

```
> restart;Digits:=30:
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
```

Identity and generators of the Cube rotation group

```
> R:= i -> matrix(3,3,[c1,c2,c3,c4,c5,c6,c7,c8,c9]):
> R(1) := matrix(3,3,[1,0,0,0,1,0,0,0,1]):
> R(2) := matrix(3,3,[0,-1,0,1,0,0,0,0,1]):
> R(3) := matrix(3,3,[1,0,0,0,0,-1,0,1,0]):
```

Construction of the elements of the group by recursive multiplication

```
> enne:=4:
> jtmp:=3:
> jtot:=jtmp:
> for i from 1 to enne do;
>   for j from 2 to jtmp do;
>     j;
>     Rnew:=multiply(R(j),R(2));
>     dupl:=0;
>     for k from 1 to jtot do;
>       check:=simplify(matadd(Rnew,-R(k)));
>       if (iszero(check)) then;
>         dupl:=1;
>         fi;
>       od;
>       if (dupl=0) then jtot:=jtot+1;
>         R(jtot):=Rnew();
>         fi;
>       Rnew:=multiply(R(j),R(3));
>       dupl:=0;
>       for k from 1 to jtot do;
>         check:=simplify(matadd(Rnew,-R(k)));
>         if (iszero(check)) then;
>           dupl:=1;
>           fi;
>         od;
>         if (dupl=0) then jtot:=jtot+1;
>           R(jtot):=Rnew();
>           fi;
>       od;
>     od;
>   fi;
```

```

> od;
> jtmp:=jtot;
> od;

      jtmp:= 7
      jtmp:= 14
      jtmp:= 21
      jtmp:= 24

```

[integration extrema

```

> ell:=1/sqrt(5+2*sqrt(2)):
> x0:= ell*(1+sqrt(2)):
> y0:= ell:
> z0:= y0:
> arch0:=[x0, y0, z0]:
> arch0:=factor(simplify(scalarmul(arch0,1/norm(arch0,2)))):
> simplify(norm(arch0,2)):

```

line 1 = [1 16] line 2 = [1 14];

```

> extr:=[1,14];

      extr:= [1, 14]

```

```

> arch1:=multiply(R(extr[1]),arch0):
> arch2:=multiply(R(extr[2]),arch0):
> simplify(norm(arch1,2)):simplify(norm(arch2,2)):

```

[length of an edge of the Archimedean polyhedra

```

> elle:=factor(simplify(norm((arch1-arch2),2))):
> evalf(%);

      0.714813488673186511896933943311

```

```

> u1:=(arch2[1]-arch1[1])*x+arch1[1]:
> u2:=(arch2[2]-arch1[2])*x+arch1[2]:
> u3:=(arch2[3]-arch1[3])*x+arch1[3]:
> U:=[u1, u2, u3]:
> intpotline:=0:
> for i from 2 to jtot do:
> dist:=simplify(sqrt(innerprod( U-multiply(R(i),U),U-multiply(R(i),U)
  ))):
> intp:=simplify(int(1/dist,x)): intp0:=eval(intp,x=0):
  intp1:=eval(intp,x=1): intpart:=intp1-intp0:
> intpotline := intpotline + intpart:
> od:

```

expression of the integral sigma_2 in terms of elementary functions

`> sigma_2:=factor(simplify(intpotline));`

$$\begin{aligned}
 \text{sigma}_2 := & -\frac{1}{4\sqrt{2+\sqrt{2}}\sqrt{5+2\sqrt{2}}}\left(\left(-13+10\operatorname{arcsinh}\left(\frac{\sqrt{2}(-2+\sqrt{2})\sqrt{3}}{6(1+\sqrt{2})}\right)\sqrt{2+\sqrt{2}}\right.\right. \\
 & -28\operatorname{arcsinh}\left(\frac{\sqrt{2}}{2+\sqrt{2}}\right)\sqrt{2+\sqrt{2}}-10\operatorname{arcsinh}\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)\sqrt{2+\sqrt{2}} \\
 & -4\operatorname{arcsinh}\left(\frac{\sqrt{2}(14+9\sqrt{2})}{2(5+2\sqrt{2})}\right)\sqrt{2+\sqrt{2}}\sqrt{2}+4\operatorname{arcsinh}\left(\frac{1}{3}\sqrt{3}\right)\sqrt{2+\sqrt{2}}\sqrt{2} \\
 & -4\operatorname{arcsinh}\left(\frac{1}{6}\sqrt{2}\sqrt{3}(4+\sqrt{2})\right)\sqrt{2+\sqrt{2}}\sqrt{2}-18\operatorname{arcsinh}\left(\frac{\sqrt{2}}{2+\sqrt{2}}\right)\sqrt{2+\sqrt{2}}\sqrt{2} \\
 & -10\operatorname{arcsinh}\left(\frac{\sqrt{2}(14+9\sqrt{2})}{2(5+2\sqrt{2})}\right)\sqrt{2+\sqrt{2}} \\
 & -10\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{3}(24+13\sqrt{2})}{6(5+2\sqrt{2})}\right)\sqrt{2+\sqrt{2}} \\
 & +10\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{3}(5\sqrt{2}+4)}{6(5+2\sqrt{2})}\right)\sqrt{2+\sqrt{2}} \\
 & +10\operatorname{arcsinh}\left(\frac{(-6+\sqrt{2})\sqrt{3}\sqrt{2}}{6(7\sqrt{2}+9)}\right)\sqrt{2+\sqrt{2}} \\
 & -10\operatorname{arcsinh}\left(\frac{(14+9\sqrt{2})\sqrt{3}\sqrt{2}}{6(7\sqrt{2}+9)}\right)\sqrt{2+\sqrt{2}}-12\sqrt{2}+10\operatorname{arcsinh}\left(\frac{1}{3}\sqrt{3}\right)\sqrt{2+\sqrt{2}} \\
 & -18\ln(1+\sqrt{2})\sqrt{2+\sqrt{2}}-10\operatorname{arcsinh}\left(\frac{\sqrt{2}(2+\sqrt{2})\sqrt{3}}{6(1+\sqrt{2})}\right)\sqrt{2+\sqrt{2}} \\
 & -10\operatorname{arcsinh}\left(\frac{1}{6}\sqrt{2}\sqrt{3}(4+\sqrt{2})\right)\sqrt{2+\sqrt{2}} \\
 & +4\operatorname{arcsinh}\left(\frac{(-6+\sqrt{2})\sqrt{3}\sqrt{2}}{6(7\sqrt{2}+9)}\right)\sqrt{2+\sqrt{2}}\sqrt{2} \\
 & +4\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{3}(5\sqrt{2}+4)}{6(5+2\sqrt{2})}\right)\sqrt{2+\sqrt{2}}\sqrt{2} \\
 & -4\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{3}(24+13\sqrt{2})}{6(5+2\sqrt{2})}\right)\sqrt{2+\sqrt{2}}\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
& - 4 \operatorname{arcsinh} \left(\frac{(14 + 9\sqrt{2})\sqrt{3}\sqrt{2}}{6(7\sqrt{2} + 9)} \right) \sqrt{2 + \sqrt{2}}\sqrt{2} - 4 \operatorname{arcsinh} \left(\frac{\sqrt{2}}{1 + \sqrt{2}} \right) \sqrt{2 + \sqrt{2}}\sqrt{2} \\
& - 4 \operatorname{arcsinh} \left(\frac{\sqrt{2}(2 + \sqrt{2})\sqrt{3}}{6(1 + \sqrt{2})} \right) \sqrt{2 + \sqrt{2}}\sqrt{2} \\
& + 4 \operatorname{arcsinh} \left(\frac{\sqrt{2}(-2 + \sqrt{2})\sqrt{3}}{6(1 + \sqrt{2})} \right) \sqrt{2 + \sqrt{2}}\sqrt{2} - 14 \ln(1 + \sqrt{2}) \sqrt{2 + \sqrt{2}}\sqrt{2} \Big) \sqrt{2}
\end{aligned}$$

```
> ipotval:=evalf(%);
```

```
ipotval:= 19.7399475276818694556141122794
```

```
> intpot:=jtot*intpotline/2:
```

```
> ipot:=evalf(%);
```

```
ipot:= 236.879370332182433467369347351
```

```
> writeto(latex_hexa);
```

```
> latex(intpotline);
```

```
> latex(ipotlineval);
```

```
> latex(ipotval);
```

```
> latex(elleval);
```

```
> latex(elle);
```

```
> writeto(terminal);
```

```
>
```

```
>
```

