

Leandro Arosio: *A wandering Short C^2*

The study of holomorphic dynamics in the complex space C^2 focused mainly on polynomial automorphisms, and it is an open question whether wandering Fatou components can exist for such maps. In the larger class of holomorphic automorphisms it is known that wandering components may occur, and all known examples are biholomorphic to C^2 . In this talk I will show how to construct a holomorphic automorphism of C^2 with a wandering Fatou component biholomorphic to a Short C^2 . Such domains were originally constructed by Fornæss as nonautonomous basins of attraction and are not biholomorphic to C^2 since they carry a nonconstant bounded plurisubharmonic function. To obtain an automorphism with a wandering Short C^2 we replicate the nonautonomous behaviour along an oscillating orbit of a single automorphism. This is a joint work with Luka Boc Thaler and Han Peters.

Matthias Aschenbrenner: *Hardy fields and transseries*

The objects in the title are two among several approaches to enrich the real continuum by infinitesimal and infinite quantities. Both come with natural notions of ordering and differentiation. In joint work with van den Dries and van der Hoeven we developed a framework of ‘asymptotic differential algebra’ to unify these approaches. In this talk aimed at a non-specialist audience, I will put this project into context, explain how mathematical logic contributes to this endeavor, and outline current challenges and further possibilities.

Filippo Bracci: *Fantastic Fatou components and where to find them*

The classification of Fatou components for automorphisms of the complex space of dimension greater than 1 is an interesting and difficult task. Recent deep results prove that the one-dimensional setting is deeply different from the higher dimensional one. Given an automorphism F of C^k , the first bricks in the theory that one would like to understand are invariant Fatou components, namely, those connected open sets U , completely invariant under F , where the dynamics of F is not chaotic. Among those, we consider ‘attracting’ Fatou components, that is, those components on which the iterates of F converge to a single point. Attracting Fatou components can be recurrent, if the limit point is inside the component or non-recurrent. Recurrent attracting Fatou components are always biholomorphic to C^k , since it was proved by H. Peters, L. Vivas and E. F. Wold that in such a case the point is an attracting (hyperbolic) fixed point, and the Fatou component coincides with the global basin of attraction. Also, as a consequence of works of Ueda and Peters-Lyubich, it is known that all attracting non-recurrent Fatou components of polynomial automorphisms of C^2 are biholomorphic to C^2 . One can quite easily find non-simply connected non-recurrent attracting Fatou components in C^3 (mixing a two-dimensional dynamics with a dynamics with non-isolated fixed points in one-variable). In this talk I will explain how to construct a non-recurrent attracting Fatou component in C^2 which is biholomorphic to $C \times C^*$. This ‘fantastic beast’ is obtained by globalizing, using a result of F. Forstneric, a local construction due to the

speaker and Zaitsev, which allows to create a global basin of attraction for an automorphism, and a Fatou coordinate on it. The Fatou coordinate turns out to be a fiber bundle map on C , whose fiber is C^* , then the global basin is bi-holomorphic to $C \times C^*$. The most subtle point is to show that such a basin is indeed a Fatou component. This is done exploiting Poschel's results about existence of local Siegel discs and suitable estimates for the Kobayashi distance. Since attracting Fatou components are Runge, it turns out that this construction gives also an example of a Runge embedding of $C \times C^*$ into C^2 . Moreover, this example shows an automorphism of C^2 leaving invariant two analytic discs intersecting transversally at the origin. I will also describe briefly the construction of another 'fantastic' invariant non-recurrent Fatou component: a Siegel cylinder. The first part of the talk is based on a joint work with J. Raissy and B. Stenones (JEMS, 2019 to appear) and the second one on a work in progress with Luka Boc-Thaler.

Zoé Chatzidakis: *Algebraic dynamics, difference fields and model theory*

In this talk I will explain how some problems about algebraic dynamics translate into problems of difference algebra. I will mention two applications, one using model theory (by Hrushovski and myself), the other mainly using difference algebra (by Medvedev and Scanlon).

Luigi Chierchia: *The topology of simple resonances in nearly-integrable Hamiltonian systems*

Simple resonances play a fundamental role in perturbation theory and in particular introduce "robust" nonlinear effects. We describe the analytic and topological structure of simple resonances for nearly-integrable analytic Hamiltonian system for holomorphic generic perturbations.

Dikran Dikranjan: *Normed monoid entropy of semigroup actions*

We discuss two entropies defined for semigroup action on a normed monoid. When the acting semigroup is amenable and cancellative, we make use of a recent analogue of Ornstein-Weiss lemma obtained in [1], otherwise, when the semigroup is finitely generated we follow an approach due to Ghys, Langevin and Walczak [2], as well as to Hofmann and Stoyanov [3]. Using appropriate functors to the category of normed monoids, one can obtain from these two entropies:

- the already known topological entropy, or metric entropy, or algebraic entropy of a semigroup action on a compact topological space, on a probability measure space, or an abelian group, respectively;
- the already known receptive topological entropy (defined in [2] and [3]), as well as receptive metric entropy (defined in [4]) and receptive algebraic entropy, defined analogously.

We compare these two entropies trying to motivate the use of the receptive entropy.

[1] T. Ceccherini-Silberstein, M. Coornaert, F. Krieger, An analogue of Fekete’s lemma for subadditive functions on cancellative amenable semigroups, *J. Anal. Math.* 124 (2014) 59–81.

[2] É. Ghys, R. Langevin, P. Walczak, Entropie géométrique des feuilletages, *Acta Math.* 160 (1988) 105–142.

[3] K.-H. Hofmann, L.N. Stoyanov, Topological entropy of group and semi-group actions, *Adv. Math.* 115 (1995) 54–98.

[4] L. Todorovich, Entropy of \mathbb{Z}^N -actions, Honours Thesis supervised by L. Stoyanov, University of Western Australia, 2009.

Antongiulio Fornasiero: *Algebraic entropy of generalized Bernoulli shifts*

Let u be an action of an Amenable group G on a set X . For every Abelian group A , u then induces two canonical ”shifts” actions of G on A^X and on $A^{(X)}$. We study the algebraic entropy of such shifts, with a generalization to the case when G is an amenable cancellative semigroup.

Giorgio Fusco: *Periodic motions for multy-wells potentials and layers dynamic for the vector Allen-Cahn equation*

Let $W : \mathbb{R}^m \rightarrow \mathbb{R}$ a potential that satisfies

$$0 = W(a) < W(u), \quad a \in A, \quad u \in \mathbb{R}^m \setminus A,$$

where $A \subset \mathbb{R}^m$ is a finite set with at least two distinct elements. We consider the Hamiltonian system

$$u'' = W_u(u), \quad W_u(u) = \left(\frac{\partial W}{\partial u_1}(u), \dots, \frac{\partial W}{\partial u_m}(u) \right)^\top,$$

and, given a small number $\delta > 0$ and $N \geq 2$ points $a_1, \dots, a_N \in A$ with $a_j \neq a_{j+1}$, $j = 1, \dots, N - 1$ and $a_N \neq a_1$ we study the existence of periodic solutions $u_\delta : \mathbb{R} \rightarrow \mathbb{R}^m$ that satisfies

$$|u_\delta(s_j^\delta) - a_j| < \delta, \quad j = 1, \dots, N,$$

for some $s_1^\delta < \dots < s_N^\delta \in [0, T^\delta)$, $T^\delta > 0$ the period of u^δ .

We also discuss the problem of layers dynamics for the vector Allen-Cahn equation

$$u_t = \epsilon^2 u_{xx} - W_u(u), \quad x \in (0, 1),$$

with periodic boundary conditions.

Paolo Giulietti: *Parabolic dynamics via anisotropic spaces*

I will present a strategy which allows to study ergodic averages and cohomological equations related to parabolic dynamics by means of hyperbolic renormalizations, exploiting transfer operator techniques on anisotropic Banach spaces. After reporting on some cases where this strategy has been successfully applied, I will present recent work both on the horocycle flow and on parabolic

flows on surfaces with singularities. Some results are joint work with C. Liverani and some with M. Artigiani.

Marc Kessebohmer: *Multifractal Decompositions of Transient Dynamics*

We develop a new thermodynamic formalism to investigate the transient behaviour of maps on the real line which are skew-periodic \mathbb{Z} -extensions of expanding interval maps. Our main focus lies in the dimensional analysis of the recurrent and transient sets as well as in determining the whole dimension spectrum with respect to the α -escaping sets. Our results provide a one-dimensional model for the phenomenon of dimension gaps which occur for limit sets of Kleinian groups. In particular, we are able to precisely quantify the height of the dimension drop in this setting. (Joint work with Maik Gröger and Johannes Jaerisch).

Salma Kuhlmann: *Automorphism groups of Hahn groups and fields*

Inspired by Schilling's work [O. F. G. Schilling. *Automorphisms of fields of formal power series*. Bull. Amer. Math. Soc. 50.12 (1944), pp. 892-901] on the study of internal automorphisms of the field of Laurent series, we extend our investigation to ordered group automorphisms, respectively ordered field automorphisms of Hahn groups respectively Hahn fields.

Carlangelo Liverani: *Quantitative statistical properties of partially hyperbolic systems*

Partially hyperbolic systems have been the object of intense study and very important results concerning ergodicity and mixing have been obtained. However very few results are available on quantitative statistical properties (such as a precise descriptions of the SRB measure and explicit bounds on the decay of correlations). Indeed it is unclear which strategies are best suited to yield such results. To help forward this sorry state of affairs I will discuss a seemingly simple partially hyperbolic system and discuss what can be proven and what are the open problems.

Lorena Lopez-Hernanz: *Stable manifolds for germs of biholomorphisms in $\{\mathbb{C}\}^n$.*

The existence of stable manifolds for germs of biholomorphisms in dimension one is a well-known problem, which depends essentially on the linear part of the map. I will present a result for germs of biholomorphisms in higher dimension that guarantees, assuming the existence of a formal invariant curve and imposing only the natural conditions that arise from the one-dimensional case, the existence of stable manifolds where the orbits are 'infinitely tangent' to the formal curve. This is a joint work with F. Sanz, J. Ribón and L. Vivas.

Valerio Lucarini: *Stochastic Dynamics of a Bistable Climate Model: Melancholia States and Instantons*

The climate of planet Earth is in a regime of bistability. The current astro-

nomical and astrophysical conditions, together with the chemistry of the planet, are such that two competing attracting states are present. Such a regime is not unique to the present conditions, but is instead realised for a vast range of values of the incoming solar radiation. One state is the one we live in and that is - currently - supportive of complex life; the other state is characterised by global glaciation and by conditions that can hardly support any life form. This is a robust finding confirmed across a hierarchy of climate models. Paleoclimatic evidences suggest that, indeed, our planet has flipped between the two states. The main physical mechanism responsible for the existence of such a regime is the ice-albedo feedback. Using a climate model of intermediate complexity, comprising of a fast component given by a three-dimensional atmosphere coupled to a slow component corresponding to a diffusive ocean, we are able to define the global stability properties of the climate, by constructing the Melancholia states that sit between the two competing attractors. Such states are embedded in the boundaries between the two basins of attraction and are characterised by having extensive glaciation down to relatively low latitudes. We then perturb the system by adding a random modulation to the incoming solar radiation, which introduces a multiplicative noise impacting the system's evolution. As a result, we observe noise-induced transitions between the competing basins of attractions. Large deviations laws written in terms of a pseudo-potential define the natural measure and the statistics of escape times. By empirically constructing the instantons connecting the attractors with the edge states, we also show that the edge state is the gateway for the noise induced transitions between the two asymptotic states. Finally, we show that, in the weak-noise limit, in the region of multiple stability, only one of the possible steady state is, in fact, realised, because of the lack of symmetry of the pseudo-potential. For low values of the solar irradiance, the noise selects as natural measure the snowball state, while for large values of the solar irradiance, the asymptotic state is the warm attractor. The changeover between the two regimes corresponds to a first order phase transition in the system, where the control parameter is the solar irradiance. Different choices of the noise law will lead to different values for the critical value of the control parameter. Taking advantage of the freedom to choose the law of the noise perturbation, we propose a new method for constructing edge states that bypasses the difficulties of the edge tracking algorithm used in the case deterministic dynamics is considered. While we analyse a specific yet important problem in paleoclimatology, our results seem to suggest a more general framework for the study of complex multistable systems.

Refs:

V. Lucarini and T. Bódai, Edge states in the climate system: exploring global instabilities and critical transitions, *Nonlinearity* 30 R32 (2017)

V. Lucarini and T. Bódai, Transitions across Melancholia States in a Climate Model: Reconciling the Deterministic and Stochastic Points of View, arXiv:1808.05098 (2018)

Luigi Marangio: *Arnold's map as a toy model for ENSO: a rigorous in-*

vestigation

The El Nino-Southern Oscillation (ENSO) is a naturally occurring phenomenon which has a major influence on climate patterns in various parts of the world; in the physical literature some toy model was proposed for its study. These models try to reproduce some of the key features of this complex dynamic using maps which involve some non-chaotic features, like frequency locking, coupled with some chaotic ones. The simplest of such mathematical models, perhaps is a map on the circle similar to the famous Arnold's map, with some additive noise.

Arnold's standard circle maps are widely use to model natural phenomena where two rationally unrelated periodicities are in competition. We consider in this work Arnold's maps with additive, uniformly distributed noise: the presence of the noise, models the (recognized) role of westerly wind bursts in the onset of El Nino.

When the map's nonlinear term, scaled by the parameter ϵ , is small, $\epsilon < 1$, the map is known to be a diffeomorphism and the rotation number $\rho = \rho(\tau)$ is a differentiable function of the driving frequency τ . Moreover, in this case, the rotation number is a monotonic function with respect to ω .

We will show that as the non linearity becomes large, the rotation number is still differentiable but is no longer a monotonic function. The first result is consequence of linear response theory, which requires a mixing hypothesis to be verified. To accomplish this task, we will rigorously compute some of the statistical properties of the Arnold's maps, through the computation of an Ulam's discretization of the transfer operator associated to the systems. Moreover this procedure leads to a certified approximation of the stationary measure in the L1 norm, which can be used to rigorously compute the rotation number, and thus to show its non monotonicity.

Mickaël Matusinski: *Non oscillating trajectories of o-minimal vector fields in dim 3*

In the context of a polynomially bounded o-minimal structure over the field of real numbers, we consider a system of two non autonomous differential equations. We show that two non oscillating solutions of such system are either interlaced, or else have their coordinates belonging to a common Hardy field. This dichotomy generalizes some of the results from F. Cano, R. Moussu and F. Sanz about non oscillating trajectories of real analytic vector fields in dimension 3 (dichotomy for integral pencil of trajectories), and from O. Le Gal, P. Speissegger and F. Sanz about solutions of o-minimal linear differential systems. After introducing the notions and context, if time permits we'll give a sketch of the proof. Joint work with O. Le Gal and F. Sanz.

Sara Munday: *A Farey-like map on the triangle*

We introduce a "slow version" of the triangle map that was first introduced by Garrity in 2001. The triangle map can be thought of as being a two-dimensional generalisation of the classical Gauss map, as such, our map is

akin to the Farey map. We observe that it is conservative, ergodic and preserves an infinite measure. We give a weak law of large numbers, using the fact that our map is pointwise dual ergodic. We also discuss a class of observables for which we can say that the Birkhoff average is almost everywhere constant.

Isaia Nisoli: *Noise induced order in the Matsumoto-Tsuda model*

In 1980, studying experimental data obtained from the Belosou-Zabotinsky chemical reaction, Matsumoto and Tsuda introduced a model: a random dynamical system consisting of a one-dimensional unimodal map with uniform additive noise. Through numerical simulations, Matsumoto and Tsuda conjectured that this maps presents a phenomenon that they called Noise Induced Order, i.e., for small noise sizes the Lyapunov exponent of the stationary measure is positive and for big noise sizes the Lyapunov exponent is negative. In this talk I will present a joint work with Galatolo and Monge, which proves this conjecture.

Giovanni Panti: *Billiards on pythagorean triples and their Minkowski functions*

It has long been known that the set of primitive pythagorean triples can be enumerated by descending certain ternary trees. We unify these treatments by considering hyperbolic billiard tables in the Poincare disk model. Our tables have $m \geq 3$ ideal vertices, and are subject to the restriction that reflections in the table walls are induced by matrices in the triangle group $PSU_{\{1,1\}}^{\pm} Z[i]$.

The resulting billiard map \tilde{B} acts on the de Sitter space $x_1^2 + x_2^2 - x_3^2 = 1$, and has a natural factor B on the unit circle, the pythagorean triples appearing as the B -preimages of fixed points. We compute the invariant densities of these maps, and prove the Lagrange and Galois theorems: a complex number of unit modulus has a preperiodic (purely periodic) B -orbit precisely when it is quadratic (and isolated from its conjugate by a billiard wall) over $Q(i)$. Time permitting, we will also discuss the Minkowski function that conjugates B with the group character $T(z) = z^{-(m-1)}$.

Marks Ruziboev: *Linear response for random dynamical systems*

We study linear response for random compositions of maps, chosen independently according to a distribution P . For a wide class of one dimensional random maps, we prove that absolutely continuous stationary measures vary smoothly when P is perturbed smoothly. We also obtain a linear response formula. We apply our results to iid compositions, with respect to various distributions P , of uniformly expanding circle maps, Gauss-Rényi maps (random continued fractions) and Pomeau-Manneville maps. Our results yield an exact formula for the invariant density of random continued fractions; while for Pomeau-Manneville maps our results provide a precise relation between their linear response under certain random perturbations and their linear response under deterministic perturbations. This is a joint work with Wael Bahsoun and Benoit Saussol.

Benoit Saussol: *Spatio-temporal Poisson process for visits to small sets in hyperbolic dynamics*

We study a recurrence property of a finite measure preserving dynamical system (X, T, m) . Given a small set A , we are interested in the process obtained from recording the successive times n of visits to A and the position $T^n(x)$ in A of the orbit, in the limit where $m(A) \rightarrow 0$. We obtain a convergence of this process, suitably normalized, to a Poisson point process in time and space under some decorrelation condition. We present some applications to hyperbolic maps and SRB measures and some billiards. We will consider the case of a balls centered at a periodic point. (joint work with Françoise Pène)

Julien Sedro: *Response for non-autonomous dynamical systems*

We will start this talk by defining response in the context of ergodic theory and giving some examples motivating this study in the context of non-autonomous systems. We will then try to explain for which models response can actually be proved, comparing the autonomous and non-autonomous cases and highlighting the connection with spectral properties of the transfer operator.

Sergei Starchenko: *O-minimal flows on compact nilmanifolds*

Let G be a real algebraic unipotent group, $\Lambda < G$, $M = G/\Lambda$ and $\pi: G \rightarrow M$ the quotient map. In this talk we consider the topological closure $\text{cl}(\pi(X))$ for a $X \subseteq G$ definable in some o-minimal expansion of the real field, and describe it in terms of images of definable families of real algebraic subgroups of G . We also discuss C-uniformity for distributions given by definable functions $f: \{R\} \rightarrow G$. (joint work with Y. Peterzil)