inoltre
$$P_{X_1}(x)\,P_{X_2}(x) = \frac{\sqrt{\det(C_1 C_2)}}{(2\pi)^N} \exp\left\{-\frac{1}{2}\left[(x-x_1^*)\cdot C_1(x-x_1^*) + (x-x_2^*)\cdot C_2(x-x_2^*)\right]\right\}$$

$$= \frac{\sqrt{\det(C_1 C_2)}}{(2\pi)^N} \exp\left\{-\frac{1}{2}\left[(x-x_0)\cdot C_d(x-x_0) + K\right]\right\}$$

$$= N(x_0,\Gamma_0)\,\frac{\sqrt{\det(C_1 C_2)}}{(2\pi)^{N/2}\sqrt{\det C_0}} \exp\left(-\frac{K}{2}\right)$$

abbiamo usato
$$N(x_0,\Gamma_0) = \frac{\sqrt{\det C_0}}{(2\pi)^{N/2}} \exp\left[-\frac{1}{2}(x-x_0)\cdot C_0(x-x_0)\right]$$

Assumiamo che C_1 o C_2 sia def. positiva, per esempio sia $C_1 > 0$.

Allora le forme quadratiche definite da C_1 e C_2 si possono diagonalizzare simultaneamente, cioè $\exists\, U$ invertibile (matrice di cambiamento di base)

tale che
$$\begin{cases} U^T C_1 U = I = \mathrm{diag}(1) \\ U^T C_2 U = \Lambda = \mathrm{diag}(\lambda_j) \end{cases}, \quad \{\lambda_j\}_{j=1,\dots,N} \text{ autovalori di } C_1^{-1} C_2$$

$$C_0 = C_1 + C_2 \qquad U^T C_0 U = \mathrm{diag}(1+\lambda_j)$$

$$C = C_2 - C_2 \Gamma_0 C_2 \qquad \text{con } \Gamma_0 = C_0^{-1}$$

$$U^T C U = U^T C_2 U - U^T C_2 \Gamma_0 C_2 U = U^T C_2 U - \underbrace{(U^T C_2 U)}_{\Lambda}\underbrace{(U^{-1}\Gamma_0 U^{-T})}_{(U^T C_0 U)^{-1} = \mathrm{diag}\left(\frac{1}{1+\lambda_j}\right)}\underbrace{(U^T C_2 U)}_{\Lambda}$$

$$= \mathrm{diag}(\lambda_j) - \mathrm{diag}(\lambda_j^2)\,\mathrm{diag}\left(\frac{1}{1+\lambda_j}\right) = \mathrm{diag}\left(\lambda_j - \frac{\lambda_j^2}{1+\lambda_j}\right)$$

$$= \mathrm{diag}\left(\frac{\lambda_j + \cancel{\lambda_j^2} - \cancel{\lambda_j^2}}{1+\lambda_j}\right) \Rightarrow \boxed{\det(U^T C U) = \prod_j \frac{\lambda_j}{1+\lambda_j}}$$