Calcolo i coefficienti delle <u>rette di regressione</u> $f^*(t) = a^* t + b^*$

$$\bar{t} = \frac{1}{m}\sum t_i \quad (\underline{\text{media}}); \qquad Var(t) = \frac{1}{m}\sum (t_i - \bar{t})^2 \quad (\underline{\text{varianza}})$$

$$Cov(t,\lambda) = \frac{1}{m}\sum (t_i - \bar{t})(\lambda_i - \bar{\lambda}) \quad (\underline{\text{covarianza}})$$

Abbiamo
$$\begin{cases} t\cdot t = m\, Var(t) + m\,\bar{t}^2 \\ t\cdot\lambda = m\, Cov(t,\lambda) + m\,\bar{t}\,\bar{\lambda} \end{cases}$$

infatti:
$$m\, Var(t) = \sum t_i^2 + m\bar{t}^2 - 2\left(\sum t_i\right)\bar{t}$$
$$= t\cdot t + m\bar{t}^2 - 2m\bar{t}^2 = t\cdot t - m\bar{t}^2$$

$$m\, Cov(t,\lambda) = \sum t_i\lambda_i - \bar{\lambda}\sum t_i - \bar{t}\sum \lambda_i + m\,\bar{t}\,\bar{\lambda}$$
$$= t\cdot\lambda - 2m\,\bar{t}\,\bar{\lambda} + m\,\bar{t}\,\bar{\lambda} = t\cdot\lambda - m\,\bar{t}\,\bar{\lambda}$$

quindi
$$G = m\begin{bmatrix} Var(t) + \bar{t}^2 & \bar{t} \\ \bar{t} & 1 \end{bmatrix}$$

termine noto dell'eq. normale
$$D = -B^T\lambda = \begin{bmatrix} t_1 & \dots & t_m \\ 1 & \dots & 1 \end{bmatrix}\begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_m \end{pmatrix} = \begin{pmatrix} t\cdot\lambda \\ m\bar{\lambda} \end{pmatrix} = m\begin{pmatrix} Cov(t,\lambda) + \bar{t}\,\bar{\lambda} \\ \bar{\lambda} \end{pmatrix}$$

Se $\det G = m^2 Var(t) > 0$ allora
$$\Gamma = \frac{1}{m\, Var(t)}\begin{bmatrix} 1 & -\bar{t} \\ -\bar{t} & Var(t) + \bar{t}^2 \end{bmatrix}$$

I coeff. delle rette di regressione sono

$$\begin{pmatrix} a^* \\ b^* \end{pmatrix} = -\Gamma B^T\lambda = \Gamma D = \frac{\cancel{m}}{\cancel{m}\, Var(t)}\begin{bmatrix} 1 & -\bar{t} \\ -\bar{t} & Var(t) + \bar{t}^2 \end{bmatrix}\begin{pmatrix} Cov(t,\lambda) + \bar{t}\,\bar{\lambda} \\ \bar{\lambda} \end{pmatrix}$$

$$= \frac{1}{Var(t)}\begin{pmatrix} Cov(t,\lambda) \\ -\bar{t}\,Cov(t,\lambda) + \bar{\lambda}\, Var(t) \end{pmatrix} = \begin{pmatrix} Cov(t,\lambda)/Var(t) \\ \bar{\lambda} - \bar{t}\, Cov(t,\lambda)/Var(t) \end{pmatrix} \Rightarrow \boxed{b^* = \bar{\lambda} - a^*\bar{t}}$$