On the numerical integration of variational equations

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Abstract. We investigate the efficiency of different numerical approaches for the integration of the variational equations of Hamiltonian systems, whose kinetic energy is quadratic in the generalized momenta and whose potential is a function of the generalized positions. In our study we consider both symplectic and non-symplectic integrations schemes, like the SABA integrators and the DOP853 method respectively. We apply these techniques to the case of the two degrees of freedom Hénon-Heiles Hamiltonian system. The numerical verification of well-known properties like the fact that, the set of Lyapunov Characteristic Exponents (LCEs) consists of pairs of values having opposite signs, with two of them being equal to zero, and that the time evolution of the Generalized Alignment Indices (GALIs) exhibits particular laws for regular and chaotic orbits, is used for characterizing the efficiency of the studied integration schemes.