1^{st} MORE TEAM MEETING

Roma, 26-27 February 2007

THE BEPICOLOMBO ORBIT DETERMINATION PROBLEM: PRESENT, PAST AND FUTURE

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PLAN

1. Present understanding of the problem
2. Past simulations
3. Future software system

1 The MORE orbit determination problem and how we currently understand it

1.1 MORE: science goals

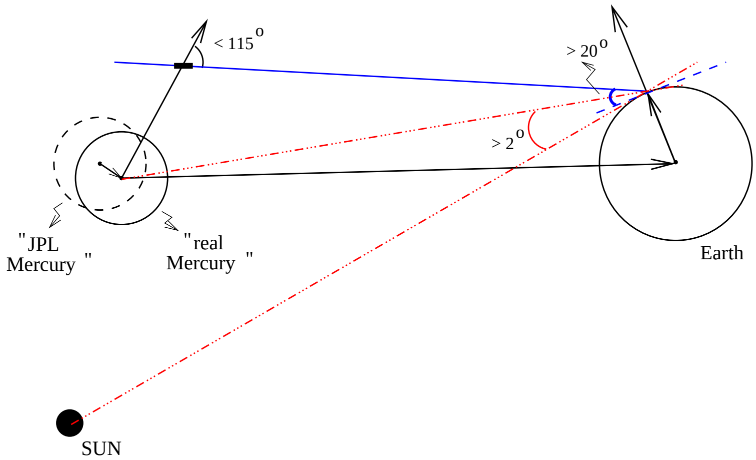
S1 To measure the rotation state of the planet Mercury, to constrain the dimension and the state of the core (ROTATION EXPERIMENT).

S2 To measure the global gravity field of Mercury, to constrain the deep internal structure; to measure the local gravitational anomalies, to constrain the mantle structure, the mantle-crust interface, the mascon (GRAVIMETRY EXPERIMENT).

S3 To measure the orbit of Mercury and the propagation of radio-waves between Earth and Mercury to test the theory of General Relativity, constrain possible alternative theories and provide an improved dynamical model for the Solar System (RELATIVITY EXPERIMENT).

s4 To determine the MPO orbit around Mercury for the duration of the Mercury orbit mission (auxiliary goal, required by the previous ones, by the altimetry experiment and with possible operational implications).

1.2 Dynamics



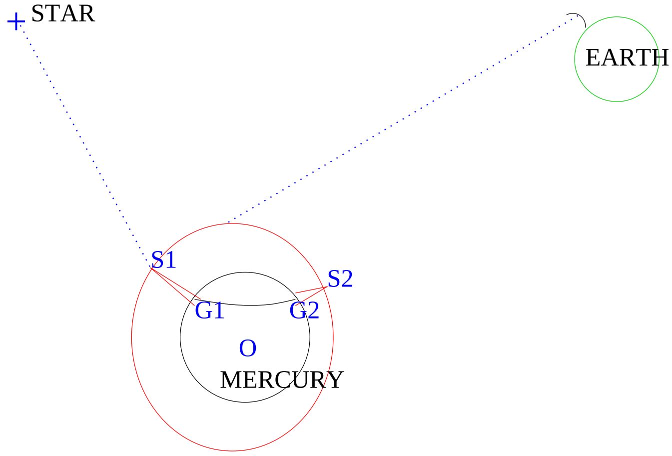
D1 The MPO orbit around Mercury, with perturbations: gravitational and non gravitational, tides, relativistic and using the ISA accelerometry data.

D2 The orbit of Mercury and of the Earth, taking into account the current model for the other planets and the Moon, in a fully relativistic framework with all the Post-Newtonian parameters and the Sun's gravitational parameters (including J_2 of the Sun).

D3 The rotation of Mercury, including the spin-orbit resonance, obliquity, libration in longitude and possible other deviations from Cassini's laws (affects the S/C mercury-centric orbit).

d4 The rotation of the Earth, according to the most up to date IERS model.

1.3 Measurements



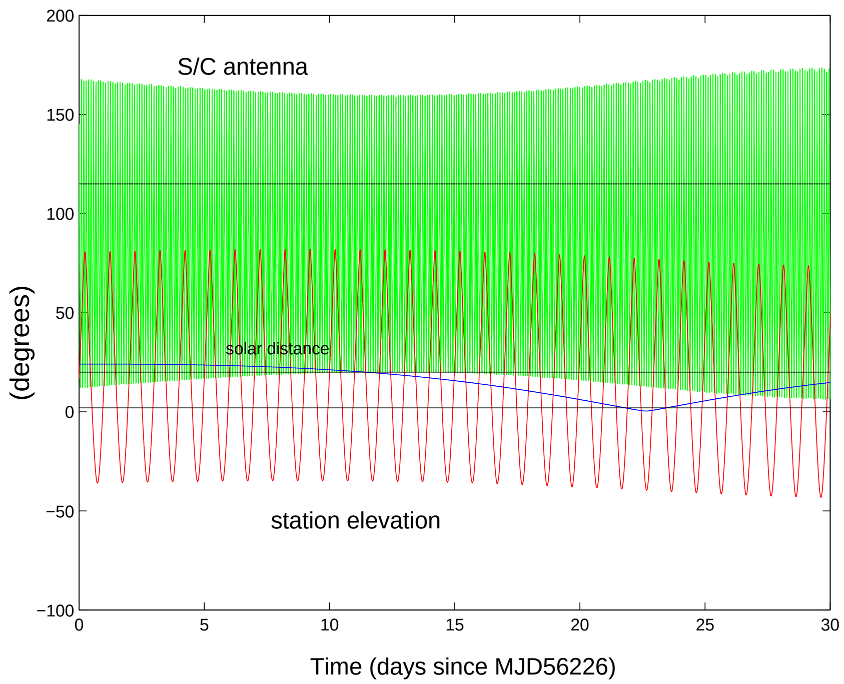
R1 Range and range rate between the ground station(s) and the Mercury Planetary Orbiter (MPO) S/C, removing plasma effects by the multi-frequency link and taking into account spacetime curvature.

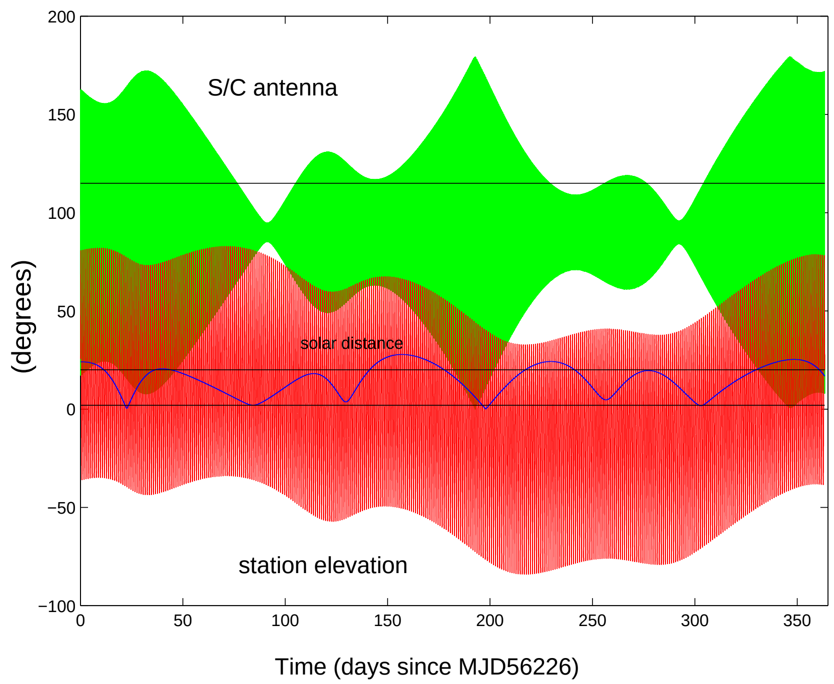
R2 Non gravitational perturbations acting on the S/C, measured by ISA (with calibration problems).

R3 The angles between a number of reference points on Mercury's surface, as seen from the MPO, and the axes of an inertial reference system.

The measurements R3 are obtained by combining suitably processed high resolution camera images (repeated on the same area of the surface) and S/C attitude data deduced from the star trackers measurements.

1.4 Visibility conditions





1.5 Parameters to be solved (Goals)

- P1 Coefficients of the spherical harmonics of the gravity field of Mercury, static part; of degrees from 2 to at least 25 (possibly 30); goal S2.
- P2 Dynamical Love number k_2 for the second harmonic solar tides, possibly phase delay; goal S2.
- P3 The main deviations of the rotation state of Mercury from the resonant Cassini state, e.g., obliquity and amplitude of libration in longitude; goal S1.
- P4 Post-Newtonian Parameters for the Sun's gravity field, e.g., γ, β, also mass, J_2 of the Sun; goal S3.
- P5 Initial conditions for the orbits of Mercury and Earth; goal: S3 (solar system ephemerides).

These parameters are **global**, that is constant over the mission, and are by themselves mission goals (even if the conclusions, to be drawn in geophysics and fundamental physics, may be obtained by further processing; e.g., computing the moments of inertia of Mercury, the main moment of inertia of the mantle).

1.6 Parameters to be solved (Auxiliary)

p6 Initial conditions for the S/C mercury-centric orbit, for each observed arc ($\simeq$ one observing session, 1–2 per day).

p7 Accelerometer calibration coefficients, for each observed arc.

These parameters are **local**, that is variable from one observed arc to the next. A delicate problem is the knowledge of the correlations between values of the local parameters in nearby observed arcs.

The accelerometer calibration may be assisted by thermal information (provided by the accelerometer auxiliary instrumentation); however, a posteriori (digital) calibration is needed anyway.

The S/C initial conditions for each day may be initialized from some long arc solution, which is anyway of lower accuracy (for lack of accelerometer a posteriori calibration).

These data are not primary science goals by themselves, although they are important for other reasons, e.g., the S/C orbit is an essential input for laser altimetry.

1.7 Rank deficiency

If the BepiColombo orbit determination is summarized in a **normal equation** relating the corrections ΔX of the parameters vector X to the normalized residuals vector Ξ

$$B^T\, B\, \Delta X = -B^T\, \Xi$$

where $B = \partial\Xi/\partial X$, then the normal matrix $C = B^T\, B$ is numerically singular, the **covariance matrix** $\Gamma = C^{-1}$ cannot be reliably computed and anyway the confidence ellipsoid (with matrix C) is huge. If the differential corrections are iterated, the procedure diverges. If the iterations are stopped, the solution obtained is disastrously wrong.

This disaster is **rank deficiency**, **exact** if C is singular, **approximate** if C is very badly conditioned (ratio of largest to smallest eigenvalue $>$ inverse of machine accuracy).

Total failure of 3 BC experiments (MORE, ISA, BELA)? No, experts know **recipes** to remove rank deficiency, but this is kraftmanship, not science. Let us build a theory.

There are only 3 ways to stabilize a problem with either rank deficiency or a very badly conditioned normal matrix: **descoping**, **additional observations** and **constrained solution**. For all three there is a **legitimacy problem**.

1.8 Rank deficiency and symmetries

Theorem: There is an effective one-parameter group of exact symmetries of all the observations $\Longrightarrow$ the $N \times N$ normal matrix C has rank $N-1$. There is an effective dim. d Lie group of exact symmetries $\Longrightarrow$ C has rank $N-d$.

Exact symmetry means the residuals are exactly the same. **Approximate symmetry** means that a value ε of the symmetry parameter changes the residuals by $O(\varepsilon^2)$. The converse is not always true, symmetries can be only approximate (unless some other hypothesis is available).

Classical examples: in the n-body problem, if the observations are only range and/or range-rate between planets (e.g., radar), the group $SO(3)$ of rotations is an exact and effective group of symmetries, of dimension 3. If the observations are angles only, the changes of scale by λ in length and μ in mass are exact symmetries for $\lambda^3 = \mu$.

Application to BepiColombo: initial conditions for Earth and Mercury, mass of the Sun cannot be adjusted at once (approximate symmetry, due to weak coupling with other planets). 4 constraints are needed.

2 The MORE simulations performed in the definition phase of BepiColombo

2.1 The BepiColombo simulations

Our group has performed three cycles of simulations of the BepiColombo Radioscience Experiment (now MORE).

- The first cycle (1999-2000) was focused on the Gravimetry Experiment. Results published in Milani, A., Rossi, A., Vockrouhlicky, D., Villani, D., Bonanno, C. 2001, *Gravity field and rotation state of Mercury from the BepiColombo Radio Science Experiments*, Planetary and Space Science, **49**, 1579–1596.
- The second cycle (2001) was focused on the Relativity Experiment. Milani, A, Vokrouhlický, D., Villani, D., Bonanno, C. & Rossi, A. 2002 *Testing general relativity with the BepiColombo radio science experiment*, Physical Review D, **66**, 082001-082012.
- The third cycle (2002-2003) adressed the Gravimetry Experiment, with special emphasis on the accelerometer calibration problem, the Conjunction Experiment (defined later) and the interaction between the Gravimetry and the Rotation experiments. The results have appeared only in Contractual Reports to ESA, like Milani, A, Rossi, A. & Villani, D. *The BepiColombo Radio Science Simulations*, Version 2, 11 April 2003.

2.2 Simplifying Assumptions

The simulations were performed without having a purpose-built interplanetary orbit determination software system (no resources; ESA does not support research as such). To be able to adapt a software system designed for satellite geodesy of the Earth we have adopted the following shortcuts and compromises.

The short arc technique was used, with uncorrelated observed arcs: the local parameters (initial conditions, one accelerometer constant for each axis) were processed as if there was no dependence of the observables from the state in a previous/later arc (also to decrease the computational resources).

The Relativity Experiment was performed separately, by solving for a local correction to Mercury's orbit for each arc, then fitting these corrections to a solar system model. Also the Rotation Experiment was assumed to be performed separately, by using the S/C orbit as reference.

The symmetries were controlled by assuming an a priori knowledge of the S/C orbit to $\simeq 3\ m$ due to hypothetical long arc solutions.

These simplifications, because of the overparametrization, weaken the fit and therefore the simulations provide a lower bound to the results which should be achieved with the experiment.

2.3 Parameters and symmetries

For the simulations of the Gravimetry Experiment, with local corrections to the orbit of Mercury and separate rotation experiment, we have the following **parameters count**:

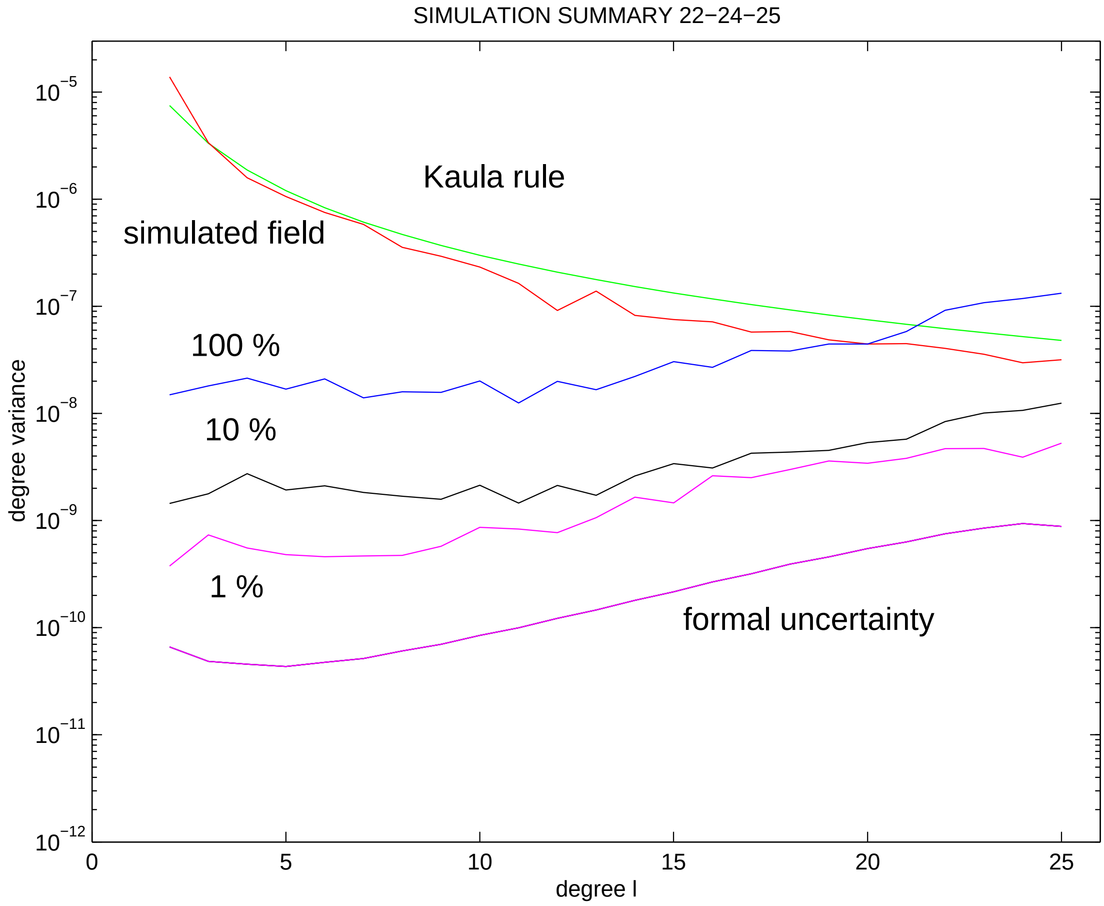
Local parameters: 6 MPO initial conditions for the arc, 3 calibration coefficients for the accelerometer, 2 corrections to heliocentric Mercury.

Global parameters: $26^2 - 3$ harmonics of Mercury's potential, Mercury's dynamical Love number k_2.

If the arc is short, ranging to the MPO only provides an estimated **correction to the range to Mercury's center of mass (CoM)**. Range-rate to MPO allows to correct **range-rate to Mercury CoM**. Thus the number of local parameters is 11, not 15, per arc, for a total of $365 \times 11 + 674 = 4,689$ local and global parameters.

One **exact symmetry** is well known in the limit case for distance $\rightarrow +\infty$ (Extrasolar planet): if the orbit of a satellite is rotated around the fixed direction from the Earth to the central body (assumed to be spherical), the residuals are the same. We assume that long arc, less accurate, orbit solutions can be used to **constrain** the short arc orbit (the symmetry is approximate with as small parameter the angle by which the Earth-Mercury direction moves during one short arc).

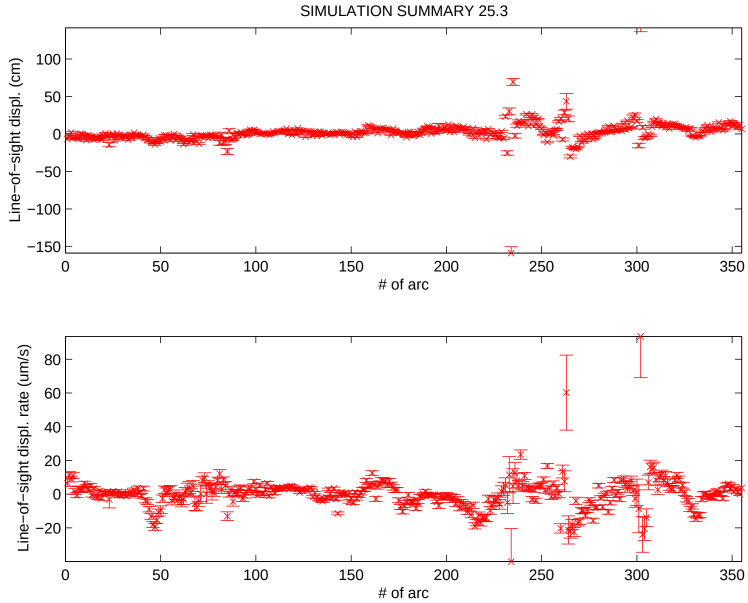
2.4 Gravimetry Experiment



The figure shows, as a function of the degree ℓ, from the top: the signal (simulated, Kaula's rule); the actual error assuming the accelerometer data have no a priori calibration, 10% and 1% calibration from thermometers; the formal error.

Other parameters: the k_2 Love number has an actual error of $0.067, 0.011, 0.004$ in the three assumptions on accelerometer calibration (simulated value 0.25; formal error 0.0035).

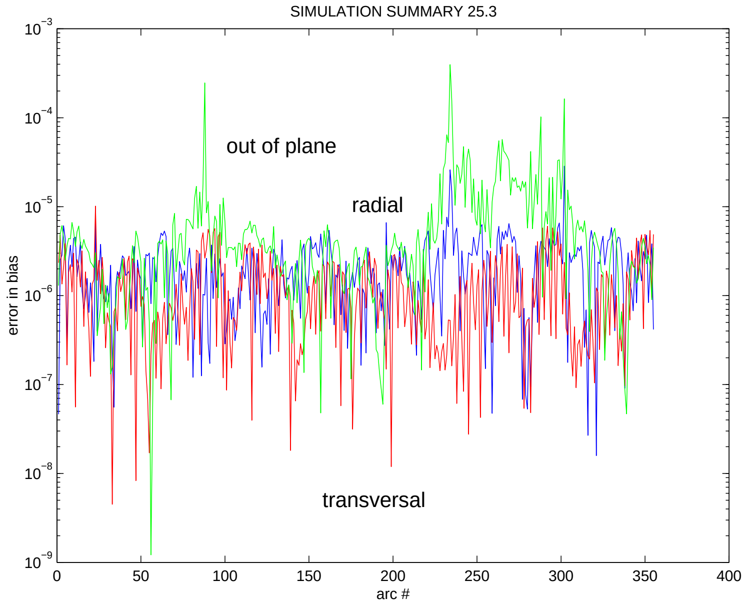
2.5 Local Parameters



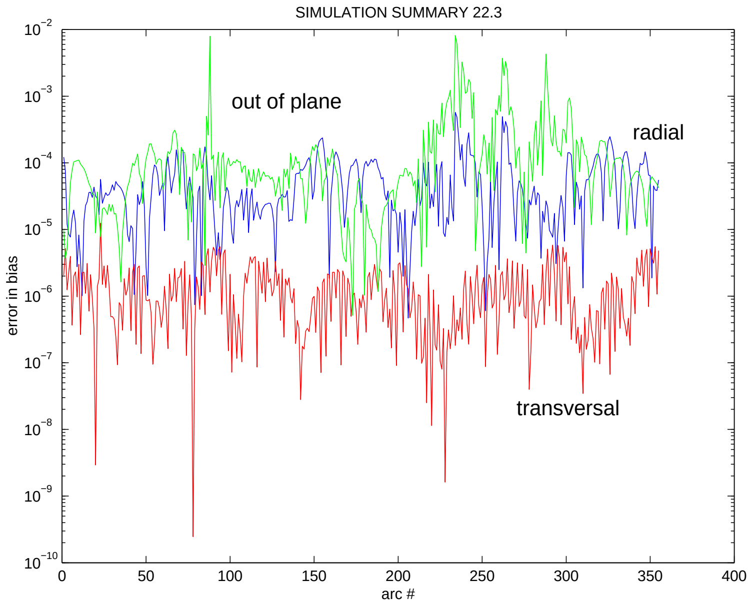
Errors in the range and range-rate corrections to the orbit of Mercury.

Errors in the position part of S/C initial conditions, χ value (top), length of position displacement (bottom, circles) and long semiaxis of confidence ellipsoid (bottom, plus).

2.6 Accelerometer Calibrations



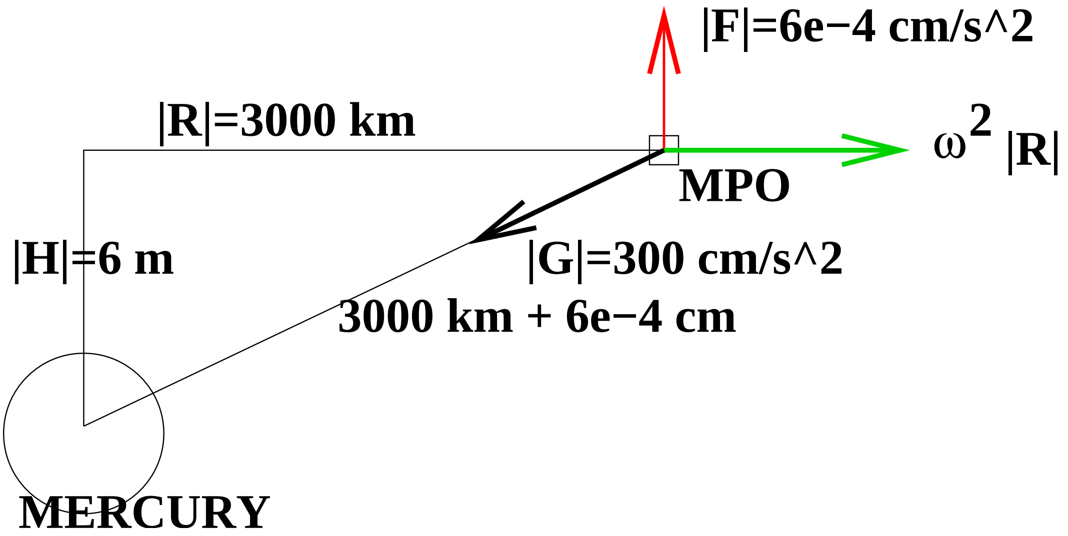
Errors in the three accelerometer a posteriori calibrations, case with 1% a priori calibration.



Errors in the three accelerometer a posteriori calibrations, case with no a priori calibration.

2.7 The photo-gravitational symmetry

The three accelerometer axes are orthogonal and oriented radially (to Mercury's CoM), in plane and out of plane. Let there be a constant calibration f along the out of plane axis. It is the same as thinking there is a constant radiation pressure acceleration and no accelerometer.



$$\vec{0} = \omega^2 \vec{R} + \vec{A} + \vec{F}$$

$$\vec{A} = -\frac{GM\,(\vec{R}+\vec{H})}{(r^2+h^2)^{3/2}} \;;\; \omega^2\, r = \frac{GM\, r}{(r^2+h^2)^{3/2}}$$

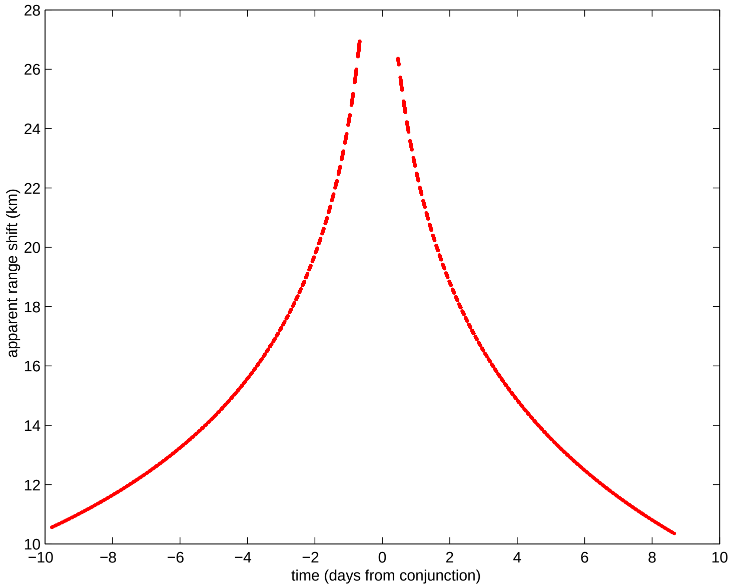
$$f = \frac{GM\, h}{(r^2+h^2)^{3/2}} \Longrightarrow \frac{h}{r} = \frac{f}{\omega^2\, r}$$

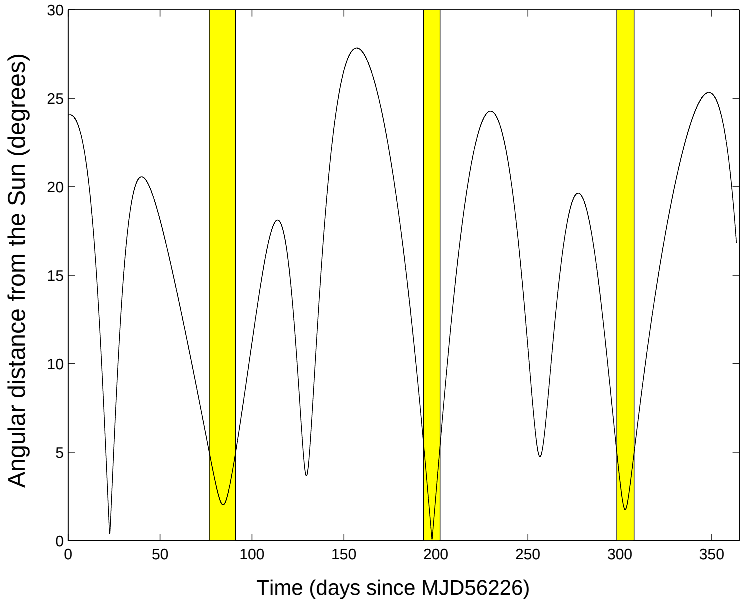
$$\frac{M'}{M} = \left(1+\frac{h^2}{r^2}\right)^{-3/2} \Longrightarrow M' = M\,(1-6\times 10^{-12})$$

2.8 Superior Conjunction Experiment (SCE)

Light propagation in curved spacetime (Shapiro effect)

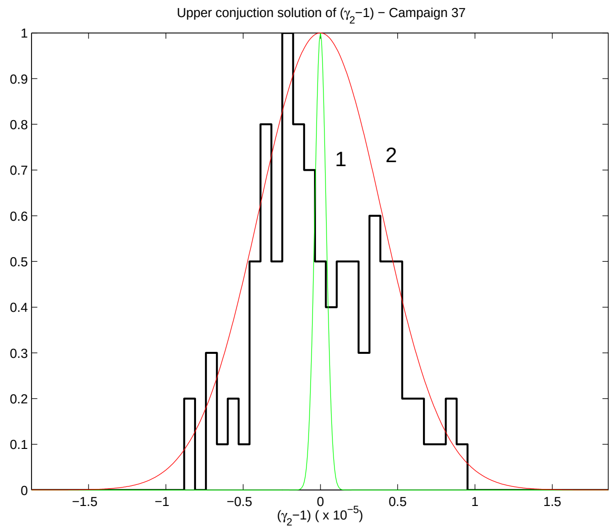
$$\Delta R = \gamma \frac{GM_S}{c^2} \ln \left(\frac{r_1 + r_2 + r_{12}}{r_1 + r_2 - r_{12}} \right)$$





2.9 Conjunction Experiment Results

Results from the 2003 work, Simulation 37 (1% accelerometer a priori calibration, 3 ground stations, corona degradation, best Sup. Conj.).



The results from the SCE ($RMS(\gamma) = 4 \times 10^{-6}$) can be used to constrain γ *a priori* with respect to the solution for the orbit of Mercury, but not as well as we assumed in the 2002 paper (probably due to a more realistic model of accelerometer calibration problems).

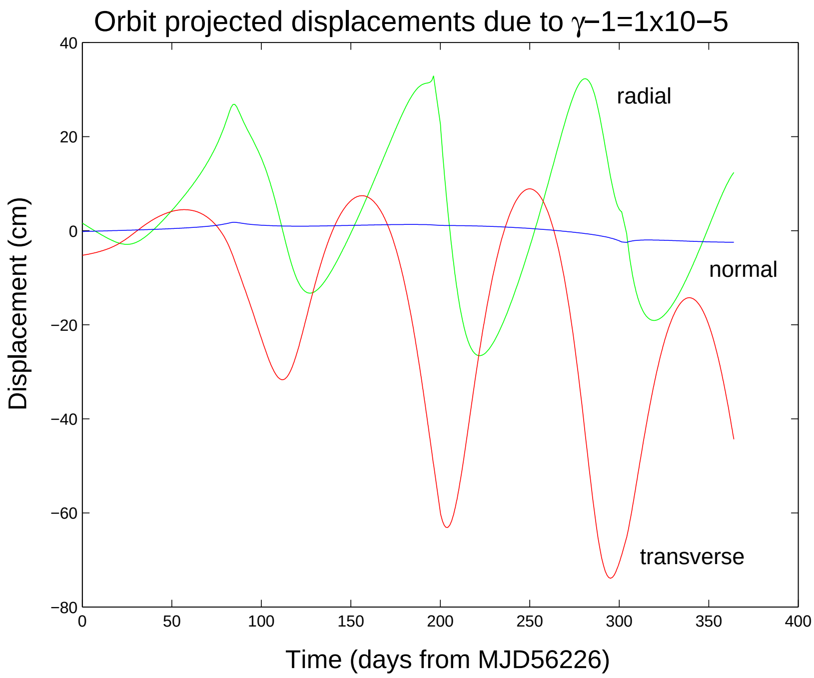
In an SCE the contribution of range-rate is just 1% of the normal matrix. Thus by repeating the same cruise phase SCE as Cassini, but with ranging in Ka band, we can gain a factor 10 in accuracy. The cruise phase SCE is much easier because the S/C is in a very stable, quiescent state, radiation pressure and thermal effects are stationary. Anyway a long (weeks) cruise test for instrument calibration is needed.

2.10 Relativity Experiment

The PPN corrections to the orbit of Mercury (and Earth) can be described by an additive term in the Lagrange function; e.g., for γ

$$L_\gamma = \frac{1}{2c^2}\gamma \sum_{A \neq B} \frac{GM_A M_B}{r_{AB}} \mathbf{v}_{AB}^2$$

and their effect on Mercury is



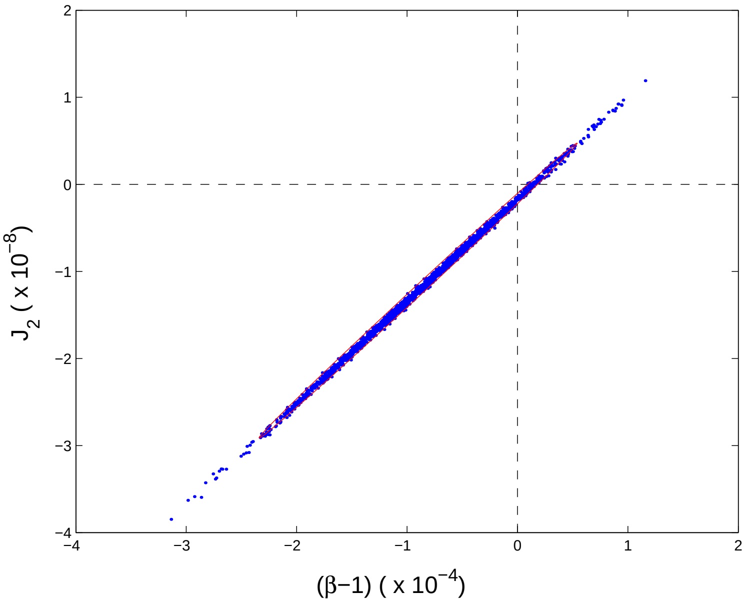
Similarly for $\beta, J_{2\odot}$, preferred frame parameters α_1, α_2 and $\dot{G}/G$. A little more complicated the discussion about the strong equivalence principle (SEP) violation η parameter. Thus all these could be measured from the Earth-Mercury distance (range-rate does not significantly contribute, timescales months).

2.11 The PPN Corrector

We have performed (in 2000-2001) a full differential correction with the PPN and the initial conditions for Earth and Mercury (minus 4 constraints to remove symmetries), using daily ranges to Mercury with $10\ cm$ random error and systematic error growing to $50\ cm$ in a one year nominal mission. The results depend upon choices of the PPN parameters, especially from the possible use of the Nordtvedt equation, assuming a metric theory:

$$\eta = 4(\beta - 1) - (\gamma - 1) - \alpha_1 - \frac{2}{3}\alpha_2$$

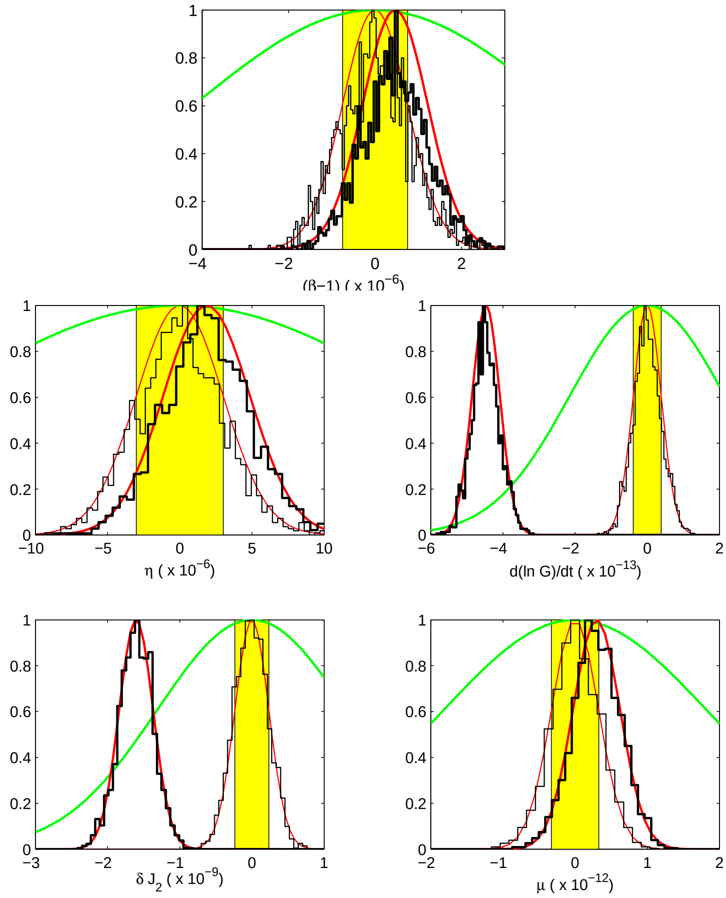
which removes the approximate symmetry $\beta - J_{2\odot}$ due to the angle between spin axis of the Sun and orbital angular momentum of Mercury being only $\varepsilon = 3.^\circ 3$, thus $\cos\varepsilon = 0.998$ and $Corr(\beta, J_{2\odot}) = 0.997$ in the solution without Nordtvedt eq.



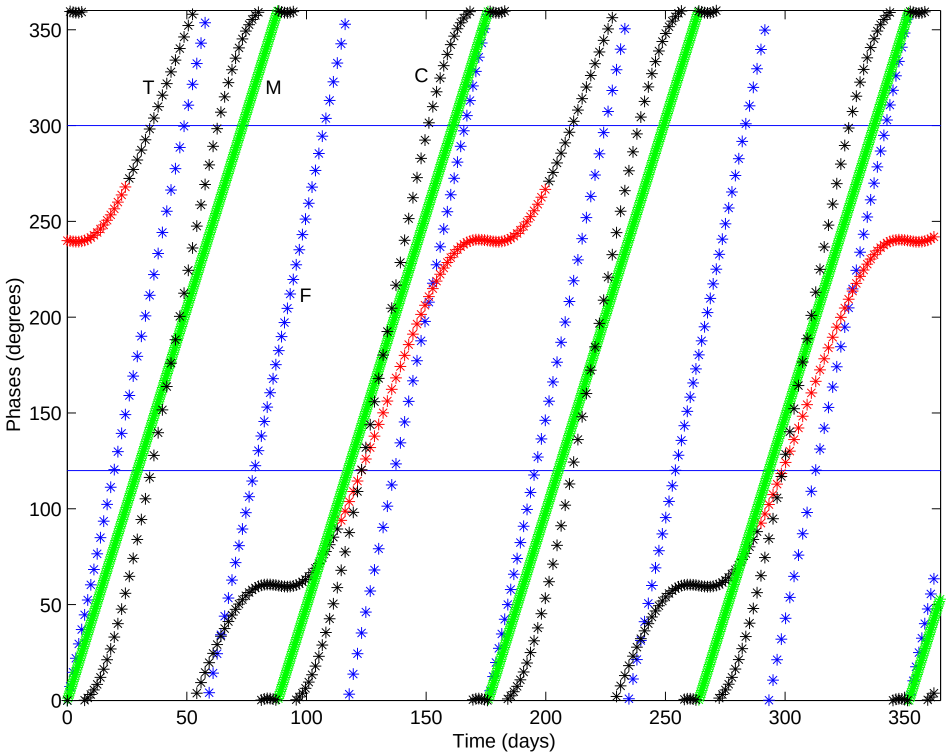
2.12 Relativity Experiment Results

Par.	Exp A (non-metric) RMS	Exp A (non-metric) Real.	Exp B (metric) RMS	Exp B (metric) Real.
$\beta - 1$	6.7 (-5)	**2.2 (-4)**	7.5 (-7)	**2.0 (-6)**
η	4.4 (-6)	**1.5 (-5)**	3.0 (-6)	**7.9 (-6)**
$\frac{d(\ln G)}{dt}$	4.0 (-14)	**5.2 (-13)**	3.9 (-14)	**5.3 (-13)**
δJ_2	7.9 (-9)	**2.8 (-8)**	2.4 (-10)	**2.1 (-9)**
$\delta\mu$	1.9 (-12)	**5.9 (-12)**	3.3 (-13)	**1.0 (-12)**

Assuming $RMS(\gamma) = 2 \times 10^{-6}$ and Nordtvedt equation.



2.13 Mercury Rotation Theory



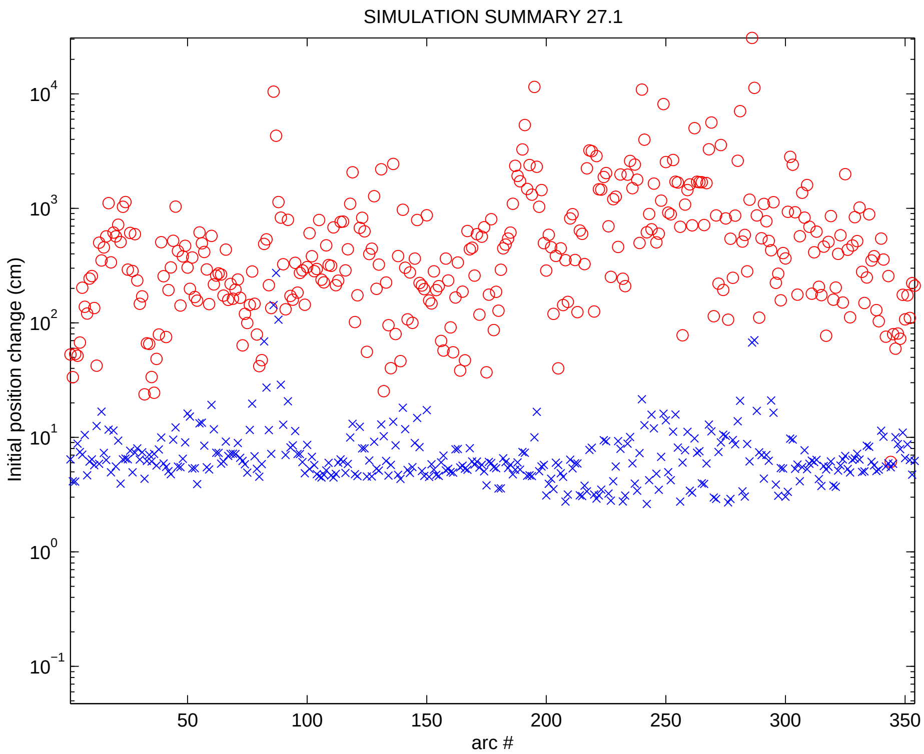
The angles relevant for the discussion of Mercury libration in longitude, all expressed in degrees and over a time span of one year (with initial time corresponding to Mercury's perihelion).

M is the mean anomaly of Mercury's heliocentric orbit. F is the phase of the planet's spin, with respect to an inertial reference system. T is the local solar time (in degrees): the illumination (at some longitude) is marked in red; note that the Sun can go back in the sky as seen from Mercury near perihelion. C is the phase of the torque applied by the Sun on Mercury equatorial bulge.

2.14 S/C Orbit as Reference for Rotation Experiment

Error budget for rotation experiment (if performed separately from gravimetry): RMS sum of 4 terms

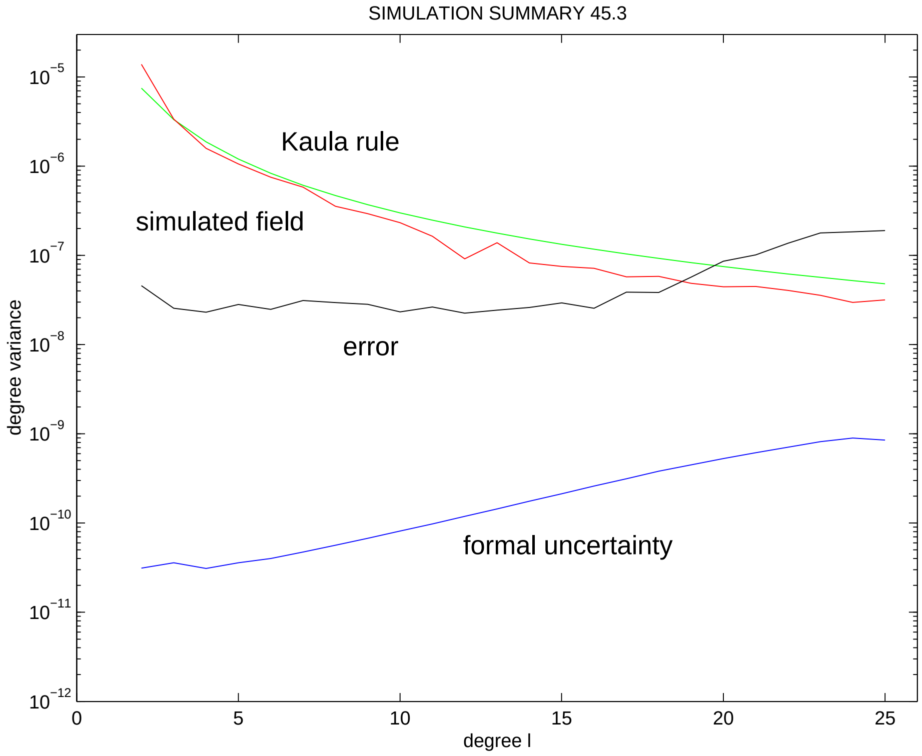
1. Error in S/C position
2. Error in relative position of surface reference points
3. Error in the S/C attitude (star mappers)
4. Thermo-mechanical stability



Uncertainty in the S/C position 24 hours after the observed arc: formal error (crosses), mean value $9.1\ cm$; actual error (circles), mean value $3.8\ m$. This is good enough for the preliminary error budget.

2.15 Rotation experiment by Gravimetry Only

Alternate hypothesis: gravimetry+rotation from tracking.



Adding to the best case (1% accelerometer calibration) just two parameters (obliquity, long. libration amplitude) the error in the gravity field increases by a factor $\simeq 100$.

Newton's principle (no mass distribution from gravimetry alone) is preserved by some **hidden symmetry**! An unknown combination of changes in initial conditions, in harmonic coefficients, in calibrations and in Mercury's CoM reproduces a different rotation of Mercury.

New result: with rotation parameters and gravity harmonic coefficients only there is no symmetry!

3 The software for MORE data processing and how to build it properly

3.1 ORBIT14: top level specifications

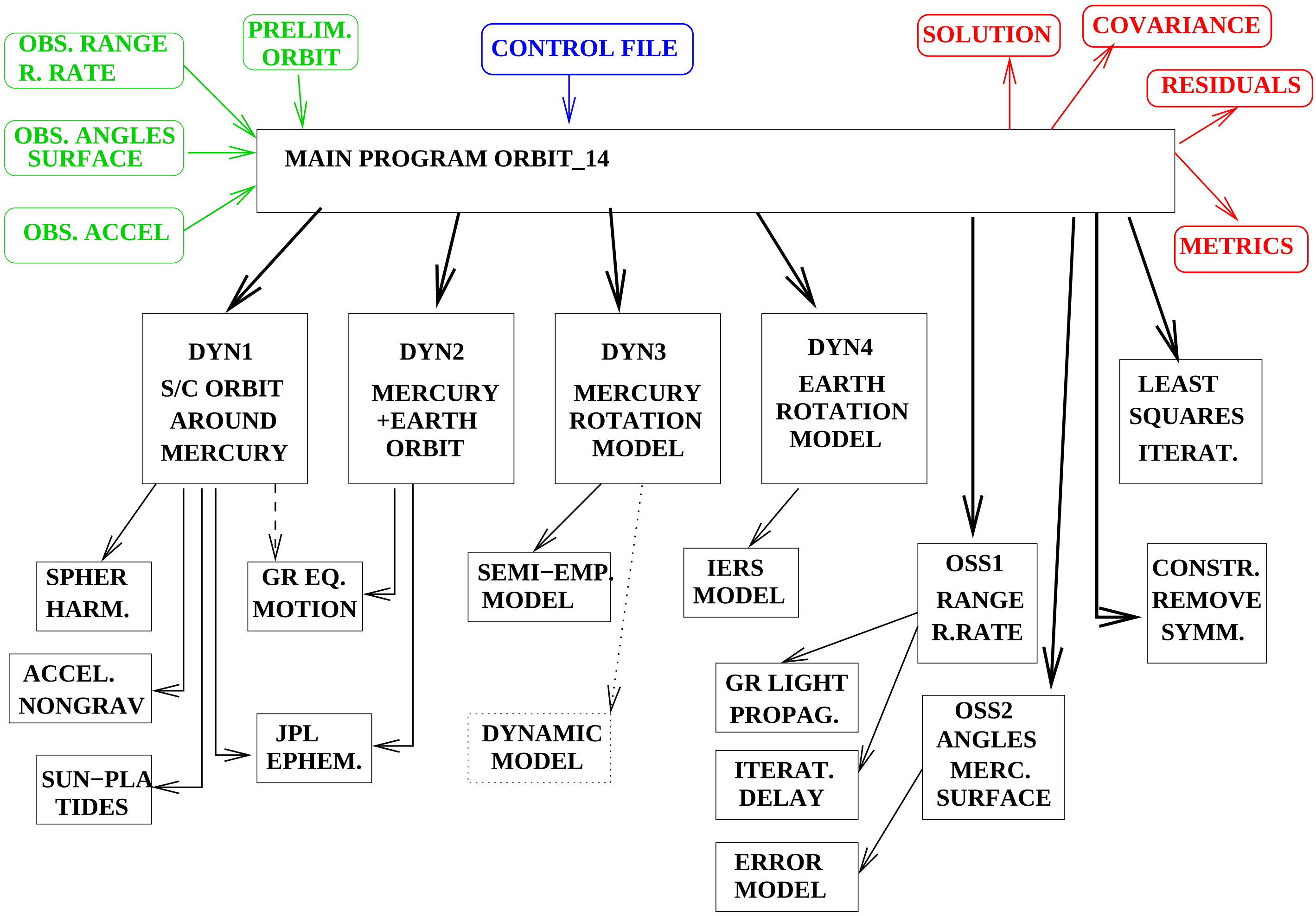
The software system ORBIT14 has the goal of determining all the parameters affecting in a significant way the measurements R1, R2 and R3, in such a way to achieve the scientific goals S1, S2, S3 and also s4. This software is meant for the BepiColombo MORE experiment, could have other applications, such as Don Quijote and Juno.

As opposed to the software used in the previous simulations, funded by ESA only for the purpose of feasibility assessment, this effort is funded by ASI as an essential part of the MORE experiment, to achieve the best possible scientific results: no compromises and no shortcuts.

To understand the level of inheritance it is enough to consider the number 14. Starting from ORBIT1/2 (1978), passing through ORBIT5 (Project LONGSTOP), ORBIT7 (Project SPACEGUARD), ORBIT8/9 (AstDys), ORBIT10 (LAGEOS), ORBIT11 (NEODyS), ORBIT12 (BepiColombo simulations), ORBIT13 (GOCE and LDIM simulations), our group has accumulated enough experience.

Anyway we intend to restart the software design from the top and from scratch, by using modern methods and programming style. We shall use the language Fortran 95 and a style based on data abstraction. The main system component are: I/O and control system, dynamics, observations, least squares and symmetry control.

3.2 ORBIT14: top level block diagram



3.3 ORBIT14: non critical modules

ORBIT14 shall be entirely redesigned (with respect to ORBIT12/13), of course reusing some existing modules with comparatively simple modifications (including changes of language and programming style), and anyway using available know-how, maybe based upon the experience in other projects (e.g., asteroid radar astrometry). A tentative list of these non-critical modules could be as follows.

- N1 Control and I/O.
- N2 Least squares iterative procedures.
- N3 Angular observations.
- N4 Range and range-rate observations (at interplanetary distances, with iterative algorithms).
- N5 Planetary gravity field and solid tides.
- N6 Non gravitational accelerations, including accelerometer data and calibrations.
- N7 Numerical propagators for equations of motion and variational equations.
- N8 Planetary perturbations.

Of course the work needed to rewrite all the above is by no means small, but is just a matter of time and manpower.

3.4 ORBIT14: critical modules

Problems arise when there is a critical module, either not available or not of the appropriate accuracy level in the previous versions. More so when the expertise for it is not available in our sub-team. A tentative list of critical modules is as follows.

C1 The relativistic dynamical model for the motion of the planets and the Sun.

C2 The model for relativistic propagation of radiowaves (not including the plasma and tropospheric effects, supposedly corrected in a preprocessing phase).

C3 The definition and conversion algorithms for the space-time reference system applicable in C1, C2.

C4 The dynamic model for the rotation of Mercury.

C5 The model for the rotation of the Earth, expressed in the space-time reference of C3.

C6 The algorithms to identify approximate symmetries and to constrain them to avoid weak solutions.

C7 Error model for angular observations, including contributions from image processing and attitude.

3.5 ORBIT14: work sharing

Our subgroup shall be entirely responsible for the integration of the orbit determination software and shall take care of the non-critical modules, provided there is uninterrupted support from ASI.

We shall also perform simulations to update the assessment of the achievable performances (with respect to the old simulations mentioned before). However, there are problems in doing this before the new software is at least partially operational.

Most critical modules require help from other subgroups of the MORE team. The main reason is that on some subjects we are not involved in research inside our subgroup, thus we need contributions from the scientists in charge.

This help may consist in supplying either ready-to-compile modules or well documented algorithms which we can implement; in revising, correcting and taking responsibility for the scientific accuracy of some code we have implemented. Some examples follow.

3.6 ORBIT14: specific requests for help

For the general relativity issues (C1 and C2) we need expert advice, especially since we believe that the $O(v^2/c^2)$ approximations easily found in the literature (e.g., Moyers) are not enough.

For the reference systems (C3 and C5) we need expert advice, also because of the recent IAU/IERS changes of standard and the not fully satisfactory situation with planetary ephemerides.

For Mercury's rotation (C4) we are prepared to use a semi-empirical model, but this may not be enough also due to the oscillating attitudes of the best known experts. A self-consistent model is needed, and this cannot be our work.

Symmetries and constraints (C6) are our own research work, although of course we cannot promise to solve all the problems.

To synthesize the camera (and star mapper) data into angular observable is the task of the camera team, which should include a proper statistical error model (C7).

3.7 MORE Sub-team in Pisa

The subgroup working in Pisa to the MORE experiment includes the following people:

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3. Alessando Rossi (ISTI, CNR, Pisa) alessandro.rossi@ @isti.cnr.it
4. Giacomo Tommei (to be employed at Dept. Mathematics, Univ. Pisa) tommei@mail.dm.unipi.it
5. XXXXXXX (to be employed at Dept. Mathematics, Univ. Pisa)
6. YYYYYYY (graduate student 2008-2010, Dept. Mathematics, Univ. Pisa)