forward the computability horizon inherent in the least squares orbit determination, in this work we tested - in the same case of the standard map the constrained multi-arc method. This method entails the determination of an orbit of a dynamical system when observations are grouped in separate observed arcs. For each arc a set of initial conditions is determined and during the orbit determination process all subsequent arcs are constrained to belong to the same trajectory. We show that using this technique in place of the standard least squares method has considerable advantage, allowing to perform accurate numerical calculations well beyond the computability horizon. Along with the numerical experiments, we also present an analytical proof related to the results obtained in (Spoto and Milani, 2016) for the ordered case. We suppose that the orbit we want to determine belongs to a Moser invariant curve, the number of observations tends to infinity and the partial approximations of the solution tend to the invariant curve with a certain rate.

Joint work with Federica Spoto, Stefano Marò and Andrea Milani

Author: V. SIDORENKO
Keldysh Institute of Applied Mathematics RAS (Russia)
Title: *“Dynamics of ”jumping” Trojans: a perturbative treatment”*
Abstract. The term ”jumping” Trojan was introduced by Tsiganis et al. (2000) in their studies of long-term dynamics exhibited by the asteroid (1868) Thersites: as it turned out, this asteroid may pass from the librations around L4 to the librations around L5. One more example of a ”jumping” Trojan was found by Connors et al. (2011): librations of the asteroid 2010 TK7 around Earth’s libration point L4 preceded by its librations around L5. We explore the dynamics of ”jumping” Trojans under the scope of the restricted planar elliptical three-body problem. Via double numerical averaging, we construct evolutionary equations which allow to analyze the details of the transition from one regime of the orbital motion to another.

Author: D. SKOULIDOU
Aristotle University of Thessaloniki (Greece)
Title: *"The orbital dynamics environment about Earth: application to passive debris removal"*

Abstract. The dynamical environment occupied by artificial celestial bodies of the Earth are subject to motions that are widely separated in frequency: the earthly day, the lunar month, the solar year, and various precession frequencies ranging from a few years to 10^5 years for the ecliptic. This great disparity of timescales involved, on account of the basic degeneracy of the Kepler problem, gives rise to a remarkably diverse collection of resonant phenomena associated with satellite orbital motions. Those satellites that satisfy such commensurability conditions can suffer quasi-secular or dramatic variations in their orbital elements as a result of the oscillation of large periods in the resonance terms. An understanding of their implications for Earth-orbiting space debris has led to a resurgence of interest in artificial satellite theory, with great import for celestial mechanics in general. It has been realized in the last few years that the satellite navigation constellations in medium-Earth orbits (MEOs) exist in a background of third-body secular resonances stemming from the perturbing gravitational effects of the Moon and the Sun. The resulting instabilities, brought on by these commensurabilities, induce especially strong changes on the orbital eccentricity, which can be transported to Earth-reentry values. These results have put forward the idea that similar phenomena could manifest themselves throughout all circumterrestrial space regions, from very low-altitude orbits up to the geostationary region and beyond. Here, we characterize the (drag free) dynamical architecture of the Earth-orbiting environment through a cartography of stability maps, using a suitably modified version of the SWIFT symplectic integration scheme to account for the coupled gravitational and radiation pressure perturbations. We find that many of the semi-secular resonances studied over a decade ago by Breiter become particularly important at the transition region between LEO and MEO.

Joint work with Aaron J. Rosengren, Kleomenis Tsiganis and George Voyatzis.

Author: E. SMIRNOV
Pulkovo Observatory (Russia)
Title: *"The machine-learning methods in the asteroids dynamics"*
Abstract. In asteroid dynamics, many problems require numerical integration of asteroids orbits. This approach consumes enough computer resources, especially when we try to analyse the dynamics of hundreds of thousands of asteroids. Any improvement in the orbit of the asteroid requires additional computer resources to be applied. Therefore, within the context of the increasing volume of new information, fast new methods should be applied to work with big data.

Artificial intelligence and machine-learning (ML) methods have become popular in recent years. In this study we apply the modern ML methods to the classical problems of the dynamics of the asteroid: the identification of the resonances, families, non-regular objects. It is shown, that such methods provides acceptable accuracy and requires much less computational resources.

Author: F. SPOTO
Observatoire de la Côte d'Azur (France)
Title: *"The contribution of Gaia to asteroid dynamics"*
Abstract. The ESA mission Gaia, which is currently surveying the sky from the Sun-Earth L2 Lagrangian Point, is providing astrometry of stars and asteroids at few mas at V=20. The first Gaia Data Release (GDR1), which includes 2 millions of stars with position, parallaxes, and proper motion and a position catalog of 1.1 billion of stars, has already shown the strength of this unprecedented power of investigation.

We have already exploited the contribution of the first Gaia data release to asteroid dynamics. We have found that now the stellar occultations represent a new approach to asteroid astrometry, and this will further be strenghtned by future Gaia data releases (see Spoto et al. 2017). But we have also also to deal with the combination of ground-based astrometry with Gaia observations.

This is a crucial aspect in the orbit determination and in the dynamics itself, because starting from the next data release in April 2018, asteroid observations will be included in the catalog. Thus, the full strength and revolution of Gaia won't be really appreciated if we cannot combine Gaia and ground-based observations.

We have already analyzed the contribution of 5 years of Gaia mission to the improvement of asteroid orbits, working with a sample of simulated Gaia observations. In preparation to the incoming data release, we have studied the impact of the combination of ground-based and Gaia observations to the detection of the Yarkovsky effect or to the next release of the INPOP ephemerides.

Joint work with F. Mignard, P. Tanga, A. Fienga.

Author: B. STEVES
Glasgow Caledonian University (United Kingdom)
Title: *"Analytical Stability Criteria for the Caledonian Symmetric Four and Five Body Problems"*

Abstract. Analytical studies of the stability of three or more body hierarchical systems are difficult because of the greater number of variables involved with each increasing number of bodies and the limitation of 10 integrals that exist in the gravitational n-body problem. Utilisation of symmetries or the neglecting of the masses of some of the bodies compared to others can simplify the dynamical problem and enable global analytical stability solutions to be derived. These symmetric and restricted few body systems with their analytical stability criterion can then provide useful information on the stability of the general few body system when near symmetry or the restricted situation. Even with symmetrical reductions, analytical stability derivations for four and five body problems are rare.

In this paper, we develop an analytical stability criterion for a five body symmetrical system, called the Caledonian Symmetric Five Body Problem (CS5BP) which has two pairs of equal masses and a fifth mass located at the centre of mass. The CS5BP is a planar problem that is configured to utilise past-future symmetry and dynamical symmetry. The introduction of symmetries greatly reduces the dimensions of the five body problem. Sundman's inequality is applied to derive boundary surfaces to the allowed real motion of the system. This enables the derivation of a stability criterion valid for all time for the hierarchical stability of the CS5BP and its subset the Caledonian Symmetric Four Body Problem (CSFBP), where the central mass is taken equal to zero. We show that the hierarchical stability depends solely on the Szebehely constant C_0 which is a function of the total energy H and angular momentum c. We then explore the effect on the stability of the whole system of adding an increasing massive central body. It is shown both analytically and numerically that the CS5BP is hierarchically stable for $C_0 > 0.0659$ and the CSFBP is hierarchically stable for $C_0 > 0.0465$. The stability criterion can be applied to the exploration of the stability of quintuple and quadruple hierarchical stellar systems and symmetrical planetary systems.

Joint work with M. Shoaib and W. Sweatman.

Key words: four body problem, five body problem, hierarchical stability, celestial mechanics, stellar dynamics.

Author: M. SUVAKOV
Institute of Physics - Belgrade (Serbia)
Title: *"Numerical search for periodic solutions of the planar three-body problem in the pairwise strong potential"*

Abstract. We searched numerically for periodic solutions of the planar three-body problem in the pairwise strong potential $O(1/r^2)$ with equal masses and zero angular momentum. Phase space of such system is six dimensional. Additional constraints to the periodic solutions of Hamilton's equations define a three-dimensional subspace of phase space in which up to the scaling all periodic solutions belong. We use dynamical system defined in that subspace as a framework for numerical search for periodic solutions. We report new orbits and study topological influence on period and action values of solutions.

Author: A. TARTAGLIA
Istituto Superiore "Mario Boella" and INdAM (Italy)
Title: *"Using the Sun-Earth Lagrange points for fundamental physics and space navigation"*

Abstract. The Lagrange points of the Sun-Earth system may be used as surveying points of a reference frame co-moving with the Earth around the Sun. Locating transponders in L1,L2,L4 and L5 would allow to measure the times of flight of electromagnetic signals traveling round on a closed (in the Lagrangian reference frame) path. The asymmetry between the times of flight of right- respectively left-handed signals could evidence the gravito-magnetic field of the Sun, i.e. a general relativistic effect of the angular momentum of the star. The expected time of flight difference is in the order of 10^{-13} s, well within the range of measurability by the present time metrology devices. Active periodic emitters in the mentioned points would also constitute beacons ("artificial pulsars") for a Relativistic Positioning System for space navigation in the inner solar system. Crucial for these applications is the accurate description of the Lissajous orbits about L1 and L2 and of the libration motion around L4 and L5.

Author: P. TEOFILATTO
Università di Roma "La Sapienza" (Italy)
Title: *"Long-Term Capture Orbits for Low-Energy Space Missions"*
Abstract. In the last decades, low-energy trajectories in multibody environments have attracted an increasing interest by the scientific community, and some actual space missions have profited from the results of the studies on this subject. This research aims at ascertaining the existence and characteristics of natural long-term capture orbits around a celestial body of potential interest, such as an outer planet of the solar system. The problem is investigated in the dynamical framework of the three-dimensional circular restricted three-body problem. Previous numerical work on this subject has shown that two-dimensional, long-term capture orbits are topologically located in the proximity of asymptotic trajectories that converge toward libration periodic orbits. This numerical evidence substantiates Conley's theorem on the topology of capture trajectories. Moreover, topological methods lead to classifying all of the trajectories that belong to the plane of the two primaries. This work intends to extend the previous investigations to three-dimensional paths. In this dynamical context, several special trajectories exist, such as quasiperiodic orbits. These can be found as special solutions to the linear expansion of the dynamics equations, and have already been proven to exist even using the (complete) nonlinear equations of motion. The nature of long-term capture orbits is thus investigated in relation to the dynamical conditions that correspond to asymptotic trajectories converging into quasiperiodic orbits. Spacecraft dedicated to long-term planetary exploration could greatly benefit from the existence of similar capture orbits, because the propellant amount needed for long-term orbit maintenance would be reduced considerably.

Joint work with Mauro Pontani and Stefano Carletta.

Author: S. TERRACINI
Università di Torino (Italy)
Title: *"Parabolic trajectories and symbolic dynamics: a survey on the variational approach to the N-body and N-centre problem"*

Abstract. In its full generality, the N-body problem of Celestial Mechanics has challenged many generations of mathematicians. It is commonly accepted, since the early works by H. Poincaré, that the periodic problem, through its associated action spectrum, carries precious information on the whole dynamics of a Hamiltonian system. Therefore, the problem of the existence and the qualitative properties of periodic and other selected trajectories for the N-body problem (from the classical celestial mechanics point of view to more recent advances in molecular and quantum models) has been extensively studied over the decades, and, more recently, new tools and approaches have given a significant boost to the field. We shall review some old an new results on the existence and classification of selected trajectories of the classical N-centre and N-body problem, with an emphasis on new analytical and geometrical techniques.

Author: F. TOPPUTO

Politecnico di Milano / TU Delft (Italy)

Title: *"Design and Validation of Ultra Low Thrust Transfers to the Sun-Earth Saddle Point with Application to LISA Pathfinder Mission Extension"*

Abstract. Flying in highly nonlinear gravitational fields is becoming more and more appealing due to the unique features that can be achieved in these models (i.e., Lagrange point orbits, ballistic capture, low-energy transfers). These orbits require less Δv than the equivalent high-energy orbits, which is achieved by a wise exploitation of the high sensitivity in initial conditions. This makes it possible for spacecraft characterized by very limited thrust authority to accomplish such transfers. More recently, attention has been paid to the exploration of the Saddle Points (SP) within the Solar System [1]. These are locations where the net gravitational accelerations balance. Regions about the SP present clean, close-to-zero background acceleration environments where possible deviations from the General Relativity can be tested and quantified. In particular, evidence is mounting that the MOND/TeVeS theory can be valid for accelerations below 1-10 m/s^2 [2]. Among the SP in the Solar System, the Sun-Earth one seems particularly appealing due to its relatively easy accessibility: it is located at a distance of approximately 258 800 km from the Earth, along the Sun-Earth line, between the Sun and the Earth. Although they seem to be remarkable locations in the Solar System, SP are still unexplored. Their location and the non-equilibrium nature suggest that flying by the SP can be done by using highly nonlinear, under-actuated orbits as opportunistic mission extension of spacecraft already about the Lagrange points. The orbits of interest being highly sensitive and having limited control authority, questions raise about their applicability in real scenarios. For this reason, a validation analysis is mandatory to assess the feasibility of flying such orbits with a special focus on their navigability. In this paper we present methods and concepts to design and validate the orbits that experience one or multiple passage through the SP. A parametric analysis is first performed to consider spacecraft initially on a number of Lagrange point orbits having different out-of-plane amplitudes. Orbits are first designed in a co-planar, circular restricted four-body problem having the Sun, the Earth, and the Moon as primaries. Preliminary solutions are then later refined in a full-ephemeris, three-dimensional restricted n-body model stated in a roto-pulsating frame, whose dynamics considers also non-gravitational forces. Both impulsive and finite-burn maneuvers are considered. The focus is on solutions with very low Δv budget (1-10 m/s), ultra low thrust (0.1-10 mN), and low/medium resources spacecraft (200-2000 kg). The validation analysis considers instead geometrical constraints (ground station visibility, occultation, conjunction, etc.) as well as sensitivity of the orbit with respect to the maneuvers execution (variations in the thrust magnitude, pointing angle, delay, interrupted burn, etc.). Moreover, a navigation tool has been developed to infer the flyability of the orbits. This carries out the orbit determination process from range and range rate simulated measurements and considers the presence of noises in both the maneuver execution and orbit propagation. The mission extension of LISA Pathfinder is considered as case study. LISA Pathfinder embarks a payload with two free-falling masses whose position is measured through laser interferometry with picometer resolution, which is ideal to experiment the MOND/TeVeS theory. In this case the available estimation for the end-of-operation state is taken as initial condition,

and the cold gas propulsion available on the LISA Pathfinder Science Module is considered as primary propulsion. Several mission extension options are designed and validated into a high-fidelity model, and their implementation is discussed.

Author: K. TSIGANIS
Aristotle University of Thessaloniki (Greece)
Title: *"Dynamical formation of the Asteroid Belt"*
Abstract. The origin of the orbital structure in the asteroid main belt has been a puzzling problem for decades; an initially "flat" disc of circular and co-planar orbits cannot evolve into the current distribution within $\sim$ 4 Gy, if the giant planets always follow the same orbits. Hence, the asteroid belt had to be depleted in mass and excited in inclination (up to 30 deg) during the early post-formation stages of the solar system and before the giant planets reached their final configuration. Recent works suggest that this is extremely difficult to reconcile with the standard scenario of terrestrial planet formation. On the other hand, recent works on the final stages of planetesimal-driven migration of the giant planets (e.g. "Nice model") suggest that an already excited belt would indeed survive, if planet migration was very swift, likely following a brief but effective instability episode. These results put constraints on the timing of formation of the asteroid belt that are met by the "Grand Tack" scenario, which attributes the redistribution of planetesimals in the inner solar system to a specific, early (gas-driven) migration pattern of the giant planets. An alternate model, assuming a more main-stream, gas-driven migration pattern that places the giant planets in a multi-resonant configuration, attributes the excitation of the belt to the giant planets having actually mildly chaotic orbits. This orbital state, which can last from a few to several hundred My, leads to consecutive crossings of secular resonances throughout the belt that can excite asteroids to high eccentricities and inclinations, given enough time. In this talk we are going to review these different dynamical formation models and discuss the specific conditions under which the formation of the belt can fit to a concrete scenario of solar system evolution.

Author: A. UREÑA
Universidad de Granada (Spain)
Title: *"Instability of all closed orbits obtained by symmetric minimization"*
Abstract. We study the dynamics around closed orbits of autonomous lagrangian systems having an involutory symmetry. Our main result states that if a symmetric orbit minimizes the free-period action functional, then it must be orbitally unstable. This applies to possibly degenerate, or even nonisolated minimizers; in the nondegenerate case, minimizers must be hyperbolic. Furthermore, no symmetries are needed in the special case of the configuration manifold being an orientable surface. Applications to geodesics and Celestial Mechanics are given.

Author: T. VAILLANT
IMCCE, Observatoire de Paris (France)
Title: *"Long term motion and rotation of Ceres and Vesta"*
Abstract. The spacecraft Dawn has studied Ceres and Vesta, the heaviest asteroids of the main belt, and has allowed to refine their physical characteristics and their rotation motions. Dawn should also constraint the ice distribution on the surface of Ceres but this requires to know the long term orbital and rotation motions of Ceres.

A long term integration of the Solar system (Laskar et al. 2011) has allowed to obtain a long term solution for Ceres and Vesta and has shown that their orbital motions are chaotic. From this integration, we obtain a secular solution for Ceres and Vesta, where the secular frequencies, which play an important part in the long term dynamics, can be identified. The rotation motion is then deduced from this orbital solution. We obtain the evolution of the angular momenta of Ceres and Vesta with a secular model, where the fast rotational motion of the planet is averaged. This computation needs the constant precessions, which are expressed as a function of some physical characteristics of Ceres and Vesta. Dawn has determined their gravitational flattenings but the polar moments of inertia, which depend on the internal structure, are not well constrained. From this model, we can study the long term stability of the angular momenta and the influence of the planetary perturbations on the long term rotation.

Joint work with Jacques Laskar and Nicolas Rambaux.

Author: G. VALSECCHI
IAPS-INAF (Italy)
Title: *"The evolution of the Line of Variation at close encounters"*
Abstract. The outcome of the first planetary fly-by occurring after the start of the dynamical evolution of a planet crossing small body strongly depends on its coordinates on the target plane of the encounter. The uncertainty associated to these coordinates is a function of the uncertainty in the orbital elements at the time of the encounter, and it is easy to show that in most cases of interest it is dominated by the uncertainty in the time of closest approach. A suitable choice of the target plane coordinates is such that one coordinate represents the minimum distance between the orbit of the small body and that of the planet, and the other is proportional to the timing of the encounter. In this way, the uncertainty is mostly along a line parallel to one of the coordinate axes, the so-called Line of Variation (LoV). The LoV approach is a crucial ingredient of the Impact Monitoring software developed at the University of Pisa and at the JPL, whose outputs are available on the risk pages of NEODyS (`http://newton.dm.unipi.it/neodys/index.php?pc=4.1`) and Sentry (`http://neo.jpl.nasa.gov/risk/`). In this study the post-encounter evolution of fictitious small bodies belonging to the LoV is studied in the framework of the analytic theory of close encounters, and the results are compared, whenever possible, with those of numerical integrations.

Author: M. VASILE
University of Strathclyde (United Kingdom)
Title: *"Uncertainty Propagation in Orbital Mechanics with Generalised Polynomial Algebra"*

Abstract. This paper presents a generalised polynomial algebra to propagate uncertainty sets through dynamical systems. It is assumed that the dynamics is dependent on a number of model parameters and the state of the system evolves from some initial conditions. One or more initial conditions and model parameters can vary within a set Ω. The paper presents an approach to approximate the set Ω with a polynomial expansion and to propagate, under some regularity assumptions, the polynomial representation through the dynamical system. The approach is based on a generalised polynomial algebra that replaces algebraic operators between real numbers with operators between polynomials. The paper first details the proposed technique and then compares, both theoretically and experimentally, its time complexity, for the same accuracy, against its non-intrusive counterpart. Two examples with increasing complexity are used to illustrate the practical applicability of the proposed approach to Orbital Mechanics.

Joint work with: Annalisa Riccardi and Carlos Ortega

Author: M. VOLPI
University of Namur (Belgium)
Title: *"On the stability of 3D planetary system configurations"*
Abstract. To date, more than 600 multiple planet systems have been discovered. Due to the limitations of the detection methods, our knowledge of the systems is usually far from complete. In particular, for planetary systems discovered with the radial velocity technique, the inclinations of the orbital planes (and thus their mutual inclinations and masses) are unknown. Our work aims to constrain the observations of several non-resonant extrasolar systems. Through analytical analysis based on a first-order secular hamiltonian expansion and numerical explorations performed with a chaos detector (MEGNO), we identify ranges of values for the mutual inclinations which ensure the long-term stability of the system. Particular attention is also given to determine the possibility of the detected extrasolar systems to be in a Lidov-Kozai resonant state.

Joint work with A.-S. Libert.

Author: G. VOYATZIS
Aristotle University of Thessaloniki (Greece)
Title: *"Inclined Asymmetric librations in planetary dynamics"*

Abstract. Librating motion between two celestial bodies is generally associated with the existence of resonances and it is signified by the oscillation of the resonant angles. When such an oscillation takes place around a value different than 0 or π the libration is called asymmetric. The first asymmetric librations were found by Message (1958) in the planar circular restricted three body problem and for the $1:2$ exterior resonance. The centers of such librations in phase space consist of linearly stable asymmetric periodic orbits which form a whole family. Markellos (1978) showed that orbits of the above family which are vertically unstable generate families of spatial asymmetric periodic orbits. Beaugé (1994) showed the existence of planar asymmetric librations in all exterior resonances of the form $1:p$. In this work, we study the resonances $1:p$ in the spatial restricted three body problem searching for families of linearly stable asymmetric periodic orbits from low up to high inclinations. Our study is applied to the orbital dynamics of trans-Neptunian objects and Centaurs.

Author: J. WALDVOGEL
ETH Zurich (Switzerland)
Title: *"Jost Bürgi's Artificium of 1586 in modern view, an ingenious algorithm for calculating tables of the sine function."*

Abstract. In the years of 1586 to 1592 the Swiss instrument maker and mathematician Jost Bürgi devised and documented an ingenious algorithm for efficiently and precisely calculating tables of the sine function. The manuscript *Fundamentum Astronomiæ* explaining this method and Bürgi's tables had been considered as lost, but have been rediscovered in 2013 by Menso Folkerts in the University Library of Wroclaw (Poland). In this presentation we explain and discuss Bürgi's algorithm, referred to as *Artificium* or *Kunstweg*, with the tools of modern Linear Algebra. By considering the difference table of the sine function and by using matrices and eigenvalue problems, we develop a theory of the algorithm and discuss the rate of convergence.

Author: D. WILCZAK
Jagiellonian University (Poland)
Title: *"Continuation and bifurcations of Halo orbits - computer-assisted proof"*

Abstract. We propose an algorithm for rigorous validation that a family of periodic orbits preserving some symmetries undergoes various types of bifurcations, including period doubling and tripling. We apply the method to the Restricted Circular Three Body Problem, giving a computer-assisted proof that the family of Halo orbits bifurcates from the family of Lyapunov orbits for wide range of the parameters μ. For μ corresponding to the Sun-Jupiter system we give a proof of the existence of a wide continuous branch of Halo orbits that undergoes period doubling bifurcation and period tripling bifurcation. The computer-assisted proof uses rigorous ODE solvers and algorithms for computation of Poincare maps and their derivatives from the CAPD library.

Joint work with Irmina Walawska.

Author: P. ZGLICZYNSKI
`Jagiellonian University (Poland)`
Title: *"Melnikov-type method for splitting of separatrices for an explicit range of small parameter"*

Abstract. We present a Melnikov type approach for establishing transversal intersections of stable/unstable manifolds of perturbed normally hyperbolic manifolds. The method is based on geometric estimates on the manifolds to obtain bounds on their first and second derivatives, and on rigorous, interval arithmetic integration of ODEs. The benefit from our approach is the following. We do not need to know the explicit formulas for the homoclinic orbits prior to the perturbation. We also do not need to compute any integrals along such homoclinics. All needed bounds are established using rigorous computer assisted numerics. Lastly, and most importantly, we establish intersections for an explicit range of parameters, and not only perturbations that are "small enough", as is the case in the classical approach.

Joint work with Maciej Capinski.