

```
> restart;Digits:=30:
```

```
> with(linalg):
```

```
Warning, the protected names norm and trace have been redefined and unprotected
```

```
Identity and generators of the Cube rotation group
```

```
> R:= i -> matrix(3,3,[c1,c2,c3,c4,c5,c6,c7,c8,c9]):
```

```
> R(1) := matrix(3,3,[1,0,0,0,1,0,0,0,1]):
```

```
> R(2) := matrix(3,3,[0,-1,0,1,0,0,0,0,1]):
```

```
> R(3) := matrix(3,3,[1,0,0,0,0,-1,0,1,0]):
```

```
Construction of the elements of the group by recursive multiplication
```

```
> enne:=4:
```

```
> jtmp:=3:
```

```
> jtot:=jtmp:
```

```
> for i from 1 to enne do;
```

```
>   for j from 2 to jtmp do;
```

```
>     j;
```

```
>     Rnew:=multiply(R(j),R(2));
```

```
>     dupl:=0;
```

```
>     for k from 1 to jtot do;
```

```
>       check:=simplify(matadd(Rnew,-R(k)));
```

```
>       if (iszero(check)) then;
```

```
>         dupl:=1;
```

```
>         fi;
```

```
>       od;
```

```
>       if (dupl=0) then jtot:=jtot+1;
```

```
>         R(jtot):=Rnew();
```

```
>       fi;
```

```
>       Rnew:=multiply(R(j),R(3));
```

```
>       dupl:=0;
```

```
>       for k from 1 to jtot do;
```

```
>         check:=simplify(matadd(Rnew,-R(k)));
```

```
>         if (iszero(check)) then;
```

```
>           dupl:=1;
```

```
>           fi;
```

```
>         od;
```

```
>         if (dupl=0) then jtot:=jtot+1;
```

```
>           R(jtot):=Rnew();
```

```
>         fi;
```

```

> od;
> jtmp:=jtot;
> od;

      jtmp:= 7
      jtmp:= 14
      jtmp:= 21
      jtmp:= 24

```

[integration extrema

```

> ell:=1/sqrt(5+2*sqrt(2)):
> x0:= ell*(1+sqrt(2)):
> y0:= ell:
> z0:= y0:
> arch0:=[x0, y0, z0]:
> arch0:=factor(simplify(scalarmul(arch0,1/norm(arch0,2)))):
> simplify(norm(arch0,2)):

```

line 1 = [1 16] line 2 = [1 14];

```

> extr:=[1,16];

      extr:= [1, 16]

> arch1:=multiply(R(extr[1]),arch0):
> arch2:=multiply(R(extr[2]),arch0):
> simplify(norm(arch1,2)):simplify(norm(arch2,2)):

```

[length of an edge of the Archimedean polyhedra

```

> elle:=factor(simplify(norm((arch1-arch2),2))):
> evalf(%);

      0.714813488673186511896933943311

```

```

> u1:=(arch2[1]-arch1[1])*x+arch1[1]:
> u2:=(arch2[2]-arch1[2])*x+arch1[2]:
> u3:=(arch2[3]-arch1[3])*x+arch1[3]:
> U:=[u1, u2, u3]:
> intpotline:=0:
> for i from 2 to jtot do:
> dist:=simplify(sqrt(innerprod( U-multiply(R(i),U),U-multiply(R(i),U)
  ))):
> intp:=simplify(int(1/dist,x)): intp0:=eval(intp,x=0):
  intp1:=eval(intp,x=1): intpart:=intp1-intp0:
> intpotline := intpotline + intpart:
> od:

```

expression of the integral sigma\_1 in terms of elementary functions

```
> sigma_1:=factor(simplify(intpotline));
```

$$\begin{aligned}
 \text{sigma}_1 := & -\frac{1}{12\sqrt{14+9\sqrt{2}}} \left( \begin{aligned}
 & 18\sqrt{5+2\sqrt{2}} \ln(1+\sqrt{2})\sqrt{14+9\sqrt{2}} \\
 & -24\sqrt{5+2\sqrt{2}} \operatorname{arcsinh}\left(\frac{1}{2}\sqrt{2}(2+\sqrt{2})\right)\sqrt{14+9\sqrt{2}} \\
 & +6\sqrt{2}\sqrt{5+2\sqrt{2}} \ln(1+\sqrt{2})\sqrt{14+9\sqrt{2}} \\
 & -6\sqrt{2}\sqrt{5+2\sqrt{2}} \operatorname{arcsinh}\left(\frac{\sqrt{2}(14+9\sqrt{2})}{2(5+2\sqrt{2})}\right)\sqrt{14+9\sqrt{2}} \\
 & -8\sqrt{5+2\sqrt{2}}\sqrt{3} \operatorname{arcsinh}(\sqrt{3})\sqrt{14+9\sqrt{2}} \\
 & -8\sqrt{5+2\sqrt{2}}\sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{3}}{2\sqrt{2}+1}\right)\sqrt{14+9\sqrt{2}} \\
 & +4\sqrt{5+2\sqrt{2}}\sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right)\sqrt{14+9\sqrt{2}} \\
 & -4\sqrt{5+2\sqrt{2}}\sqrt{3} \operatorname{arcsinh}\left(\frac{2+\sqrt{2}}{1+\sqrt{2}}\right)\sqrt{14+9\sqrt{2}} \\
 & -4\sqrt{5+2\sqrt{2}}\sqrt{3} \operatorname{arcsinh}\left(\frac{1}{3+\sqrt{2}}\right)\sqrt{14+9\sqrt{2}} \\
 & -4\sqrt{5+2\sqrt{2}}\sqrt{3} \operatorname{arcsinh}\left(\frac{2}{3+\sqrt{2}}\right)\sqrt{14+9\sqrt{2}} \\
 & +24\sqrt{5+2\sqrt{2}} \operatorname{arcsinh}\left(\frac{(\sqrt{2}-1)\sqrt{3}}{3(1+\sqrt{2})}\right)\sqrt{14+9\sqrt{2}} \\
 & -24\sqrt{5+2\sqrt{2}} \operatorname{arcsinh}\left(\frac{1}{3}\sqrt{3}\right)\sqrt{14+9\sqrt{2}} \\
 & -6\sqrt{5+2\sqrt{2}} \operatorname{arcsinh}\left(\frac{\sqrt{2}}{2+\sqrt{2}}\right)\sqrt{14+9\sqrt{2}} \\
 & -12\sqrt{5+2\sqrt{2}}\sqrt{2} \operatorname{arcsinh}\left(\frac{\sqrt{2}}{2+\sqrt{2}}\right)\sqrt{14+9\sqrt{2}} - 15\sqrt{2} - 12
 \end{aligned} \right)
 \end{aligned}$$

```
> ipotval:=evalf(%);
```

```
ipotval:= 20.3224403607839568410788152152
```

```
> intpot:=jtot*intpotline/2:
```

```
> ipot:=evalf(%);
```

*ipot*:= 243.869284329407482092945782582

> writeto(latex\_hexa);

> latex(intpotline);

> latex(ipotlineval);

> latex(ipotval);

> latex(elleval);

> latex(elle);

> writeto(terminal);

>

>