

residui:

$$\xi = \lambda - a^* t - \beta^* \mathbf{1} \quad \text{sono tali che } \bar{\xi} = \bar{\lambda} - a^* \bar{t} - \beta^* = 0$$

è il vettore $(1, \dots, 1)^T$
m volte

cioè la media dei residui
è nulla.

$$\text{Var}(\xi) = \frac{1}{m} \sum (\xi_i - \bar{\xi})^2 \quad \text{per cui}$$

$$m \text{Var}(\xi) = \xi \cdot \xi = \lambda \cdot \lambda + (a^*)^2 t \cdot t + (\beta^*)^2 m - 2a^* \lambda \cdot t$$

$$- 2\beta^* m \bar{\lambda} + 2a^* \beta^* m \bar{t} = \lambda \cdot \lambda + (a^*)^2 t \cdot t + (\bar{\lambda} - a^* \bar{t})^2 m$$

$$- 2a^* (m \text{Cov}(t, \lambda) + m \bar{t} \bar{\lambda}) - 2(\bar{\lambda} - a^* \bar{t}) m \bar{\lambda} + 2a^* (\bar{\lambda} - a^* \bar{t}) m \bar{t}$$

$$= m \text{Var}(\lambda) + m \bar{\lambda}^2 + (a^*)^2 (m \text{Var}(t) + m \bar{t}^2) + \bar{\lambda}^2 m + (a^*)^2 \bar{t}^2 m$$

$$- 2a^* \bar{t} \bar{\lambda} m + 2a^* m \text{Cov}(t, \lambda) - 2a^* m \bar{t} \bar{\lambda} - 2m \bar{\lambda}^2 + 2a^* \bar{t} \bar{\lambda} m$$

$$+ 2a^* \bar{t} \bar{\lambda} m - 2(a^*)^2 m \bar{t}^2$$

$$= m \text{Var}(\lambda) + \frac{\text{Cov}^2(t, \lambda)}{\text{Var}^2(t)} m \text{Var}(t) - 2 \frac{\text{Cov}(t, \lambda) m}{\text{Var}(t)}$$

$$= m \left[\text{Var}(\lambda) - \frac{\text{Cov}^2(t, \lambda)}{\text{Var}(t)} \right] = m \text{Var}(\lambda) \left[1 - \text{Corr}^2(t, \lambda) \right]$$

$$\text{dove } \text{Corr}(t, \lambda) = \frac{\text{Cov}(t, \lambda)}{\sqrt{\text{Var}(t) \text{Var}(\lambda)}} \in [-1, 1] \quad \text{Correlazione}$$