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Title: *"Analytical Stability Criteria for the Caledonian Symmetric Four and Five Body Problems"*

Abstract. Analytical studies of the stability of three or more body hierarchical systems are difficult because of the greater number of variables involved with each increasing number of bodies and the limitation of 10 integrals that exist in the gravitational n-body problem. Utilisation of symmetries or the neglecting of the masses of some of the bodies compared to others can simplify the dynamical problem and enable global analytical stability solutions to be derived. These symmetric and restricted few body systems with their analytical stability criterion can then provide useful information on the stability of the general few body system when near symmetry or the restricted situation. Even with symmetrical reductions, analytical stability derivations for four and five body problems are rare.

In this paper, we develop an analytical stability criterion for a five body symmetrical system, called the Caledonian Symmetric Five Body Problem (CS5BP) which has two pairs of equal masses and a fifth mass located at the centre of mass. The CS5BP is a planar problem that is configured to utilise past-future symmetry and dynamical symmetry. The introduction of symmetries greatly reduces the dimensions of the five body problem. Sundman's inequality is applied to derive boundary surfaces to the allowed real motion of the system. This enables the derivation of a stability criterion valid for all time for the hierarchical stability of the CS5BP and its subset the Caledonian Symmetric Four Body Problem (CSFBP), where the central mass is taken equal to zero. We show that the hierarchical stability depends solely on the Szebehely constant C_0 which is a function of the total energy H and angular momentum c. We then explore the effect on the stability of the whole system of adding an increasing massive central body. It is shown both analytically and numerically that the CS5BP is hierarchically stable for $C_0 > 0.0659$ and the CSFBP is hierarchically stable for $C_0 > 0.0465$. The stability criterion can be applied to the exploration of the stability of quintuple and quadruple hierarchical stellar systems and symmetrical planetary systems.

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Key words: four body problem, five body problem, hierarchical stability, celestial mechanics, stellar dynamics.