NOTA : COLLISIONE BINARIA UNIDIMENSIONALE

$$\begin{cases} \xi = \dfrac{(\tau^*)^2}{2!} - \dfrac{C(\tau^*)^4}{4!} + \cdots \\[2ex] t^* = \dfrac{(\tau^*)^3}{3!} - \dfrac{C(\tau^*)^5}{5!} + \dfrac{C^2(\tau^*)^7}{7!} + \cdots \end{cases}$$

$$t^* = t - t_c, \qquad \tau^* = \tau - \tau_c$$

per comodità pongo

$$s = \tau^* \quad e \quad w = t^*$$

$$w = \frac{s^3}{3!}\left(1 - \frac{3!\,C s^2}{5!} + \frac{3!\,C^2 s^4}{7!} + O(s^6)\right)$$

$$(3!\,w)^{1/3} = s\left(1 - \frac{3!\,C s^2}{5!} + \frac{3!\,C^2 s^4}{7!} + O(s^6)\right)^{1/3}$$

usando lo sviluppo binomiale

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + O(x^3)$$

si ha

$$(3!\,w)^{1/3} = s\left(1 - \frac{3!\,C s^2}{3\cdot 5!} + \underbrace{\left(\frac{3!\,C^2 s^4}{3\cdot 7!} - \frac{(3!)^2 C^2 s^4}{9\cdot(5!)^2}\right)}_{= 2C^2 s^4\left(\frac{1}{7!} - \frac{2}{(5!)^2}\right)} + O(s^6)\right)$$

$$s = (3!\,w)^{1/3} + \frac{3!\,C s^3}{3\cdot 5!} - 2C^2 s^5\left(\frac{1}{7!} - \frac{2}{(5!)^2}\right) + O(s^7)$$

Assumiamo che s sia dato da uno sviluppo in serie di potenze di $w^{1/3}$

$$s = \sum_{n\geq 0} c_n w^{n/3}$$

Si vede che $c_0 = 0$. Determiniamo c_1, c_2, c_3, c_4, c_5

$$c_1 w^{1/3} + c_2 w^{2/3} + c_3 w + c_4 w^{4/3} + c_5 w^{5/3} + \dots =$$

$$(3!\, w)^{1/3} + \frac{3!\, C}{3 \cdot 5!} \left(c_1 w^{1/3} + c_2 w^{2/3} + c_3 w + c_4 w^{4/3} + c_5 w^{5/3} + \dots \right)^3$$

$$- 2C^2 \left(\frac{1}{7!} - \frac{2}{(5!)^2} \right) \left(c_1 w^{1/3} + c_2 w^{2/3} + c_3 w + c_4 w^{4/3} + c_5 w^{5/3} + \dots \right)^5$$

$$+ O(s^7)$$

ponendo c_i $(i = 1, \dots, 5)$ uguale al coefficiente di $w^{i/3}$ che compare a destra dell'uguale si ha:

$$c_1 = (3!)^{1/3}$$

$$c_2 = 0$$

$$c_3 = \frac{3!\, C}{3 \cdot 5!} c_1^3 = \frac{(3!)^2 C}{3 \cdot 5!} = \frac{2C\, 3!}{5!}$$

$$c_4 = 0$$

$$c_5 = \frac{3!\, C}{3 \cdot 5!} 3 c_1^2 c_3 - 2C^2 \left(\frac{1}{7!} - \frac{2}{(5!)^2} \right) c_1^5$$

$2 =$

$$= \frac{3!\, C}{3 \cdot 5!} 3 (3!)^{2/3} \frac{2C\, 3!}{5!} - 2C^2 \left(\frac{1}{7!} - \frac{2}{(5!)^2} \right) (3!)^{5/3}$$

$$= 2C^2 (3!)^{5/3} \left(\frac{8}{(5!)^2} - \frac{1}{7!} \right)$$

Allora abbiamo ottenuto

$$s = (3!)^{1/3} w^{1/3} + \frac{2C\, 3!}{5!} w + 2C^2 (3!)^{5/3} \left(\frac{8}{(5!)^2} - \frac{1}{7!} \right) w^{5/3} + \dots$$

Ora vogliamo usare questo risultato in

$$\xi = \frac{s^2}{2!} - \frac{C s^4}{4!} + \dots$$

per ottenere ξ in funzione di w, cioè del tempo.

Si ha

$$s = \frac{(3!)^{2/3}}{2!} w^{2/3}$$

$$+ \left[\overbrace{\frac{1}{2!}\left(2\,(3!)^{1/3} \frac{2C\,3!}{5!} \right) - \frac{1}{4!} C\,(3!)^{4/3}}^{= C(3!)^{4/3}\left(\frac{2}{5!} - \frac{1}{4!}\right) = 6C\sqrt[3]{6}\left(-\frac{3}{120}\right) = -\frac{3C\sqrt[3]{6}}{20}} \right] w^{4/3}$$

$$+ \left[\overbrace{\frac{1}{2!}\left(\frac{4C^2(3!)^2}{(5!)^2} + 2(3!)^{1/3}\, 2C^2 (3!)^{5/3} \left(\frac{8}{(5!)^2} - \frac{1}{7!} \right)\right)}^{*} \right.$$

$$\left. \underbrace{- \frac{C}{4!}\left(4\; 3!\; \frac{2C\,3!}{5!} \right)}_{※} \right] w^2 + \dots$$

questo termine viene da

$$\left((3!)^{1/3} w^{1/3} + \frac{2C\,3!}{5!}\, w \right)^4$$

$$* = 2C^2(3!)^2 \left(\frac{9}{(5!)^2} - \frac{1}{7!} \right) = \frac{43C^2}{1400}$$

$$※ = -\frac{8C^2(3!)^2}{4!\,5!} = -\frac{C^2}{10}$$

Dunque si ha

$$\xi = \frac{(3!)^{2/3}}{2!} w^{2/3} - \frac{3C(3!)^{1/3}}{20} w^{4/3} + \overbrace{\left(\frac{43}{1400} - \frac{1}{10} \right)}^{= -\frac{97}{1400}} C^2 w^2 + \dots$$

cioè

$$\xi = \sqrt[3]{\frac{9}{2}}\,(t^*)^{2/3} - \frac{3\sqrt[3]{6}\,C}{20}(t^*)^{4/3} - \frac{97\,C^2}{1400}(t^*)^2 + \dots$$

Infine, introducendo l'energia H, con $C = -2H$, si trova

$$\xi = \sqrt[3]{\frac{9}{2}}\,(t^*)^{2/3} + \frac{3\sqrt[3]{6}\,H}{10}(t^*)^{4/3} - \frac{97\,H^2}{350}(t^*)^2 + \dots$$