2018AMC$_{70}$

BETWEEN MATHEMATICS AND ASTRONOMY

A WORKSHOP IN HONOR OF ANDREA MILANI COMPARETTI
ON THE OCCASION OF HIS 70th BIRTHDAY

3-5 SEPTEMBER 2018
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF PISA

BOOK OF ABSTRACTS

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Invited speakers

Roberto Battiston (Agenzia Spaziale Italiana, Italy)
Precision Physics in Space

Space is the best suited environment to perform highly precise test of theories like general relativity and quantum mechanics. I will review some of the researches conducted at the Italian Space Agency on precision physics in space.

Fabrizio Bernardi (SpaceDyS s.r.l., Italy)
From NEODyS to SpaceDyS

I present the evolution of the team of Milani at the University of Pisa from the NEODyS experience to the establishment of the SpaceDyS company. Born in 2011, SpaceDyS is a spin-off of the University of Pisa, counting 11 partners, including Andrea Milani and his long time collaborators. The present core business is related to the provision of services and qualified consultancy to ESA for the precursor services of the Space Situational Programme of the agency relative to the NEO and asteroids expertize, and to the development of a new software for ESA aimed to provide the orbit determination and impact monitoring of asteroids for the NEO Coordination Centre system in Frascati. During these years the company has grown and expended its activities in other fields such as the space debris orbit determination, image processing, re-entry prediction, complex dynamics for radio science experiments and on board orbit determination with GNSS signals.

Alessandra Celletti (University of Rome Tor Vergata, Italy)
Normal forms & KAM theory, space debris & the rotation of the Moon

Since centuries Celestial Mechanics is a test-bench for many theories of Dynamical Systems, among which perturbation theory and KAM theory. An accurate modeling and an appropriate study of the dynamics is mandatory to get realistc results, but they often require a heavy computational effort. After an overview on normal forms and (conservative and dissipative) KAM theory, I will consider two examples in Celestial Mechanics, where normal forms and KAM theory give successful results. The first example concerns the dynamics of space debris, which is studied through averaging theory and normal forms computations. The second example concerns the rotation of the Moon, whose stability can be investigated through a computer-assisted implementation of KAM theory.

Alberto Cellino (INAF-Osservatorio Astrofisico di Torino, Italy)
Finding asteroid families with AMC

I had the chance to collaborate with AMC for a long time in the field of identification of asteroid families. This is a field in which many important results have been obtained by AMC and his friends and collaborators. In particular, the identification of families is a natural development of the computation of the orbital proper elements of asteroids, a traditional field of excellence in which the contribution of AMC has been absolutely fundamental. Of course, as many friends of Andrea, I had the opportunity of experiencing his explosive changes of mood, a side of his personality for which he is well known and with good reason.

Rudolf Dvorak (Univ. of Vienna, Institute for Astrophysics, Austria)
On the Formation of Terrestrial Planets

In this talk we deal with the formation of terrestrial planets in our Solar System and also in the interesting exosystem HD141399 with 4 gasgiants. Starting with several hundred small bodies in the size of the Moon we follow their evolution on one hand in our SS where already Jupiter and Saturn are in their present position and on the other hand in the formentioned planetary system. The growing of the planets through collisions and merging is treated with simple numerical integrations but also with sophisticated SPH codes. We show how terrestrial planets accumulate and deal also with the role of the water during these collisions.

Davide Farnocchia (JPL-Caltech, California, USA)
The trajectory of interstellar visitor 'Oumuamua

The object now known as 1I/'Oumuamua was discovered on 2017 October 19 by the Pan-STARRS1 survey. Within a few days of discovery, additional observations collected with ESA's Optical Ground Station telescope and other observatories, together with pre-discovery data from Pan-STARRS1, showed that the orbit of 'Oumuamua was hyperbolic with an eccentricity of 1.2, identifying the object as originating from outside the Solar System and approaching from the direction of the constellation Lyra, with an asymptotic inbound velocity of about 26 km/s. A variety of physical observations collected during the visit of 'Oumuamua to the Solar System show that the object has an unusually elongated shape and a tumbling rotation state, while its surface physical properties resemble those of cometary nuclei, even though 'Oumuamua showed no evidence of cometary activity. With the goal of further constraining the trajectory of 'Oumuamua, we collected high-quality astrometry with the Canada-France-Hawaii Telescope, the ESO Very Large Telescope, and the Hubble Space Telescope. The resulting dataset provides dense coverage from discovery to 2018 January 2, when 'Oumuamua became fainter than $V = 27$ at a heliocentric distance of 2.9 au. We present the results of our trajectory analysis and discuss the implications on the nature and origin of this interstellar visitor.

Sylvio Ferraz-Mello (Universidade de São Paulo, Brasil)
Tidal synchronization of close-in terrestrial planets and planetary satellites

The application of the creep tide theory (Ferraz-Mello et al.) and of the Maxwell model (Correia et al.) to the rotation of close-in terrestrial planets and planetary satellites shows that, in these cases, the rotation is damped to attractors with periods nearly commensurable with the orbital period, and the final solutions are not stationary. These attractors are forced oscillations (physical librations) around one center. For this reason, the use of some classical averaged models to study the evolution of planetary satellites and other stiff bodies is not appropriate. In order to overcome these difficulties, we adopt, for the creep tide theory, the new model proposed by Folonier et al. which allows the simultaneous calculations of the tidal deformations and the body rotation and use it to study the cases of Enceladus, Mimas and Ganymede.

Massimiliano Guzzo (Università degli studi di Padova, Italy)
Integrability of the restricted three-body problem near collisions

We present the integration of the spatial circular restricted three-body problem in a neighbourhood of its collision singularities by extending an idea of Tullio Levi-Civita.

Robert Jedicke (University of Hawaii, USA)
The Panoramic Survey Telescope and Rapid Response System

I will provide an overview of the design, goals, and construction of the Pan-STARRS survey system focussing on Andrea Milani's contributions to its Moving Object Processing System (MOPS).

Jacques Laskar (IMCCE, Observatoire de Paris, France)
Long term motion of the Solar System

Anne Lemaître (Institute naXys - University of Namur, Belgium)
40 years of collaboration between Namur and Pisa

The talk will summarize the collaborations between our two teams during 40 years, and will be the opportunity to thank Andrea Milani for his faithfull friendship.

Andrea Milani Comparetti (University of Pisa, Italy)
Studying for 52 years: what can be learned from such a long experience

Alessandro Morbidelli (CNRS/OCA, France)
Looking for very old, dispersed asteroid families

The original size distribution of asteroids is not known. It is possible that most small asteroids ($D < 50$ km) are collisional fragments (Bottke et al., 2005). However, only a small fraction of the asteroid population at these sizes belong to known families, identified by the HCM. It is probably that the collisional generation of these asteroids occurred during the early times. The asteroid belt should have lived through three different dynamical phases: (i) a phase of dynamical excitation/depletion/mixing associated to planet formation; (ii) a phase of dispersion in eccentricity and inclination (but not semi major axis) during the dynamical instability that characterized the giant planet evolution, sometimes after the disappearence of the gas from the protoplanetary disk and

(iii) the current phase with the planets on known orbits. The families identifiable by the HCM could only have formed during phase (iii). Thus, we have designed a method (Bolin et al., 2017) that can allow us to identify old and big families that formed after phase (i) but before phase (ii). With this method we have identified a family of primitive bodies formed in the inner asteroid belt (Delbo et al., 2017). This allowed us to give some constraints on the size distribution of asteroids after phase (i) and show that it was much shallower than the current one, as predicted in Bottke et al. (2005). This supports the idea that asteroids formed big. The hunt for old pre-phase(ii) families has just started. It could potentially provide strict constraints on the dynamical instability of the giant planets and its chronology.

Ettore Perozzi (Agenzia Spaziale Italiana, Italy)
SSA & AMC: a history of acronyms

Ten years ago ESA started the Space Situational Awareness (SSA) programme which soon turned out to be the right institutional framework for further developing what was being done at the university of Pisa on NEO impact monitoring. This eventually allowed the establishment of the ESA NEO Coordination Centre at ESRIN and the migration of NEODyS into the ESA NEO System. Nevertheless, negotiations were often difficult, sometimes painful, yet always rewarding. A review of the development of the ESA SSA NEO Segment is presented throughout acronyms, anecdotes, science, technology, and geographical returns.

Alessandro Rossi (IFAC-CNR, Italy)
Space debris mitigation: history and perspectives

Since the realization of the danger represented by the space debris it became clear that an effective prevention and mitigation action was needed to control the proliferation of the Earth orbiting population of objects. Those actions were indeed able to partly stabilize the growth of the population, but more aggressive measures should be undertaken in the future decades, also in view of the proposed launch of the mega constellations of satellites in Low Earth Orbit (LEO) which are prone to change in radical way the traffic and operations in this already crowded region of space. It is now clear that the impact of orbital debris on the space activities should be reduced tackling the problem from different points of view, including prevention, mitigation and protection. These goals can only be achieved through a global approach that considers, from the outset of a space mission, opposing and challenging constraints for the space environment preservation, the spacecraft survivability in the harsh space environment and the safety of humans on ground. Hence a new paradigm in the planning of space missions has to be considered, where the space debris issue is central, from different perspectives: theoretical, technological (hardware and software) and political. In the talk, first, a brief history of the past mitigation measures and of their main effects will be presented. Then a summary of the most recent findings and proposals in the field, mainly obtained in the framework of the H2020 ReDSHIFT project, will be presented.

Federica Spoto (IMCCE, Observatoire de Paris, France)
Gaia DR2: the data release in which I have worked and AMC has not

The second data release of Gaia includes for the first time a sample of Solar System objects. The first preliminary results have already shown the impact that the mission will have on our knowledge of the Solar System. I will review the main content of Gaia asteroid observations, from their properties to the applications. A special attention will be given to the 290 Gaia DR2 observations of the main belt asteroid 4701.

Paolo Tortora (University of Bologna, Italy)
Orbit determination and the Pioneer Anomaly: a long-range test of gravity in the Solar System

The present work consists of the investigation of the navigation anomaly of Pioneer 10 and 11 probes which became known as the "Pioneer Anomaly": the trajectories followed by the two spacecraft did not match the ones retrieved with standard navigation software. The mismatch appeared as a linear drift in the Doppler data received by the probes, which has been ascribed to an approximately constant sunward acceleration of about 8.5×10^{-10} m/s^2. The study carried out at University of Bologna yields a convincing explanation to this discrepancy. The study is based on the analysis of Doppler tracking data through the ODP (Orbit Determination Program), developed by NASA/JPL. The research was carried out through the following steps: seek for any kind of physics affecting the dynamics of the spacecraft or the propagation of radiometric data, which may have not been properly taken into account in previous analyses, and check whether or not these might rule out the anomaly. In this respect, a major effort has been put to build a thermal model of the probes in order to predict the recoil force due to anisotropic thermal radiation, since this non-gravitational force is not natively included in the ODP. Tracking data encompassing more than twenty years of Pioneer 10 interplanetary cruise, plus twelve years of Pioneer 11 were analyzed. The processing of the data was carried out in light of the results of the thermal model. Different strategies of orbit determination have been implemented, encompassing single arc, multi arc and stochastic filters, and their performance compared. Orbital solutions have been obtained without the need for any acceleration other than the thermal recoil one indicating that this is most likely the only responsible for the observed linear drift in the Doppler data. As a further support to this, we checked that inclusion of an additional constant acceleration does not improve the quality of the orbital solutions. All tests performed led to the conclusion that no anomalous acceleration was acting on Pioneers spacecraft.

Kleomenis Tsiganis (Aristotle University of Thessaloniki, Greece)
Dynamical excitation in the primordial asteroid belt

The dynamical evolution of the primordial distribution of asteroids towards its present state – in particular the inclination distribution – is still a subject open to debate. An initially 'flat' disk could not evolve into the present distribution under 4.5 Gy of planetary perturbations, if the planets did not migrate. Hence, the asteroid belt had to be both depleted in mass and dynamically excited, sometime during the early post-formation stages of solar system evolution. Of course, different migration scenarios invoke different excitation mechanisms, with different degrees of success in shaping the asteroid belt. In this talk we are going to review some of these mechanisms and present a new one, in which inclination excitation is facilitated by the sweeping of secular resonances that may appear as consequence of gravitational interactions between the giant planets and the proto-planetary disk.

Massimiliano Vasile (University of Strathclyde, UK)
Quantification of Epistemic Uncertainty in Orbital Mechanics

The talk will introduce the concept of uncertainty quantification in orbital mechanics and will discuss a well-known example. The talk will then explain what epistemic uncertainty is and how that can be propagated and quantified in orbital mechanics. A few examples and techniques will complete the talk.

David Vokrouhlický (Institute of Astronomy, Charles University, Prague)
Young asteroid families: following AMC inspiration

In this talk I will describe some of the recent work that allowed to determine ages of especially young asteroid families, clusters and pairs. The idea to meaningfully constrain the age of an asteroid family dates back to Andrea Milani's paper with Paolo Farinella on Veritas in 1994. Since then a couple different techniques were developed and applied to mainly young families, and these results helped to establish bridges to new applications in planetary science. The ambition to discover ever younger clusters has also lead to entirely new phenomena related to small asteroids.

Regular Speakers

Santiago Barbieri (Università degli studi di Padova, Italy)
Sharp Nekhoroshev estimates for the three-body problem around a periodic torus

Nekhoroshev theorem on quasi-integrable hamiltonian systems has been widely studied in the past decades and many are its applications in celestial mechanics. However, when considering Nekhoroshev stability for a perturbed hamiltonian around a periodic torus, the proofs which are available in the literature contain non-sharp estimates and constants. Moreover, a rigorous estimate on the analyticity domain for the three-body planetary hamiltonian lacks when considering concrete applications, thus leaving uncertainty when considering explicit values for the analyticity widths. In this work, we construct a sharp Nekhoroshev-like stability result for the planetary three-body problem around a periodic torus and we make use of a recent rigorous result by T. Castan on the analyticity domain of its hamiltonian. Moreover, a suitable application to the restricted three-body problem is also considered. Indeed, such systems are simple enough to allow for a disentanglement of the physical thresholds for the applicability of the theorem, so that the aim of this study is to enlight the role that complex singularities, the number of averaging steps and the use of Cauchy estimates play in ensuring Nekhoroshev stability. The difficulties in reaching physical values for the magnitude of the perturbation when applying such mathematical result to concrete astronomical examples - namely the Sun-Jupiter-Saturn system for the planetary case and the Kirkwood region for the restricted one - are pointed out as well as the possible ways to overcome them. Finally, since the proof is constructive, the techniques we introduce in order to have sharp Nekhoroshev stability around a periodic torus can be generalized to any finite-dimensional quasi-integrable system.

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Steven R. Chesley (JPL-Caltech, California, USA)
The Yarkovsky Effect Caught in the Act

The Yarkovsky effect is a slight nongravitational acceleration that is important understanding the past and futures of many asteroids. I will summarize work on detecting the Yarkovsky Effect among the near-Earth asteroids, which first started in a collaboration in Pisa in 1999, and has since led to many papers from many groups. The first detection was for 6489 Golevka in 2003, and we now have nearly 100 near-Earth asteroids for which the Yarkovsky effect is an important part of the dynamical model. Detections of the Yarkovsky effect also provide important clues to the physical and dynamical properties of asteroids, for instance their obliquity and even, in the case of 101955 Bennu, their mass.

Lorenzo Cibin (GGS SpA, Italy)
The Fly-Eye Telescope - The Innovative Optical Technology for SSA

The possibility to implement a Ground Optical Observation Network for the cataloguing and monitoring of Space Debris, previously considered as too challenging to be studied, was first demonstrated by the ESA SARA Project in 2010, based on twelve key elements that shall be satisfied by the physical apparatus needed to implement it, namely: 1) Large Field of View, 2) Quick Motion Telescope, 3) Quick Readout Camera, 4) High Resolution Camera, 5) Correlated Fill Factor, 6) Random Fill Factor, 7) Large Aperture Telescope, 8) Image Processing for Trails, 9) Accurate Astrometric Reduction, 10) Correlation and Orbit Determination, 11) Advanced Survey Scheduling, 12) Network of Optical Sensors. The Instrumental solution necessary to implement the innovative Ground Optical Observation Network is the Fly-Eye telescope we are currently developing for the European Space Agency (ESA). The Fly-Eye Telescope is an innovative Optical Instrument based on a novel technology allowing to merge some functional characteristics which are extremely difficult to implement together in a traditional Telescope, due to physical reasons. In particular, the application of the Fly-Eye technology allows to obtain a good optical resolution in a huge field of view, though using a relatively large mirror to collect light. As the name suggests, to obtain such a combination of advanced performances, the Instrument mimics the eye of the Insects. This is obtained by combining together many simple eyes to create a composed eye, as it happens for the eyes of the flies. The explanation of the key features is not only due to the presence of an instrument with Large field of View and large Aperture, but to the combination of all the elements that contribute to the result such as a Quick Motion capability, a Quick Readout and a High Resolution of the CCD camera with an optimized and random Fill Factor, in the presence of an adequate Network of Optical sensors. Advanced algorithms are used to process the trails generated by debris and capable of an accurate Astrometric reduction. Finally, advanced and highly efficient methods of orbit determination and correlation allow to support Survey and Tasking applications even combined, applicable to all orbit areas. The activity of Prof. Andrea Milani, pioneering the research for SSA Ground Optical Observation methodologies, was fundamental in the definition of the actual performances requested for its implementation. Thanks to his engagement in the SARA program, in fact, it was possible to clearly highlight all the key elements and to verify both the unprecedented effectiveness and the real feasibility of the SSA Optical Ground Network, based on such performances. His original and innovative ideas paved the way for a completely new approach in SSA field, that are now considered as reference points by the ESA and the SSA Community.

Stefano Cicaló (SpaceDyS s.r.l., Italy)
Radar-based Re-Entry Predictions with very limited tracking capabilities: the GOCE case study

The problem of the re-entry predictions of GOCE has been deeply investigated in the literature, due to the large amount of data, mainly radar and GPS, available until re-entry. The accurate GPS and attitude measurements are used to compute a precise reference orbit for the three weeks of decay, and to extrapolate the ballistic coefficient evolution of the object. In previous works, the capabilities of radar-based solutions for the re-entry predictions of GOCE and of similar objects were investigated, focusing on the german TIRA radar. In this work we have performed additional analysis, focusing on the northern european radar EISCAT-UHF, located in Tromsø, Norway. This sensor, conceived for atmospheric studies, has recently been considered for space debris applications. Due to its limited tracking capabilities, we are interested in testing its effectiveness in supporting re-entry predictions. We have simulated reliable re-entry prediction scenarios, with different availability of data from EISCAT and TIRA. The main conclusion is that, provided a suitable amount of observations, EISCAT-based re-entry predictions are comparable to TIRA-based, but also to GPS and TLE-based, corresponding ones. In general, EISCAT is not able to determine an orbit with the same accuracy of TIRA, but the results are equivalent in terms of re-entry predictions, if we consider the relevant parameters involved and their effects on the re-entry time. What is very important is the difficulty in predicting atmospheric and attitude significant variations in between the current epoch and the actual re-entry. Thus, it is not easy to keep the accuracy of predictions much lower than 10% of the residual lifetime, apart from cases with constant area to mass ratio, and low atmospheric variations with respect to the models. An experiment with real data is presented for the object 2012-006K, with consistent results.

Camilla Colombo (Politecnico di Milano, Italy)
Close encounter characterisation and deflection design for planetary protection and defence

Space benefits mankind through the services it provides to Earth. Future space activities progress thanks to space transfer and are safeguarded by space situational awareness that detects, predicts and assesses the risk due to man-made space debris and potentially hazardous asteroids and comets. When looking at the Earth's vicinity in the Solar System, Near Earth Objects, whose orbit come close to that of Earth's may pose a potential threat for our planet. At the same time, the probability that rocket upper stages or spacecraft during science missions or at the end of their life will hit the Earth or any other protected natural body needs to be assessed. NEOs and interplanetary man-made debris have in common the complex dynamics subject to the n-body problem and other forces and frequent close encounters with the Earth and other bodies, which makes the orbit prediction a challenging task. This talk with show the similarities between the two problems and the methods developed to tackle them within the COMPASS project. The ESA and NASA Planetary Protection requirement for space mission requires the spacecraft or upper stage have a less than 10-4 chance of impacting planets and moons within 100 years after launch. To verify compliance to this requirement, the distribution of launcher injection errors or the uncertainty in the end-of-life manoeuvre are mapped over 50 or 100 years at least. The line sampling method for the identification of impact will be shown together with the characterisation of close-encounters in the b-plane. On the other side, the design of deflection missions for NEOs and the planning of end-of-life manoeuvres for Libration point missions can take advantage of the b-plane representation and the keyholes characterisation to design robust deflection manoeuvres that increases the missed-distance on the b-plane, or maximise the minimum orbit intersection distance to obtain a robust disposal and reduce the risk of a future resonant returns of the NEO to the Earth.

Pierfrancesco Di Cintio (CNR-IFAC, INFN Firenze, Italy)
Noise and discreteness effects and the Radial Orbit Instability in collisionless anisotropic spherical systems

By means of N-body simulations we investigate the role of Poissonian noise and discreteness effects in the onset of the Radial Orbit Instability (ROI) in spherical collisionless self gravitating models with Osipkov-Merritt anisotropy. Moreover, we investigate the interplay of anisotropy and collective effects linked to the long-range nature of the $1/r^2$ gravitational force in ROI by building self consistent equilibrium models with more general $1/r^\alpha$ forces and studying their stability properties as function of their degree of anisotropy.

Pierluigi Di Lizia (Politecnico di Milano, Italy)
Orbit determination of resident space objects with the multibeam radar sensor BIRALES

The estimated number of resident space objects is progressively increasing over the years. These objects include fragments generated from the break-up of operational satellites or abandoned upper stages of launchers. This scenario poses serious risks for space activities. Consequently, monitoring and predicting the orbits of the resident objects is of paramount importance for the safe utilization of the space environment surrounding Earth. The aim of this talk is to describe the contribution to this task provided by the novel multibeam BIstatic RAdar for LEo Survey (BIRALES) sensor. BIRALES is part of the Italian contribution to the European Space Surveillance and Tracking Framework. It uses a radio frequency transmitter located at the Italian Joint Test Range of Salto di Quirra in Sardinia as transmitter and part of the Northern Cross (NC) radiotelescope located in Medicina (Bologna, Italy) as a multibeam receiver. The transmitter is made of an amplifier that is able to supply a maximum power of 10 kW in the bandwidth 410-415 MHz. Pertaining to the receiving segment, part of the North-South arm of the Northern Cross radiotelescope has been refurbished to mount 4 receivers per antenna. The resulting field of view (FoV) is divided in 24 independent beams. When an object transits inside the antenna FoV, the beams are illuminated by the reflected radio wave. Consequently, besides the classical range, Doppler shift and Signal-to-Noise Ratio (SNR) measurements, the beam illumination sequence can be exploited to obtain an estimate of the angular deviation profiles of the scattering object with respect to the nominal receiver pointing direction. This higher level of detail with respect to a single-beam system grants precious information for both initial and accurate orbit determination. The data received from BIRALES are provided to a tailored orbit determination (OD) algorithm. The OD process is divided in two phases. First, the angular profiles are estimated starting from the SNR data available from each beam. By identifying the different peak SNR values measured by each illuminated beam, a curve fit aimed at minimizing the angular displacement from the center of each beam peak provides a first guess for the profiles. This first guess is then refined by matching the generated SNR profiles with the measurements. This provides, for each observation instant, the estimated angular deviation profiles of the object with respect to the nominal pointing direction of the receiver. During the second phase, the object state at the first observation epoch is estimated by matching the generated orbital trajectory with the available measurements, i.e. the slant range, Doppler shift and the estimated angular deviations. Within this talk, the performance of the sensor will be illustrated. The sensor performance is assessed considering the cases of catalogued and uncatalogued objects, single and repeated passages, and different sensor configurations. For all cases, the effect of measurement noise on each available measurement is investigated.

Alessandro Fortunati (University of Bristol, UK)
Perturbation methods and Liapunov functions

The aim of the talk is to present the application of perturbative methods commonly used in Celestial Mechanics to the construction of Liapunov functions for autonomous and non-autonomous systems and the estimate of the basin of attraction of asymptotically stable equilibria. A joint work with S. Wiggins.

Catalin Gales (University of Iasi, Romania)
Dynamical effects of tesseral resonances in the LEO region

More than half of human-made objects are distributed within the Low-Earth-Orbit (LEO) region, which is ranging between 90 to 2000 km in altitude. At these altitudes, the orbital motion of a satellite is perturbed by both dissipative forces, such as the atmospheric drag, whose main effect is the decay of the semi-major axis, and conservative forces, like the oblateness of the Earth, gravitational attraction of the Sun and Moon, solar radiation pressure, which basically influence the long-term evolution of the other orbital elements.
Although the atmospheric drag provokes the orbital decay of human-made objects, there are some cases when this behavior can be balanced by some effects due to conservative forces. An example is provided by tesseral resonances which occur whenever there is a commensurability between the orbital period of the satellite and the period of Earth's rotation.
This talk aims at describing the dynamical effects of these resonances in the LEO region. By constructing a toy model, we provide a sound analytical support to the numerical investigation of the problem. We are able to discuss the interplay between the conservative and dissipative effects and to draw conclusions about

the role of the dissipation and the existence, location and stability character of the equilibria. In particular, we provide examples showing the occurrence of various dynamical phenomena such as: passage through the resonance, temporary capture into resonance, escape motions, trapped motions, shift of the equilibria during a Solar cycle, etc.

This talk refers to some works in collaboration with Alessandra Celletti.

Juan Luis Gonzalo Gomez (Politecnico di Milano, Italy)
Collision avoidance manoeuvre design and application to passive deorbiting missions

The ever-increasing proliferation of objects in Earth orbit, both operational satellites and space debris, poses a critical threat to the safe and sustainable use of space. An international effort is being undertaken to tackle this key issue, such as implementing end-of-life deorbiting manoeuvres or use of passive deorbiting devices, such as drag or solar sails. For objects already in orbit, these solutions could also be applied through so called de-orbiting kits deployed by a servicing spacecraft. On the downside, their relatively large cross-sectional area appreciably increases the risk of collision with other spacecraft or space debris. This work deals with the design of collision avoidance manoeuvres (CAMs) for end-of-life deorbiting missions using drag or solar sails, considering the possibility of manoeuvrings either the de-orbiting satellite or the incoming object. By leveraging proximal motion equations, the optimal direction of the deviating actions in the impulsive and continuous-thrust cases is obtained, either by maximising the total deviation projected on the b-plane of the nominal conjunction or by minimising the collision probability at the conjunction. A set of representative cases is proposed by using ESA's MASTER tool to estimate the relative velocity of a possible conjunction at any given point of the deorbiting trajectory. Furthermore, the evolution of the covariance matrix with the time to impact is also considered, to check whether the growth of the uncertainties introduces a practical limit to the lead-time for the CAM.

Giovanni F. Gronchi (University of Pisa, Italy)
Playing with polynomials for the computation of orbits

We review a recent method for the computation of preliminary orbit of asteroids that uses two short arcs of optical observations. The method is based on the conservation laws of the Kepler problem. Playing with these relations we show that we are led to a polynomial equation of degree 9 for the topocentric distance of the asteroid, and that this degree is optimal in a sense that will be specified.

Rüdiger Jehn (ESA/ESOC, Germany)
Andrea Milani and the Gretchen Question

Whenever a spacecraft travels to an interplanetary target radio science is one of the major science objectives. Even MORE so with BepiColombo, when in Dec 2025 the European spacecraft will reach Mercury, which due to its proximity to the Sun is the perfect location to measure relativistic effects. Shortly after the BepiColombo mission was approved, the engineers at ESA and industry came to the Mission Analysis Section at ESOC asking: "Can you check if these fantastic results that Prof Milani promises are realistic?" Having just a basic orbit determination and navigation tool INTNAV that calculates the orbit knowledge as function of Doppler and range measurements, we decided to jump into new waters and develop GRETCHEN. The Gretchen question in Goethes Faust raises fundamental doubts and thus the acronym was there, but the full title became a bit constructed: General RElativity Test based on Changes of the HErmian (=Mercury) motioN. In February 2004 Deimos was given a two-year contract to upgrade the software for the orbit, gravity field and post-Newtonian relativity parameter estimation. In the talk the outcome and Andrea's comments will be reported. Having developed such a powerful software tool also allowed to study the gravity field and orbit determination of a mission to asteroid Ryugu or 1999 JU3. On 27 June 2018 Hyabusa 2 arrived at this 900 m sized asteroid and thanks to GRETCHEN we could predict how long it takes to get a reasonable gravity field estimation. The first stunning pictures from asteroid Ryugu have just arrived. As the launch of BepiColombo finally came closer and a number of engineering problems popped up, a continuous question was, how are the radio science results affected if: 1) the reaction wheel off-loadings will be more frequent, 2) the target orbit cannot be reached due to lack of fuel and BepiColombo will be inserted into a higher orbit, 3) or if the Ka-band transponder has to be switched off at perihelion due to power limitations. A few software updates were made in the course of a Master thesis in 2015/16 and all questions could be answered with satisfaction. But it was not only BepiColombo were we had the privilege to work with Andrea Milani. In the early days of space debris research our paths crossed and later on as the outcome of a study with the University of Pisa we published a nice paper together in the Monthly Notices of the Royal Astronomical Society ("Correlation and Orbit Determination of Space Objects Based on Sparse Optical Data"). It was always demanding but at the same time a pleasure to work with Andrea and we hope we can keep collaborating for many years to come as we do now in field of Near-Earth Objects.

Zoran Knežević (Serbian Academy of Sciences and Arts, Serbia)
50 papers with Andrea Milani (and counting)

From 1987, when Andrea Milani and myself commenced our long and fruitful scientific collaboration, we published nearly 50 papers in common. In this talk I shall briefly overview the past 30+ years of working together, the main topics of our research and some of the most important results we have achieved. Jointly we developed the analytical and synthetic theories of asteroid motion and methods to compute asteroid mean and proper elements, continuing up to now to provide these useful dynamical parameters to the community. For the European Space Agency we devised an analytical perturbation theory, based on selenocentric proper elements, and implemented it for mission analysis of

a low lunar polar orbiter. Next, we worked together on the problems of orbit determination for the next generation observational surveys, in particular on the orbit determination with very short arcs. Finally, in the last decade, our common research was focused on the identification, dynamical modeling an collisional interpretation of asteroid families, with a special emphasize on the determination of their ages.
As an illustration of our continuing reasearch, I will show some examples of our most recent results on cratering families, and on the novel computation of asteroid synthetic frequencies to reliably assess the locations of secular resonances.

Giacomo Lari (University of Pisa, Italy)
Modeling the long-term dynamics of the Galilean satellites

The Galilean satellites are the four biggest satellites of Jupiter; they are the scene of spectacular phenomena, such as volcanoes and concealed oceans, and the motion of the three inner moons is characterized by a three-body mean motion resonance, called Laplace resonance. Due to the presence of resonant angles, the construction of a long-term dynamical model can become cumbersome, because of the number of terms that must be considered in the expansion of the perturbations. In this talk, we present a new semi-analytical model that describes the resonant and secular motion of the Galilean satellites, defined by a Hamiltonian depending on slow angles only, which was obtained with an analytical expansion of the perturbing functions and an averaging operation. We consider aspects and forces that in similar studies of the past were neglected, such as the satellites' inclinations and the Sun's perturbation. Through a comparison with the ephemerides of the moons, we will show that our model captures well all the features of the actual motion of the Galilean satellites. Moreover, we investigate the evolution of the system due to the tidal dissipation.

David M. Lucchesi (IAPS/INAF, Italy)
A measurement of the Earth's Gravitomagnetic field in the centennial of the Lense-Thirring effect

One hundred years ago, a collaboration between the astronomer and mathematician Josef Lense, and the physicist Hans Thirring, opened the experimental verification to one new, and very important, prediction of Einstein's newly born theory of General Relativity (GR): Gravitomagnetism. The problem was posed by Thirring in terms of the integration of the equations of motion of a test body into the field of a rotating mass by means of the perturbation theory, a branch of mathematical physics in which Lense was very expert. Their results were applied to the orbit of the planets and of their moons. Lense and Thirring pointed out that, in 1918, the new effects of Einstein's theory were too small to be measured accurately within the solar system. Today, 100-yr after the original papers of Lense and Thirring, despite the improvements in the knowledge of the ephemerides of the orbits of planets and moons, the measurement of the Lense-Thirring effect, albeit noticeably improved, has not yet been achieved with sufficient precision and accuracy in the solar system. Vice versa, more precise results have been obtained in the field of the Earth by measuring the distance of passive geodetic satellites from a network of ground stations using the Satellite Laser Ranging technique. However, a refined and robust error budget, based on a reliable assessment of the main systematic sources of perturbation due to both conservative and non-conservative forces, has not yet been fully achieved. In this talk, the current results obtained by the LAser RAnged Satellites Experiment (LARASE) for the measurement of the Lense-Thirring precession on the combined orbits of the LAGEOS, LAGEOS II and LARES satellites will be presented. In GR, Gravitomagnetism is tightly related with the concept of inertia and of the origin of the local inertial (apparent) forces. Ultimately, Gravitomagnetism is connected to Mach's Principle and the way of including it in Einstein's theory of gravitation. Gravitomagnetism plays a fundamental key role in the strong field regime in the astrophysics of compact objects, as well as in cosmology. Definitely, the accurate measurement of the Lense-Thirring effect in the weak-field and slow-motion limit of GR represents a weak general relativistic interpretation of Einstein's ideas about Mach's principle.

Stefano Marò (Instituto de Ciencias Matematicas, Spain)
Long term dynamics in mean motion resonances with crossing singularities

We consider the long term dynamics of the restricted N body problem, modeling the dynamics of an asteroid moving in the gravitational field of the Solar system. In particular we deal with the case of a mean motion resonance with a planet. The asteroid evolution is computed by averaging over the fast angles using standard techniques of perturbation theory. We focus on the critical case where the trajectory of the asteroid crosses the one of a planet during the evolution. This produces a singularity in the averaged equations of motion. We prove that the averaged vector field can be extended to two Lipschitz continuous vector fields in a neighborhood of crossing configurations and we define generalized solutions, going beyond this singularity. Moreover, we prove that the orbit distance between the asteroid and a planet is differentiable also in case of crossings. Some applications will be given considering mean motion resonances with Jupiter and crossing with the Earth. This work extends the results in (Gronchi-Tardioli 2013) to the resonant case. We conclude with a comparison with other models present in literature.

Paolo Paolicchi (University of Pisa, Italy)
Age of asteroid families with the YORP-eye method

The YORP-eye theory has been defined in a recent paper (Paolicchi and Knežević, Icarus 274,314 (2016)): the YORP efect affects the orientation of the spin axis, thus accelerating the Yarkovsky migration along the semimajor axis of the members of an asteroid dynamical family. In the "V-plot" semimajor axis a vs magnitude H (or $1/D$) an empty region appears in the central part (the "eye"). Our method allows to identify footprints of the effect even in the cases for which the "eye" is not immediately detected, and to estimate the age of the family. In this paper we perform a direct comparison with the ages estimated according to the

Yarkovsky effect, finding, in most cases, a rather good agreement. In principle our method can allow a preliminary estimate of the age also for those families for which no previous estimate is present.

Jérôme Perez (ENSTA ParisTech, Italy)
Isochrony in 3D radial potentials. From Michel Hénon ideas to isochrone relativity: classification, interpretation and applications.

Revisiting and extending an old idea of Michel Hénon, we geometrically and algebraically characterize the whole set of isochrone potentials. Such potentials are fundamental in the potential theory. They appear in spherically symmetrical systems formed by a large amount of charges (electric or gravitational) of the same type considered in a mean-field theory. Such potentials are defined by the fact that the radial period of any test charges in such potentials, provided that it exists, depends only on its energy and not on its angular momentum. Our characterization of the isochrone set is based on the action of a real afine subgroup on isochrone potentials related to parabolas in the $\mathbb{R}^2$ plane. Furthermore, any isochrone orbits are mapped to associated keplerian elliptic ones by a generalization of the Bohlin transformation. This mapping allows us to understand the isochrony property of a given potential as relative to the reference frame in which its parabola is represented. We detail this isochrone relativity in the special relativity formalism. We eventually exploit the completeness of our characterization and the relativity of isochrony to propose a deeper understanding of general symmetries such as Kepler's Third Law and Bertrand's theorem.

Giovanni B. Valsecchi (IAPS-INAF, IFAC-CNR, Italy)
Comet encounters with the planets: an analytical approach

Close encounters with the giant planets play an important role in the evolution of the orbits of comets, and many numerical treatments of the issue can be found in the literature. We revisit the subject in the framework of the analytic theory of close encounters, that provides useful insight on the properties of the strong perturbations caused by planetary encounters.