Virtual Impactors: Search and Destroy

Andrea Milani†
e-mail: milani@dm.unipi.it

Steven R. Chesley†
e-mail: chesley@dm.unipi.it

Andrea Boattini‡
e-mail: boattini@ias.rm.cnr.it

Giovanni B. Valsecchi‡
e-mail: giovanni@ias.rm.cnr.it

†Dipartimento di Matematica, Università di Pisa
Via Buonarroti 2
56127 PISA, ITALY

‡IAS-Planetologia, Area di ricerca CNR
Via Fosso del Cavaliere
00133 ROMA, ITALY

revised version, November 13, 1999

Abstract

If, for an asteroid which has been observed only over a short arc then lost, there are orbits compatible with the observations resulting in collisions, recovery would be desirable to decide if it will actually impact. If recovery is essentially impractical, as is the case for many small asteroids in the 100 m to 500 m diameter range, the next best thing is to make sure that the lost asteroid is not on a collision course. We propose a method to achieve this guarantee, with an observational effort far smaller than the one required for recovery. The procedure involves the computation of an orbit which is compatible with the available observations and, by hypothesis, results in an impact at some later encounter; this we call a Virtual Impactor (VI). The collision at some future time is a strong constraint, thus the VI has a well determined orbit. We show that it is possible to compute for each given time of observation the skyprint of the VI, that is the set of astrometric positions compatible with an impact (or a near impact). The skyprint needs to be scanned by powerful enough telescopes to perform a negative observation; once this has been done for the skyprints of all VIs, collisions can be excluded even without recovery. We propose to apply this procedure to the case of the lost asteroid 1998 OX_4, for which we have found orbital solutions with impacts in the years 2014, 2038, 2044 and 2046. Suitable observing windows are found when the VI would be close to the Earth in 2001 and in 2003, and the corresponding skyprints are small enough to be covered with very few frames. This procedure might become more and more necessary in the future, as the number of discoveries of small potentially hazardous asteroids increases; we discuss the general principles and the validation procedures that should apply to such a VI removal campaign.

Who counts the wampum of the night
To see that none is due?
—Emily Dickinson, 1859, poem 128.

1 The problem

Recent progress in the computation of collision solutions for Near-Earth Asteroids (NEAs) allows one to identify future possible impacts from asteroids which have already been discovered [Milani et al. 1999]. Some of these asteroids have only been observed for a very short time, so that the uncertainty of their positions on the sky grows rapidly, and very soon they have to be considered lost. In the long run most of these lost asteroids will be recovered serendipitously, but, there is no guarantee that they would be recovered before a possible impact.

Although the best thing would be just to recover a lost potential impactor, it may turn out that the recovery is not feasible with the available telescope resources. The next best thing to do is to make sure that the asteroid is not on a collision course. Even for a lost asteroid, it remains possible to exclude a collision solution with a rather small observational effort.

To do this we need to be able to characterize the footprint on the sky that the asteroid would have if it were on a collision course, and to develop a suitable observational strategy to check whether it is or it is not there. The adoption of this procedure can eliminate a potential threat, and also alleviate a possible public perception that the astronomical community has failed in its obligation of monitoring NEAs, not only for scientific purposes, but also to guarantee the safety of the Earth. This is possible with a small observational effort because the collision at some later time can be thought of as an observation, and in that sense the possible collider has a well determined orbit, as it is typical of asteroids observed at multiple oppositions, so that its position on the sky has a small uncertainty.

The possible impact solutions for all NEAs are announced, and continuously updated, by the *NEODyS* online information service (`http://newton.dm.unipi.it/neodys`). For asteroids discovered after the start of operations of *NEODyS*, the announcement of possible impact solutions can result in quick response by observers, resulting in further observations that reduce the uncertainty region enough to eliminate the impact risk. This sequence of events took place in the recent case of 1999 RM_{45}. In this case within a couple of days of the announcement the impact solutions could be ruled out by new observations. However, a totally different situation occurs in the case of impact solutions found for a lost asteroid. Currently there is only one such asteroid for which a collision with our planet is known to be possible (although unlikely) in the next 50 years: 1998 OX_4, for which there are orbits compatible with the observations leading to collisions in 2014, 2038, 2044 and 2046. In the future, thanks to the increasing efficiency of both the telescopic searches and of the computational methods to detect possible impacts, there could be many more examples of this problem.

To explain with the necessary precision the solution we propose, we need to introduce a few concepts, in particular the notion of *Virtual Impactor* (VI); they are presented in a non-mathematical way in the following subsections. In Sec. 2 we describe the algorithms to detect the VIs and to predict where in the sky we should "search" for them; this section is rather mathematical, and is not required to understand the subsequent sections. In Sec. 3 we discuss the first real example of a lost potential impactor, namely asteroid 1998 OX_4, and suggest how the VIs associated with it can be "destroyed". In Sec. 4 we discuss the general principles and the validation procedures that should apply to a VI "search and destroy" campaign.

1.1 Virtual asteroids and lost asteroids

When an asteroid is discovered, we do not know anything like "the orbit" of the real object. There is a range of possible orbits, all compatible with the observations, forming a *region of confidence* in orbital element space. We can describe this by thinking of a swarm of *virtual asteroids*, with slightly different orbits all compatible with the observations. The reality of the asteroid is shared among the virtual ones, in the sense that only one of them is real, but we do not know which one. To make things simple, we can think that all the virtual objects are equally likely to be the real one; a more complicated probability law could be used, changing the exact values of the estimated probabilities but not the qualitative picture. Since the confidence region contains a continuum of orbits, each virtual asteroid is in turn representative of a small region, that is its orbit is also uncertain, but to a much smaller degree [1]. Note that the so called "nominal" orbit is just one of the virtual asteroids, without any special significance.

What does it mean that an asteroid is *lost*? The virtual asteroids, as seen in the sky, are initially very close together, within the range of possible errors in the observations; as time goes by, they gradually separate, mostly because of small differences in the semimajor axis; they thus become like a string of pearls, "the wampum of the night." When the length of the string, as seen in the sky, is too long to be contained in a reasonable number of fields of view of a typical telescope, the real asteroid cannot be recovered by using the ephemerides, that is a single prediction of the position based upon the nominal orbit. It could be recovered by taking many frames, but to discuss this we need to introduce the notion of *negative observation*: one telescope frame needs to be scanned to ascertain that the asteroid is not there, that is, that some virtual asteroids do not exist, before going to scan another frame. When the number of required frames is very large, the observational expense of the recovery may not be warranted, and the asteroid is considered lost.

What should be done to *recover* an asteroid when it is indeed lost, but is of special significance? The observational scanning of many frames where different virtual asteroids could be found needs not to be done with the same telescope, and not in the same night. For each virtual asteroid there are better times for observation, as an

[1] This is a non-mathematical description, which could be transformed in a rigorous definition by using the mathematical notions of *probability density function* and of *partition of unity*.

example when it is bright enough [2]. For each of these recovery opportunities the negative observations need to be recorded, and some coordinating center needs to assemble the negative observations to make sure that all virtual asteroids have been searched for ("to see that none is due") until the real one is found. It is important that the sampling of the confidence region by virtual asteroids be dense enough to ensure that the real asteroid does not slip through a gap in observation coverage. This idea was discussed by [Bowell et al. 1993], who attempted to recover the historically important asteroid (719) Albert, which is desperately lost.

1.2 Virtual impactors

In case a lost asteroid is Earth-crossing, there might be one (or more) of the virtual asteroids associated with it for which an actual collision is possible. Then there is a small nearby region filled with collision orbits still compatible with the observations. We call a virtual asteroid with an orbit in this region a *Virtual Impactor* (VI).

In practice, let us suppose that some time after the discovery apparition the orbit undergoes a close approach to the Earth; the *Modified Target Plane* (MTP) for that encounter is the plane perpendicular to the geocentric velocity at closest approach [Milani and Valsecchi 1999]. For some initial condition in the confidence region in element space there is a point on the MTP representing the close approach of the corresponding orbit. If some of the points on the MTP are inside the cross section of the Earth, these correspond to VIs.

For this paper, we have modified the procedure to find potential collisions described recently in [Milani et al. 1999, Sec. 3]. Here we find VIs in the following way:

1. We sample the set of possible orbits by computing a large number (typically of the order of a thousand) orbital solutions equally spaced along the "weak direction" [Milani 1999]—the longest axis of the ellipsoid approximating the confidence region in element space—at an epoch close to observations.

2. We then propagate each of these virtual asteroid orbits until some specified future epoch (currently we stop at 2050), and note cases which pass near the Earth.

3. We perform detailed MTP analyses on these notable cases, and compute in an iterative manner the nearby orbital solution along the weak direction on the MTP that approaches most closely to the Earth; if the approach is to within the radius of the Earth, we have found a VI.

In the neighborhood of a VI there is a small region of the element space containing other collision solutions; this we call a *Virtual Impactor Region* (VIR). To exclude the possibility of an impact (at a given encounter) we need not only to perform negative observations of the VI, but of all the orbits belonging to the VIR.

[2] Different virtual asteroids can have a different absolute magnitude, since the latter is estimated together with the orbit.

1.3 Skyprints

We want to have a reasonable "safety margin" with respect to the VIR, a margin that will be reflected in a slightly larger sky region to be observed; in these matters safety is obviously worth some extra effort. We therefore consider a rectangular region in the MTP, centered on the VI point; the exact definition is detailed in Sec. 2.5, but the idea is to ensure that points outside it are either passing at ≥ 5 Earth radii during that encounter, or belong to orbits for which $\sigma \geq 5$, that is well outside the confidence region.

We then compute the preimage of the MTP rectangle back into the confidence region in element space, thus obtaining the set of orbits of interest, which is somewhat larger than the VIR because of the safety margins. The details of this computation are described in Sec. 2.5, but a qualitative description could be as follows: this region is similar to the Cartesian product of the MTP rectangle, pulled back in the orbital element space, and of a 4-dimensional region; this is due to the fact that the orbital element space is 6-dimensional, thus each point on the 2-dimensional MTP has a preimage which is 4-dimensional.

If we apply the procedure described in [Milani 1999] to obtain the sky coordinates at a given epoch of the *observation confidence region* in which a lost asteroid should be located, we typically obtain a thin strip which can be extremely long, even more than a full tour around the sky (because sometimes the string of virtual asteroids extends over more than a full orbit).

When this observation confidence region is too large to be completely scanned, we need to identify the subset of observations corresponding to the orbits which are both compatible with the existing observations and capable of having a dangerously close approach, that is to the VIR, slightly augmented for safety. At a given time, the footprint in the sky of this region is the *skyprint* corresponding to a given VI.

If the skyprint of a given VI is, at some time, small enough to be observed at once with a single plate/CCD frame (at most with a few frames), and the corresponding virtual asteroid is bright enough at that time to be detected with certainty (with the given telescope, exposure, and detection technique), then a negative observation is possible with small resources. Again, some safety margins are required to allow us to rely on the negative observation, as discussed in Sec. 4.

The main results of Sec. 2 can now be described as follows: we have a reliable procedure to find virtual impactors, and an efficient procedure to compute the skyprint associated with them at any given time. These tools can be used to plan a campaign to search and destroy VIs. One such campaign to hunt those of 1998 OX_4 is proposed in Sec. 3.

2 Computational methods

The computation of the ephemerides of a virtual impactor to allow for a negative observation involves six steps, described in the following six subsections.

2.1 Confidence region and multiple solutions

The uncertainty associated with astrometric observations of an asteroid limits the precision with which the orbital elements of the latter can be determined. After computing, by the method of least squares, the solution that minimizes the sum of the residuals squared (the *nominal solution*), one can define a *confidence region* in the space $\mathcal{X}$ of the orbital elements at some fixed epoch t_0. Using the same notation as [Milani 1999, Sec. 2], the 6-dimensional ellipsoid (with interior) approximating the confidence region is given by

$$\Delta X \cdot C_X \, \Delta X \leq \sigma^2 \tag{1}$$

where ΔX is the change in the orbital elements $X \in \mathcal{X}$ with respect to the nominal solution, and C_X is the normal matrix. The inverse of the normal matrix is the *covariance matrix* $\Gamma_X = C_X^{-1}$. The largest eigenvalue of Γ_X is associated to a *weak direction* along which the orbital elements are poorly constrained by the available observations.

Following [Milani 1999, Sec. 5], we sample the confidence region by computing *multiple solutions* by moving along the weak direction with a small step $\Delta\sigma$; in this way we sample uniformly a line in the confidence region, the *Line Of Variations* (LOV). The resolution in the detection of possible impacts is controlled by $\Delta\sigma$; more precisely, the probability of impact above which all the impacts are certainly detected is inversely proportional to $\Delta\sigma$ [Milani et al. 1999]. We are currently using $\Delta\sigma = 0.005$, that is we sample the *line of variations* along the weak direction for $|\sigma| \leq 3$ with 1201 solutions X_i; these multiple solutions are used as virtual asteroids, that is each one of them is taken as representative of a small region in the space of initial conditions. Note that other authors, e.g., [Muinonen 1999] and [Chodas and Yeomans 1996], use different methods to sample the confidence region; these differences are important for the efficiency and the reliability of the computations, but they are not conceptually essential. We use virtual asteroids X_i uniformly spaced along a smooth line in the $\mathcal{X}$ space because this allows us to exploit the topological properties of a string, e.g., if some continuous function is positive for X_i and negative for X_{i+1} we can conclude that it has a zero along the LOV.

2.2 Scanning for close approaches

Given a catalog of virtual asteroids X_i, $i = 1, \ldots, 1201$ we propagate all the orbits from the initial epoch t_0 until some target epoch t_1 and record all the close approaches to the terrestrial planets within a distance of 0.1 AU and taking place between t_0 and t_1. Note that the list of such approaches for one solution X_i is in general different from the list for a different, but nearby one, say $X_{i\pm1}$. This arises from the MTP stretching Γ_{MTP} [Milani et al. 1999] which separates the solutions on the MTP at a given encounter by a distance

$$d = \Gamma_{\mathrm{MTP}} \; |\sigma(X_i) - \sigma(X_j)| \ .$$

For example, if $\Gamma_{\mathrm{MTP}} = 40$ AU/σ, then two consecutive solutions with $\Delta\sigma = 0.005$ are separated by $d = 0.2$ AU on the MTP, thus they cannot both pass within 0.1 AU of the Earth at that encounter. This fact dictates the resolution limit: a close approach with $\Gamma_{\rm MTP} > 40{\rm AU}/\sigma$ may not be found during the propagation if $\Delta\sigma \geq 0.005$.

The sequence of close approaches to a given planet, such as the Earth, for a given solution X_i, is controlled by the complex interplay of resonances and orbital changes resulting from close approaches, as in the case of 1999 AN_{10} discussed in [Milani et al. 1999]. It is possible *a posteriori* to explain the occurrence of a given close approach for one solution X_i as the result of a sequence of resonant and/or non-resonant returns. However, the cascade of returns is often so complex that it generates several possibilities of close approach every year; some of these possibilities have such a large stretching, hence such small probabilities of occurring, that they are not worth considering.

For this reason we have adopted a procedure which works the other way round: we use the scan of 1201 orbits to detect close approaches, in this way selecting the returns with a significant probability, then by inspecting the sequence of returns of a given solution we can easily identify the resonance mechanism which has allowed the returns to take place.

2.3 Target plane analysis

Let us assume that the orbit with initial conditions X_i undergoes a close approach to the Earth at a time $t_i \gg t_0$; the *Modified Target Plane* (MTP) for that encounter is the plane perpendicular to the geocentric velocity at closest approach. Let this plane be $\mathcal{T}$ and the points on it be $T \in \mathcal{T}$; then there is a differentiable map

$$F_T : \mathcal{X} \longrightarrow \mathcal{T}$$

which can be explicitly computed by orbit propagation from epoch t_0 to a time near t_i when the orbit crosses the MTP; the matrix DF_T of the partial derivatives of F_T is computable by means of the *state transition matrix*, the solution of the variational equation. If $\Delta X = X - X_i$ is a small change in the orbital elements with respect to the reference solution X_i, then the mapping on the target plane can be approximated by its differential

$$\Delta T = DF_T \, \Delta X \ .$$

By this linear map, the ellipsoid of Eq. (1) is mapped onto the target plane as an elliptical disk with equations [Milani and Valsecchi 1999]

$$\begin{array}{rcl} \sigma^2 & \geq & \Delta T \cdot C_T \, \Delta T \\ C_T & = & (\Gamma_T)^{-1} \\ \Gamma_T & = & DF_T \, \Gamma_X \, (DF_T)^T \ . \end{array}$$

By computing the eigenvalues $\lambda_1 > \lambda_2 > 0$ of Γ_T, we find that there is again a *weak direction* corresponding to the long axis of the MTP ellipse. As the time elapsed from t_0 increases, the confidence region becomes longer and longer in the phase space, and simultaneously thinner and thinner (this follows from Liouville's theorem, by which the phase space 6-dimensional volume is invariant). Thus, when a close approach takes place decades after the initial epoch t_0, the two eigenvalues of Γ_T have a very large ratio. An orbital solution that moves the MTP intersection along the weak direction results in a negligible change in the value of the target function, that is a negligible increase in the RMS of the observation residuals. On the contrary, a comparatively small change in the orthogonal direction, along the minor axis of the ellipse, would result in a significant increase of the residual RMS.

Thus the points on the MTP which can be reached with a negligible increase in the RMS are the points of the straight line which is the eigenspace of the larger eigenvalue λ_1; let T_{min} be the point along that line that is closest to the center of the Earth, provided it is not too far along the line (with respect to $\sqrt{\lambda_1}$). The distance d_{min} of T_{min} provides an estimate of the closest approach distance possible within the confidence ellipsoid; this estimate involves two approximations. First, the width $\sqrt{\lambda_2}$ of the ellipse is neglected; second, the nonlinear map F_T is replaced by its linearization DF_T. Both approximations can be removed, e.g., by the method of *semilinear confidence boundaries* described in [Milani and Valsecchi 1999]. However, for the purpose of finding VIs another approach is more efficient.

2.4 Finding virtual impactors

Around some times t_i many of the virtual asteroids X_i experience a close approach to the Earth; this we call a *virtual shower.* A shower can be decomposed into separate *returns*, which are continuous strings of solutions having the same close approach; they are represented by sequences of n consecutive solutions X_i, $i = k, k+1, \ldots, k+(n-1)$. It is often the case that the shower at a given time contains several different returns [Milani et al. 1999, Table 1]. For each return we want to identify the closest possible approach in order to decide if an impact is possible. Starting from the solution X_j that has the closest approach among the ones of the return, we apply corrections to it to push the approach towards a closer one; the method is a variant of Newton's method, also called *differential corrections* in the context of orbit determination. It is similar, but not identical, to one method used in [Muinonen 1999].

We begin by defining the correction we would like to apply on the MTP, which is $\Delta T = T_{min} - F_T(X_j)$. Then we need to identify a change ΔX in the orbital elements (with respect to X_j), such that

$$DF_T\,\Delta X = \Delta T\ . \tag{2}$$

To this purpose, we have to consider that DF_T is a composition of a projection $\Pi_{\mathcal{E}_T}$ onto a two-dimensional subspace $\mathcal{E}_T \in \mathcal{X}$, followed by an invertible map $A_T : \mathcal{E}_T \longrightarrow \mathcal{T}$:

$$DF_T = A_T \circ \Pi_{\mathcal{E}_T}\ .$$

Here $\mathcal{E}_T$ is the 2-dimensional subspace of $\mathcal{X}$ spanned by the two rows of DF_T. We further define $\mathcal{L}_T$ to be the 4-dimensional subspace of $\mathcal{X}$ orthogonal to $\mathcal{E}_T$, so that

$\mathcal{E}_\mathcal{T} \times \mathcal{L}_T = \mathcal{X}$. In simple terms, a change in the orbital elements along $\mathcal{L}_T$ does not change the position on the target plane, as far as the linear approximation is applicable. It is possible to perform a change of coordinates in $\mathcal{X}$, by means of an orthogonal 6×6 matrix V, in such a way that

$$\Delta X = V \left[\begin{array}{c} \Delta E_T \\ \Delta L_T \end{array} \right]$$

with $\Delta E_T \in \mathcal{E}_\mathcal{T}$ and $\Delta L_T \in \mathcal{L}_T$; then the normal matrix C_X in the new coordinate system is changed into

$$V^T\, C_X\, V = \left[\begin{array}{cc} C_{E_T} & C_{E_T L_T} \\ C_{L_T E_T} & C_{L_T} \end{array} \right]$$

in this way the contributions to Eq. (1) from the two components can be identified.

On the contrary, all changes along $\mathcal{E}_T$ map linearly into nonzero changes on the target plane; thus there is an inverse map $B_T = A_T^{-1}$. The projection $\Pi_{\mathcal{E}_T}$ is not invertible, the preimage of each point being a 4-dimensional space. However, we can select one change in orbital elements $\Delta X \in \mathcal{X}$ which has a given displacement as image on the target plane $\Delta T \in \mathcal{T}$. The portion of the preimage contained in the confidence ellipsoid (1) can be described by the inequality

$$\begin{array}{rcl} \sigma^2 & \geq & \Delta X \cdot C_X\, \Delta X \\ & = & \Delta L_T \cdot C_{L_T}\, \Delta L_T + 2 \Delta L_T \cdot C_{L_T E_T}\, \Delta E + \Delta E_T \cdot C_{E_T}\, \Delta E_T \\ & = & (\Delta L_T - L_0) \cdot C_{L_T}\, (\Delta L_T - L_0) + const \end{array}$$

where $\Delta E_T = B_T\, \Delta T$ is fixed, uniquely determined by the MTP displacement ΔT. The above inequality, seen with ΔL_T only as variable, defines a 4-dimensional ellipsoid (with interior) $Z(\Delta T)$; we are going to select as representative of $Z(\Delta T)$ the point L_0, which is the center of symmetry of this ellipsoid. L_0 can be computed in several different ways [Milani 1999, Sec. 2], e.g., by separating the terms of different degrees in ΔL, and is obtained as a function of ΔE_T as

$$L_0 = -C_{L_T}^{-1}\, C_{L_T E_T}\, \Delta E_T\ . \tag{3}$$

This allows to select

$$\Delta X = V \left[\begin{array}{c} \Delta E_T \\ -C_{L_T}^{-1}\, C_{L_T E_T}\, \Delta E_T \end{array} \right] = V \left[\begin{array}{c} I_2 \\ -C_{L_T}^{-1}\, C_{L_T E_T} \end{array} \right] B_T\, \Delta T = H_T(\Delta T) \tag{4}$$

in such a way that (2) is satisfied. Then the orbit with initial conditions $X_j + \Delta X$ is propagated to the time $\simeq t_i$ of the close approach under study, and because of the nonlinear effects

$$F_T(X_j + \delta X) \neq F_T(X_j) + \Delta T$$

but the distance between the new MTP point and T_{min} is now smaller, and the procedure can be iterated. This means we reset the 'nominal' orbit to $X_j + \Delta X$, we compute its MTP and its partial derivatives matrix DF_T, find a new weak direction, a new minimum distance point T_{min} along the weak direction, then select a new ΔX satisfying the new version of Eq. (2), and so on until convergence. If the close approach distance at convergence is less than one Earth radius, a virtual impactor has been found. If the close approach distance at convergence of Newton's method is above one Earth radius, but the width $\sqrt{\lambda_2}$ of the MTP ellipse is such that impact is possible, a VI exists, although it is not on the LOV.

2.5 Preimage of the virtual impactor into element space

We need now to describe the subset in the elements space containing the orbits compatible with the observations in the sense of (1) that lead to an impact at the same return of the VI found in the previous step. The displacement ΔT_0 of the last iteration of Newton's method is negligibly small, $\Delta T_0 \simeq \underline{0}$. The point on the MTP of convergence of Newton's method defines a 4-dimensional ellipsoid $Z(\Delta T_0) \simeq Z(\underline{0})$ of changes in orbital elements ΔX with respect to the VI, such that, as long as the linear approximation applies, $\Delta X \in Z(\underline{0})$ implies an impact, actually one passing through the same point on the target plane.

Our purpose is to define a negative observations strategy that allows us to exclude the possibility of an impact; thus we need to select a safety area on the MTP, such that the orbits passing outside are certainly not dangerous. We define a rectangle $R \subset \mathcal{T}$, centered on the MTP intercept of the VI, such that in the direction of the weak axis (eigenspace of λ_1), the rectangle extends ± 5 Earth radii, and in the direction parallel to the eigenspace of λ_2 the rectangle spans the $|\sigma| \leq 5$ region, as in Figure 1. We are moving only 5 radii along the weak direction, anyway much less than $\sqrt{\lambda_1}$ (which is typically several AU); on the contrary we do not move beyond the $\sigma = 5$ line in the direction across the axis of the MTP confidence boundary, because $\sqrt{\lambda_2}$ is typically less than one Earth radius. These conditions are chosen in such a way that an orbit that intersects the MTP outside R either fits badly the existing observations, or cannot come too close to the Earth; however, they may not be appropriate to a case in which λ_2 is of the same order of magnitude of λ_1, as it would be the case if the impact could take place soon after the discovery epoch.

Now let ΔT be variable inside the rectangle R; for each ΔT the preimage in $\mathcal{X}$ is a 4-dimensional ellipsoid $Z(\Delta T)$ (at least in the linear approximation); the preimage $X_R \subset \mathcal{X}$ of R is the union of all of the 4-dimensional ellipsoids of each point in R

$$X_R = \bigcup_{\Delta T \in R} Z(\Delta T) \ .$$

This region is a generalized cylinder, topologically (but not isometrically) equivalent to a product of a rectangle and of the interior of an ellipsoid in 4-dimensional space. To explore it fully, e.g. with a Monte Carlo method, is possible but computationally expensive, being a 6-dimensional region. Thus we resort to an approximation. The region $Z(\Delta T)$ is obtained by computing $\Delta E_T = B_T \ \Delta T$; the 4-dimensional ellipsoid

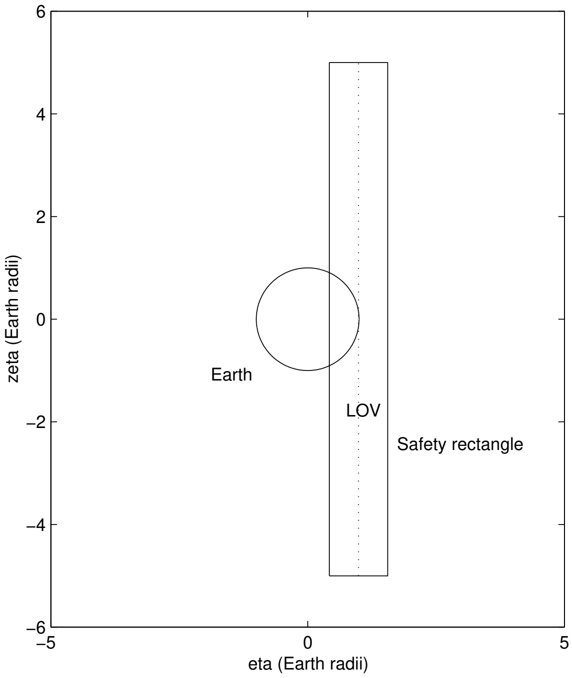

Figure 1: Safety rectangle R on the target plane, for the 2046 VI of 1998 OX_4 described in the fourth column of Table 1.

changes with ΔE_T, thus with ΔT. However, $\Delta T \in R$ is very small, and so is the corresponding ΔE_T, at least for impacts with low probability that occur many decades after the initial conditions, i.e., a small change in the initial semimajor axis moves the point on the MTP a great deal. Thus we approximate the region X_R by the Cartesian product

$$X_R \simeq H_T(R) \times Z(\underline{0})$$

which can be computed in a simple and explicit way.

2.6 Projection from element space onto the sky plane

Once the VIR, with safety margins, X_R has been computed, or at least approximated in the $\mathcal{X}$ space, we need to be able to compute the image of it onto the sky at any given time in which the observations might take place. Given the approximate representation of X_R as a product, we can separately map the two factors, which can be done in a very efficient way.

One observation at time t is mathematically described by a smooth *observation function* $F_S : \mathcal{X} \longrightarrow \mathcal{S}$, where $\mathcal{S}$ is the celestial sphere. A point on the celestial sphere is represented by two angular coordinates $S = (\alpha, \delta)$. At time t the virtual impactor has an ephemeris $S_0 \in \mathcal{S}$.

We first map the four corners R_i of R to the four initial conditions $H_T(R_i) = X_{R_i} \in \mathcal{X}, i = 1, \ldots, 4$ according to (4). For these four initial conditions, we compute four displacements with respect to the VI ephemerides $\Delta S_i = F_S(X_{R_i}) - S_0$. They mark on the sky a quadrilateral $F_s(H_T(R))$ which is almost flattened to a segment in the cases of a low probability impact at a very late epoch. (See in the Figures of the next section.)

The space of changes in orbital elements ΔX is a product $\mathcal{X} = \mathcal{E}_T \times \mathcal{L}_T$; we need to map the 4-dimensional ellipsoid $Z(\Delta T_0) \subset \mathcal{L}_T$ onto the sky plane. To do this we use the same formalism used in [Milani 1999, Sec. 3] to map the confidence ellipsoid onto the sky: the same formulas are used, but they refer to a 4-dimensional space rather than to a 6-dimensional space.

To this purpose, let us consider the differential (linearized map) DF_S of F_S, computed at the VI initial conditions. If we restrict the map F_S to the 4-dimensional subspace $\mathcal{L}_T$, we can describe DF_S by means of a 2×4 matrix of partial derivatives, with rows $\partial\alpha/\partial L_T$ and $\partial\delta/\partial L_T$. Let $\mathcal{E}_S$ be the 2-dimensional subspace of $\mathcal{L}_T$ spanned by these two gradients, and $\mathcal{L}_S$ the orthogonal space (also 2-dimensional), so that $\mathcal{L}_T = \mathcal{E}_S \times \mathcal{L}_S$.

Then DF_S restricted to $\mathcal{L}_T$ is a composition of an orthogonal projection $\Pi_{\mathcal{E}_S}$ onto $\mathcal{E}_S$, followed by an invertible map $A_S : \mathcal{E}_S \longrightarrow \mathcal{S}$:

$$DF_S = A_S \circ \Pi_{\mathcal{E}_S} \ .$$

It is possible to perform a change of coordinates in $\mathcal{L}_T$, by means of an orthogonal 4×4 matrix V_4, in such a way that

$$\Delta L_T = V_4 \left[\begin{array}{c} \Delta E_S \\ \Delta L_S \end{array} \right]$$

with $\Delta E_S \in \mathcal{E}_S$ and $\Delta L_S \in \mathcal{L}_S$; then the normal matrix C_L in the new coordinate system is changed into

$$V_4^T \, C_L \, V_4 = \left[\begin{array}{cc} C_{E_S} & C_{E_S L_S} \\ C_{L_S E_S} & C_{L_S} \end{array} \right] \ .$$

This allows to select a representative in the $\mathcal{L}_T$ space for every point on the sky near the ephemerides of the virtual impactor:

$$\Delta L_T = V_4 \left[\begin{array}{c} \Delta E_S \\ -C_{L_S}^{-1} \, C_{L_S E_S} \, \Delta E_S \end{array} \right] = V_4 \left[\begin{array}{c} I \\ -C_{L_S}^{-1} \, C_{L_S E_S} \end{array} \right] B_S \, \Delta S = H_S(\Delta S) \qquad (5)$$

where $B_S = A_S^{-1}$. Equation (5) is fundamentally the same as Eq. (4), but for the dimension of the matrices; it can be used by defining the sky ellipse $K \subset \mathcal{S}$ image of $Z(\Delta T_0)$ by the linearized map DF_S, by computing the pullback $H_S(K)$, sampled by a discrete set of points in the $\mathcal{L}_T$ space (hence in the $\mathcal{X}$ space, by adding the VI value of ΔE_T); the image of these points is the semilinear confidence boundary, which is a good approximation of $F_S(Z(\Delta T_0))$.

The last step in the procedure is to represent the image, on the sky, of the Cartesian product $H_T(R) \times Z(\Delta T_0)$ by the Cartesian product of the displacements on the sky, that is

$$F_S\left(H_T(R) \times Z(\Delta T_0)\right) \simeq F_S(H_t(R)) \times F_S(Z(\Delta T_0)) \ .$$

The left hand side is, by definition, the *skyprint* of the VI, and the right hand side is computable in practice by means of a moderate number of observations predictions, since only one-dimensional subsets of $\mathcal{X}$ have to be computed. As discussed in [Milani 1999], conceptually we need anyway to select a finite sample of some region of the $\mathcal{X}$ space of initial conditions, but in practice to sample a one dimensional line is a small computational load, to sample a 6-dimensional space is computationally very expensive.

Please note that these computations contain a number of approximations which are accurate only provided the skyprint is small. If this is the case, as in the Figures of the next section, then the product structure is clearly visible, since $F_S(H_t(R))$ is almost a segment and $F_S(Z(\Delta T_0))$ is almost an ellipse, thus the skyprint looks very much like the drawing of an ordinary 3-dimensional cylinder. If the skyprint was large, the linear, semilinear, and Cartesian product approximations used in this section would break down.

On the other hand, if the skyprint was big, to the point of being not well approximated with the linear and semilinear formalisms, then the negative observation campaign would not be worthwhile. We do not have a simple way to predict in which cases the skyprint will be small, in which cases it will be too large to be usable; this depends upon the size of the confidence region (in turn depending upon the number of observations and the length of the observed arc), the date of the possible impact, its probability, the date and the geometry of the observation. Thus we need to apply the computational procedure outlined above case by case; to this purpose we have developed rather efficient software.

3 The 1998 OX_4 virtual impactors

We apply the method described above to the lost asteroid 1998 OX_4. Attention was focused on this object after we announced that it has a remote, but not negligible impact probability. It was discovered on July 26, 1998 (UT) by Jim Scotti with the 0.9-m Spacewatch telescope on Kitt Peak [MPEC 1998-O27], and observed only over a short arc, due its intrinsic faintness and unfavorable observing conditions [MacMillan 1999]. The keplerian orbital elements with RMS uncertainty at epoch 1999 Jan 22.0 TDT are: semimajor axis 1.581 ± 0.029 AU, eccentricity 0.486 ± 0.014, inclination 4.53 ± 0.16 degrees, longitude of ascending node 300.23 ± 0.15 degrees, argument of perihelion 116.549 ± 0.035 degrees, mean anomaly 35.57 ± 0.88 degrees. The object is already lost by several degrees, and the large uncertainty in semimajor axis implies that the length of the confidence region will continue to grow rapidly.

Table 1: 1998 OX_4 Virtual Impactor Data.

Year of Impact	2014	2038	2044	2046
Date (UT)	Jan 20.539	Jan 20.203	Jan 20.686	Jan 20.264
σ	2.422	0.746	0.760	0.779
H mag	21.2	21.3	21.3	21.3
$\mathrm{MOID_{GF}}$ ($R_\oplus$)	0.480 ± 0.126	0.373 ± 0.174	0.710 ± 0.188	0.990 ± 0.189
Γ_{MTP} (AU/σ)	397	95	47	18
P_{LD}	2×10^{-6}	6×10^{-6}	20×10^{-6}	40×10^{-6}
$P_{R_\oplus}$	0.3×10^{-7}	1×10^{-7}	2×10^{-7}	2×10^{-7}
Keplerian orbital elements at epoch `MJD 51024.0 TDT`				
a (AU)	1.626809266912	1.606833539990	1.607099946443	1.607352219135
e	0.506326452475	0.497502331734	0.497622010073	0.497735281195
I (deg)	4.767779650096	4.664179526644	4.665577804419	4.666901397937
Ω (deg)	300.4692804569	300.3776807433	300.3789300531	300.3801125065
ω (deg)	116.5952500167	116.5727153899	116.5730145568	116.5732965881
M (deg)	310.6465675773	309.6526746252	309.6661466286	309.6788984818

3.1 The returns and the impacts

Our analysis of the orbital evolutions propagated from the catalog of 1201 multiple solutions shows that there are four possible collisions associated to this asteroid in the next half century: collisions are possible in January of 2014, 2038, 2044, and 2046. The existence of each of these impacting solutions has been confirmed by totally independent orbit computations (P. Chodas, private communication, 1999). Table 1 provides details for the corresponding VIs, including the MOID with gravitational focusing effects included ($\mathrm{MOID_{GF}}$), the stretching on the MTP (Γ_{MTP}), and the probabilities of encounter within the lunar distance and of Earth collision (P_{LD} and $P_{R_\oplus}$, respectively). The σ values listed in the table refer to the increase in ξ, the RMS of the observational residuals, relative to $\xi_0 = 0.528$, the RMS for the nominal orbit, according to the relation $\sigma^2 = 2N_{obs}\,(\xi^2 - \xi_0^2)$, where $N_{obs} = 21$ is the number of astrometric positions used in the fit. The keplerian orbital elements for each VI are also given. The J2000 reference frame is used throughout this paper.

These impact possibilities result from resonant returns after other close approaches closer to the present time. The 2014 VI is comparatively far from the other ones in the confidence region of element space, and undergoes a very close approach in January 2001. The three VIs for 2038, 2044, 2046 belong to virtual asteroids close together along the line of variations; all three have a shallow encounter in January 2001, and they all have a quite close approach in January 2003, after which a high order resonant return leads to the possible collision. A complete description of the cascade of returns would be more complex, because there are many shallow encounters and even non-resonant returns: this asteroid has a moderate inclination ($4^\circ.5$) and approaches within 0.1 AU can occur quite far from the descending node, and even near the ascending node.

It needs to be stressed that the use of the discrete set of 1201 virtual asteroids to represent the line of variations implies a resolution limit, thus it cannot be guaranteed that the impact possibilities listed in Table 1 are the only ones; however, the ones which might have been missed must have very small probabilities. Another possible reason of incomplete exploration of the confidence region is due to its large size, resulting in a shape significantly different from the ellipsoid described by the quadratic Eq. 1. More refined scans of the confidence region could be done, but a decision needs to be taken on the level of probability at which an impact possibility deserves a negative observation campaign.

3.2 Observing opportunities

The different orbital histories for the 2014 VI on one hand, and the 2038, 2044 and 2046 VIs on the other, lead to different observing opportunities. Given that this object is faint, the main constraint is that observations are possible only when the geocentric distance is small, e.g., less than 0.5 AU, the best opportunities corresponding to the closest approaches. For the 2014 VI there is an observing opportunity during a very close approach in January 2001. For the 2038, 2044 and 2046 VIs there are observing opportunities during a shallow encounter in February 2001, and even better ones during the January 2003 close approach. We shall discuss all three of these opportunities. Detailed ephemerides for each opportunity described below are available at `http://copernico.dm.unipi.it/neohome/virimp`

The 2014 VI in January 2001

This VI approaches the Earth in the third week of January 2001. The encounter takes place after passage of the asteroid from perihelion: the object is too close to the Sun to be observed before the encounter, and in just one day moves from the evening sky into the opposition region where it will rapidly slow down during the following days. On January 24 it will already move only one degree per day being around magnitude 14.5 V; then it will fade 0.3-0.5 magnitudes per day until the end of the month. The observing conditions on January 24 are given in Table 2, and the size and location of the region of the sky to be searched are shown in Figure 2. The region to scan is the envelope of the ellipses, and measures about 9 arcmin in right ascension and 16 arcmin in declination, thus the VI is easy to find, if it exists.

The 2038, 2044 and 2046 VIs in February 2001

The three VIs happen to be all visible at about magnitude 20.5, close to opposition, in the second part of February 2001 (see Table 3, which has been computed for February 23), although observations can start one month earlier and stop one month after that date. They will be close to the celestial equator and stay far from the Milky Way. The three VIs are less than a degree apart from each other, and the envelopes of their projected uncertainty regions are about 16 arcmin or less in right ascension and 12 arcmin or less in declination; one example of the skyprint is shown in Figure 3. Few telescope frames should be necessary to exclude them all; however, the telescopes to be used need to have rather faint limiting magnitudes.

The 2038, 2044 and 2046 VIs in February 2003

All three VIs are in favorable conditions for observation after an Earth encounter taking place in January, much like the 2001 opportunity for the 2014 VI (see Table 4 and Figure 4, all referred to February 3). At this time the three VIs are comparatively bright (about magnitude 17.5), but the proper motion has already slowed down to an almost main-belt rate. Their projected uncertainties are larger than in the previous cases, up to 1 degree for the 2046 VI, and the three regions to be searched for the three VIs are a few degrees apart. Therefore, it will be necessary to take more frames, but this is compensated for by the much brighter magnitude at which the search has to be conducted, allowing the use of comparatively small telescopes.

3.3 Observing conditions and tradeoffs

The main problem in devising a strategy for negative observations of VIs is the expected apparent magnitude; lesser problems are the proper motion, the lunar phase and the galactic latitude.

The absolute magnitude of 1998 OX_4 is estimated, by a fit to all available photometric measurements, at $H = 21.3 \pm 0.23$; however, this value is estimated for the nominal orbit, further changes by ± 0.2 occur between the different virtual asteroids spanning the confidence region; this is taken into account in the predicted magnitudes listed in Tables 2–4. The photometric data of Spacewatch are generally quite good (P. Pravec, private communication, 1999), although an absolute calibration is difficult for such faint objects, for which there are in most cases no multi-opposition orbits.

So, in this case the main source of uncertainty in the prediction of the apparent magnitude at some later apparition is not the photometric uncertainty, but the change of the luminosity with rotational phase and aspect angle. From known lightcurves the changes can be up to ± 1 magnitude, although such extreme values are unusual. Phase effects are also difficult to model reliably, if there is no available information on the physical properties of the asteroid [Bowell 1989], as in this case.

The conclusion is that in this case the uncertainty of the predicted apparent magnitude is large, and to be reasonably confident in a negative observation it is necessary that the given telescopes, exposures and detection techniques be validated for about two magnitudes fainter than the predicted values given in Tables 2–4.

The dates we have selected for the January-February 2001 and for the February 2003 opportunities have also taken into account the need to have a moderate proper motion (too high could result in too large trailing losses and/or pointing problems, too low could result in the VI hiding in the glare of a bright star), a large enough galactic latitude and an appropriate lunar phase, especially for the dimmer case. These dates can be changed for the convenience of the observers, but the observing time windows

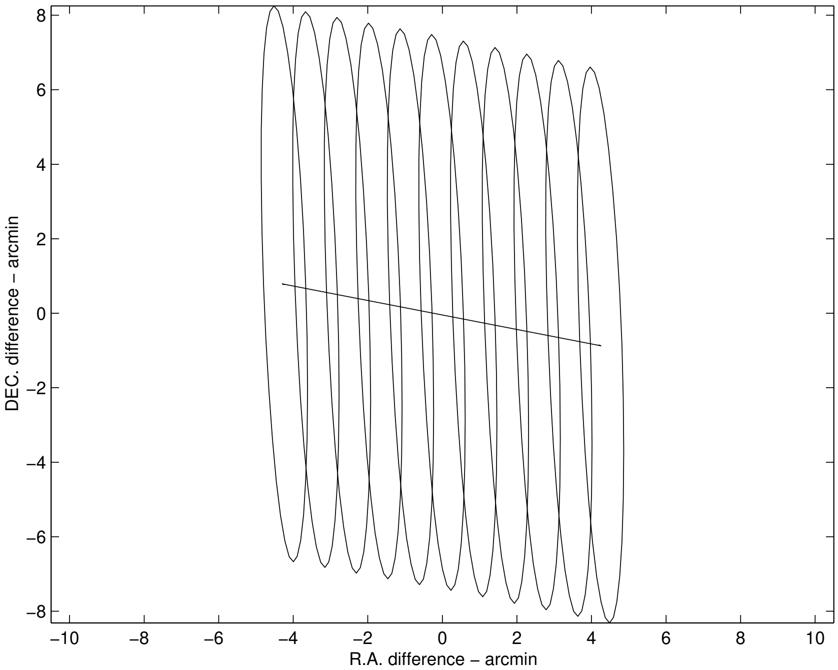

Figure 2: Skyprint for the 1998 OX_4 2014 VI on 2001 Jan. 24. See Table 2. Note that in this and subsequent figures West is on the right.

Table 2: 1998 OX_4 VI Ephemeris, Jan. 2001

Epoch: 2001 Jan. 24.0 UT	
Impact Year	2014
Right Ascension	8:42:12.1
region	$\pm 5'$
motion (deg/day)	0.817
Declination	$+5^\circ\ 41'\ 39''$
region	$\pm 8'$
motion (deg/day)	-0.371
V mag	14.2
Solar Elongation	$165^\circ.9$
Galactic latitude	$27^\circ.2$
Solar Distance (AU)	1.0108
Earth Distance (AU)	0.0272

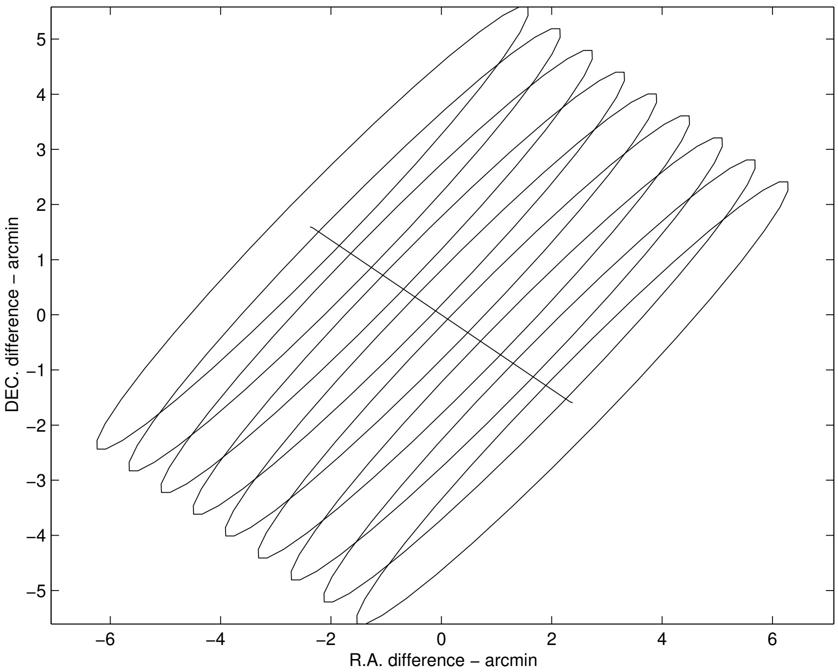

Figure 3: Skyprint for the 1998 OX_4 2046 VI on 2001 Feb. 23. See the last column of Table 3.

Table 3: 1998 OX_4 VI Ephemeris, Feb. 2001

Epoch: 2001 Feb. 23.0 UT			
Impact Year	2038	2044	2046
Right Ascension	11:06:27.8	11:05:11.8	11:03:59.3
region	$\pm 9'$	$\pm 8'$	$\pm 6'$
motion (deg/day)	-0.652	-0.650	-0.649
Declination	$-5^\circ\ 44'\ 57''$	$-5^\circ\ 38'\ 6''$	$-5^\circ\ 31'\ 32''$
region	$\pm 5'$	$\pm 5'$	$\pm 6'$
motion (deg/day)	0.230	0.230	0.231
V mag	20.5	20.5	20.5
Solar Elongation	$161^\circ.3$	$161^\circ.6$	$161^\circ.8$
Galactic latitude	$48^\circ.5$	$48^\circ.4$	$48^\circ.3$
Solar Distance (AU)	1.3451	1.3438	1.3425
Earth Distance (AU)	0.3699	0.3681	0.3665

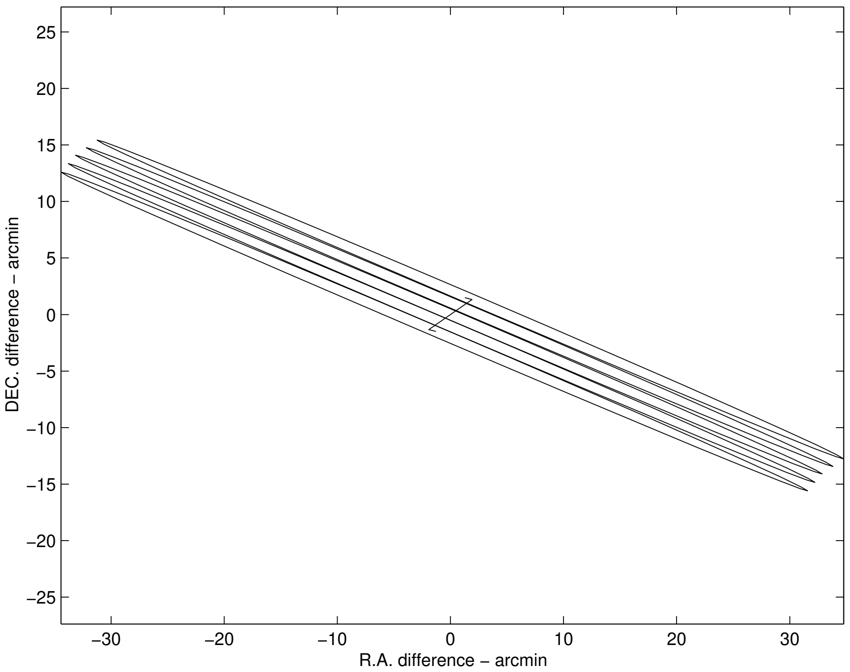

Figure 4: Skyprint for the 1998 OX_4 2046 VI on 2003 Feb. 3. See the last column of Table 4.

Table 4: 1998 OX_4 VI Ephemeris, Feb. 2003

Epoch: 2001 Feb. 3.0 UT			
Impact Year	2038	2044	2046
Right Ascension	8:35:21.0	8:20:07.6	8:05:22.4
region	$\pm 6'$	$\pm 15'$	$\pm 35'$
motion (deg/day)	0.280	0.587	0.906
Declination	$+6^\circ\ 41'\ 52''$	$+7^\circ\ 44'\ 42''$	$+8^\circ\ 41'\ 49''$
region	$\pm 8'$	$\pm 12'$	$\pm 16'$
motion (deg/day)	-0.068	-0.136	-0.200
V mag	17.1	17.1	17.2
Solar Elongation	$167^\circ.7$	$165^\circ.9$	$163^\circ.5$
Galactic latitude	$26^\circ.2$	$23^\circ.3$	$20^\circ.4$
Solar Distance (AU)	1.0827	1.0800	1.0774
Earth Distance (AU)	0.0992	0.0971	0.0954

are short (a few days only) in the cases in which very close approaches are used, as in the case of the 2014 VI; thus it is advisable to have telescope and observer time reserved at different sites, to be safe against the possibility of bad weather. There is, in fact, another observing opportunity in July 2000 for all four VIs, but observing conditions would be significantly worse, because of fainter apparent magnitudes, the lunar phase and the proximity of the galactic disk.

For the 2038, 2044 and 2046 VIs we have presented two opportunities for negative observations; the choice among the two amounts to a tradeoff between the size of the telescopes required and the difficulty of the observation; also the time we have to wait to know is an issue. The February 2001 opportunity would allow us to guarantee safety from these possible impact two years earlier. However, the telescopes needed to guarantee detection up to apparent magnitude 22.5 are not available as a matter of routine for asteroid observations: the observing nights have to be requested in advance, keeping margins for possible bad weather, and experienced asteroid observers need to be available at observatories where they may not be normally working. On the other hand the total telescope time could be small, given that the size of the regions to be scanned are small (see Figure 3 and Table 3).

Since 2038, 2044 and 2046 are far in the future, we may decide that we can wait two years longer to know if these impacts are indeed possible. The February 2003 opportunity would allow the use of telescopes of a size which is normally available for asteroid discovery and follow-up, including some good amateur sites. But, in this case the time window is more restricted and the observing conditions change rapidly, thus several observatories should be invited to join as a safeguard against bad weather at some site. Moreover most small telescopes would have to scan several frames to cover the three skyprints, which are somewhat larger than in the other case. Thus coordination is more an issue than telescope size. At this encounter it is in fact possible that 1998 OX_4 would be recovered anyway, even if it is not going to have close approaches either in 2038, on in 2044, or in 2046.

For the 2014 VI there is no other opportunity with good observing conditions (say apparent magnitude < 22) until the possible impact. Although this is the VI with lowest impact probability of the four, the earlier time of possible impact and the lack of a later opportunity, in our opinion, suggest that the opportunity given in this paper for negative observation should not be missed.

3.4 Positive Detections

So far we have discussed negative observations, but we need to discuss the consequences of a positive observation in one of the skyprints defined by some VI, even if this is unlikely to happen. It is clear that in the unfortunate event that a detection were to occur either inside or close to some VI skyprint a more serious confirmation process would become necessary. It would be essential to ascertain the correct identification of the object, at first by using its apparent motion, and then by continuing the observations over several nights. Let us assume that this confirmation procedure would be successful, and that the real asteroid was indeed very close (in the initial conditions space) to the VI.

To assess the possible consequences, we have simulated an observation only 5 seconds to the East in right ascension from the nominal prediction for the 2046 VI on February 23, 2001 (and with the same declination). As it can be see from Figure 3, this is well inside the skyprint. Using the VI orbit from the last column of Table 1 as initial guess, we have applied differential corrections and found a new nominal orbit with a close approach at 0.008 AU in January 2046. By applying the Newton's method of Sec. 2.4 we have found a VI at $\Delta\sigma = 1.29$ and with stretching 0.0045, which implies a probability of impact roughly 1/300.

We have also checked that this conclusion would be robust with respect to the statistical model of the observation errors, by the following procedure. We have removed from the fit the observation in 1998 resulting in the largest residual (more than 2 arcsec in R.A., to be compared with an overall RMS of 0.529 arcsec; it is indeed reasonable to consider it as an outlier). But, with the 2001 skyprint observation, the best fit solution still has a close approach in 2046 at 0.006 AU, and a VI at $\Delta\sigma = 0.84$, with essentially the same probability of impact.

We can conclude from this unpleasant experiment that, if the real asteroid was to be found in one of the skyprints described above, the probability of impact would immediately increase to worrisome values. On the other hand, the asteroid would not be lost anymore, and it could be followed up to further refine the orbit; in the worst case, if the impact was confirmed by further observations, there would be plenty of time to plan for deflection.

4 Conclusions

Given the importance of a VI observing campaign and the likely public interest generated by it, the organization and the validation of such a project could best be handled by a committee formed by an appropriate international organization, e.g., the International Astronomical Union. Any oversight committee must comprise experts both in computational and observational work. We propose here some general principles to be considered for such a campaign.

4.1 Coordination and Validation Issues

The organizational efforts needed for a VI campaign depend greatly on the circumstances of the apparition. In particular, the predicted brightness of the VI will be decisive in determining whether dedicated time on large telescopes will be necessary, or whether smaller instruments can be used. A comparatively large community of professional and non-professional asteroid observers has been successful in the follow-up of NEAs that are generally brighter than $V = 20$. So, when a VI is brighter than $V = 18$, this community can perform the necessary observations, but issues of validation and coordination will be more challenging than if a few experienced professionals with very large instruments are involved, as would be necessary if the VI is fainter than $V = 18$.

Given the special responsibility in the validation of a negative observation, we suggest a few general guidelines for any VI search:

1. At least two independent observing teams using two different facilities should be involved in the search for each VI around the same time. The allocation of observing time should be sufficient to take into account bad weather conditions. It may be advisable to get confirmation of the negative observations on a second night from one of the two teams.

2. The observation epoch should be chosen carefully to consider factors such as the expected galactic latitude, solar elongation, and brightness, as well as the lunar phase. Observers must be very careful if the VI is expected to move rather slowly (5-10 arcmin/day or less) to ensure that it does not remain hidden by any star glare throughout the whole or most of the exposure time. Confusion with field stars is less of an issue if multiple exposures with the same instrument are available.

3. The observers must be able to detect objects two magnitudes fainter than the nominal one expected for the VI, taking into account trailing losses due to the asteroid's apparent motion, as discussed in Sec. 3.3. This "safety cushion" should be adjusted on a case by case basis, depending on the photometric accuracy of the observers in the discovery apparition and on the magnitude consistency observed. The limiting magnitude needs to be calibrated for objects with a specific motion rate. For some long-lost objects discovered photographically it may be advisable to extend the factor of safety to three magnitudes. To validate the capability to detect at the level needed, the observer could be required to measure the position, taken at a short time interval, of a known asteroid near the VI which is somewhat fainter than the predicted VI magnitude.

4.2 Photographic Archives

The use of photographic archives to look for VIs is an interesting field of investigation, although special care is required. Because of the less reliability of photographic magnitudes, an extra magnitude of cushion is needed. This approach has two other disadvantages. First, we will rarely find opportunities to detect the small objects which will generally be the subject of VI searches, and second, even though we may find some very deep plates, the trailing loss remains a serious problem. The great advantage is that this search can be done at any time, does not require any telescope resources, and the reliability of these results is easy to verify. Close approaches of VIs, as we described in the January 2001 and February 2003 cases, represent the best examples in which the more deep and consistent surveys can give a precious contribution. By using the elements given in Table 1 one can compute ephemerides for previous VI close encounters and use them to search plate archives for promising opportunities.

4.3 Follow-up vs. negative observations

The present work does not obviate the need to follow-up NEAs with observations after they have been discovered. On the contrary, it is widely accepted that the current level of follow-up work is not yet at a satisfying level even for the northern hemisphere.

The problem is more important for smaller asteroids with absolute magnitude $20 < H < 22$, corresponding to diameters 200 m < D < 500 m for an average albedo. Most of these are discovered during close approaches, and end up being lost unless a chance rediscovery takes place in a later close approach. The strategy we have proposed is most applicable to these small bodies; for the larger objects with $H < 20$ recovery should be considered mandatory if they can have impacts upon the Earth, even with small probabilities.

For even smaller objects, in the range of $22 < H < 25$, not officially designated as Potentially Hazardous Asteroids, but still capable of causing a severe regional disaster, our technique of excluding the possibility of collisions without recovery might be the only option. Although the present method will usually allow us to deal with the hazard posed by objects down to the Tunguska size level ($H = 25$), we may not always have another chance to observe associated Virtual Impactors before a threatening event. Indeed the discovery apparition may lead to a resonant return with collision without an opportunity to rule out the collision possibility prior to the supposed impact.

Another source of concern is the following. As the discovery rate continues to increase faster than the follow-up capability, especially at faint magnitudes, there is a real danger of creating Virtual Impactors at an unacceptably high rate, to the point that negative observation campaigns, their efficiency notwithstanding, would use too much observational resources.

References

[Bowell et al. 1993] Bowell, E., L. H. Wasserman, K. Muinonen, R. H. McNaught, and R. M. West 1993. A Search for the lost Asteroid (719) Albert. *BAAS* **25**, 1118.

[Bowell 1989] Bowell, E., B. Hapke, D. Domingue, K. Lumme, J. Peltoniemi and A.W. Harris 1989. Application of photometric models to asteroids. In *Asteroids II*, (R.P. Binzel, T. Gehrels, and M.S. Matthews, Eds.) University of Arizona Press 524–556.

[Chodas and Yeomans 1996] Chodas. P and D. K. Yeomans 1996. The orbital motion and impact circumstances of Comet Shoemaker-Levy 9 and Jupiter. In *The Collision of Comet Shoemaker-Levy 9 and Jupiter*, (K.S. Noll, H.A. Weaver, and P.D. Feldman Eds.), Cambridge University Press.

[MacMillan 1999] MacMillan, R. CCNet DIGEST, June 14, 1999, `http://abob.libs.uga.edu/bobk/ccc/cc061499.html`.

[Milani 1999] Milani, A. 1999. The Asteroid Identification Problem I: recovery of lost asteroids. *Icarus* **137**, 269–292.

[Milani and Valsecchi 1999] Milani, A., and G.B. Valsecchi 1999. The Asteroid Identification Problem II: Target Plane Confidence Boundaries, *Icarus*, **140**, 408–423.

[Milani et al. 1999] Milani, A., S.R. Chesley, and G.B. Valsecchi 1999. Close approaches of asteroid 1999 AN10: resonant and non-resonant returns, *Astron. Astrophys.* **346**, L65-L68.

[MPEC 1998-O27] MPEC 1998-O27, Minor Planet Center, 31 July 1998.

[Muinonen 1999] Muinonen, K. 1999. Asteroid and comets encounters with the Earth. In *The dynamics of the small bodies of the solar system: A major key to solar system studies*(A. E. Roy and B. Steves, Eds.), Kluwer Academic Publ., pp. 127–158 .