La soluzione dell'eq. normale è $\qquad x^* = -\Gamma B^T \lambda$

Gli ellissoidi definiti da C hanno centro in x^*, cioè

$$m\, Q(x) = m\, Q^* + (x - x^*)^T C (x - x^*)$$

$Q^* = Q(x^*)$ è il valore minimo.

<u>CALCOLO di Q^*</u> $\qquad m\,Q^* = \lambda^T\lambda - 2\lambda^T B \Gamma B^T \lambda + \lambda^T B \cancel{\Gamma^T} \cancel{C} \Gamma B^T \lambda =$

$= (\lambda + B x^*)^T (\lambda + B x^*)$

$$= \lambda^T\lambda - \lambda^T B \Gamma B^T \lambda \leq \lambda^T \lambda$$

vettore dei residui dopo il fit: $\quad \underbrace{\xi = \lambda + B x^* = \lambda - B\Gamma B^T \lambda}$

da $m\,Q(x^*) = m\,Q^* = (\lambda + B x^*)^T (\lambda + B x)$

<u>ESEMPIO</u> (retta di regressione)

$f(t) = at + b \qquad t_i \quad i = 1 \dots m \qquad \xi_i = \lambda_i - f(t_i) = \lambda_i - a t_i - b$

$$Q(a,b) = \frac{1}{m} \sum_{i=1}^{m} \left[\lambda_i - a t_i - b\right]^2 = \frac{1}{m}\left[\lambda^T\lambda + 2\lambda^T B x + x^T B^T B x\right]$$

$$\begin{cases} m \dfrac{\partial Q}{\partial a} = \sum_{i=1}^{m} 2\left[\lambda_i - a t_i - b\right](-t_i) = 0 \\ m \dfrac{\partial Q}{\partial b} = \sum_{i=1}^{m} 2\left[\lambda_i - a t_i - b\right](-1) = 0 \end{cases}$$

$$\begin{cases} \sum_i \lambda_i t_i - a \sum_i t_i^2 - b \sum_i t_i = 0 \\ \sum_i \lambda_i - a \sum_i t_i - b m = 0 \end{cases}$$

$$\begin{bmatrix} \sum_i t_i^2 & \sum_i t_i \\ \sum_i t_i & m \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_i \lambda_i t_i \\ \sum_i \lambda_i \end{pmatrix}$$

$x = (a, b) \qquad B = (b_{ik}) \qquad b_{ik} = -f_k(t_i)$

$$\begin{cases} f_1(t) = t \\ f_2(t) = 1 \end{cases} \qquad B = -\begin{bmatrix} t_1 & 1 \\ \vdots & \vdots \\ t_m & 1 \end{bmatrix}$$

$$C = B^T B = \begin{bmatrix} t_1 \dots t_m \\ 1 \dots 1 \end{bmatrix} \begin{bmatrix} t_1 & 1 \\ \vdots & \vdots \\ t_m & 1 \end{bmatrix} = \begin{bmatrix} t \cdot t & t \cdot 1 \\ 1 \cdot t & m \end{bmatrix}$$

eq. normale: $\quad C x = -B^T \lambda$

$$x^* = -\Gamma B^T \lambda$$

$$C = \begin{bmatrix} \sum_i t_i^2 & \sum_i t_i \\ \sum_i t_i & m \end{bmatrix}$$

$$\Gamma = \frac{1}{\det C} \begin{bmatrix} m & -\sum_i t_i \\ -\sum_i t_i & \sum_i t_i^2 \end{bmatrix}$$