Author: R. AGGARWAL
University of Delhi (India)
Title: *"Stability Of Equilibrium Points In The Photogravitational Restricted Four-Body Problem With Variable Mass"*
E-POSTER

Abstract. This paper is the generalization of our earlier work Mittal et al. (2016), where we have studied the restricted four-body problem (R4BP) with variable mass. In this problem, we have taken all the primaries as radiated. Following the procedure given by Gascheau (1843) and Routh (1875), the conditions of linear stability of Lagrange triangle solution in the photogravitational restricted four-body problem (R4BP) are determined. The three radiated primaries having masses m_1, m_2 and m_3 form an equilateral triangle with $m_2 = m_3$ as long as they satisfy above conditions. The equations of motion of the current problem differ from the equations of motion of Mittal et al. (2016) and Papadouris and Papadakis (2013). There exist eight equilibrium points for a fixed value of parameters γ, α (the proportionality constant in Jeans' law (1928)), $\mu = 0 : 005$ (the mass parameter) and radiation parameter q_i ; (q_i ; $i = 1; 2; 3$). All the equilibrium points are non collinear if q_2 , q_3. It has been observed that the collinear and out-of-plane equilibrium points also exist for $q_2 = q_3$. All the libration points are found to be unstable. Zero velocity curves (ZVC) are also drawn and regions of motion are discussed.

Joint work with Amit Mittal, Md. Sanam Suraj.

Author: R. ALVES SILVA
University of São Paulo (Brazil)
Title: *"An analysis of the 3/1 mean-motion resonance in the Pluto-Charon-Styx system"*
TRADITIONAL POSTER

Abstract. The discovery of four small satellites, namely Styx, Nyx, Kerberos and Hydra, reveals the complexity of the Pluto-Charon dynamical architecture. Brozovic et al. (2014) presents the orbital elements and mass estimates for Charon and the other four satellites, which allow the study of the dynamics of this system. In this presentation, we analyze the dynamical configuration of the Pluto-Charon-Styx sub-system focusing on a qualitative description of the 3:1 near-resonance regime of motion. The approach used is an analytical development of the general three-body problem, in the particular case of co-planar motions. The analysis of the topology of the phase space of the system allows us to detect the regions of stable and chaotic motion. The results of the analytical study are compared to those obtained through numerical integrations of the exact motion equations of the system.

Author: M. ARORA
University of Delhi (India)
Title: *"Effect of Oblateness on the existence and location of Libration Points in R4BP"*
TRADITIONAL POSTER

Abstract. We have studied the restricted four-body problem (R4BP) when all the three primaries are oblate spheroids. The three primaries with equal masses are located at the vertices of an equilateral triangle and they move in the same plane around their centre of mass. The fourth body which is an infinitesimal mass, moves in the same plane and is acted by the attraction of the three primaries. It is assumed that the mass of the fourth body is so small that its influence on the motion of the primaries is negligible. We have assumed the condition under which the equilateral triangle configuration remains same throughout the motion from the literature of restricted three body problem. The equations of motion have also been derived and we have determined the libration points numerically for different values of the oblateness parameter. Zero velocity curves have also been drawn.

Joint work with Dr. Rajiv Aggarwal and Dr. Md. Sanam Suraj.

Author: L. BERNUS
IMCCE, Observatoire de Paris (France)
Title: *"On the equations of motion, first integrals, and numerical integration of the post-Newtonian N mass monopoles system"*
E-POSTER

Abstract. We describe how to set the relativistic barycenter of the N -body system to the origin of the coordinates using an appropriate Poincaré transformation, which lets the equations of motion invariant. Then we formulate the equations such that seven first integrals become exact.

Author: D.D. CARPINTERO
Fac. Cs. Ast. y Geof. (UNLP) - IALP (Conicet-UNLP) (Argentina)
Title: *"Solving the inverse problem of the exoplanetary transit series: the case of the Kepler 419 system"*
TRADITIONAL POSTER

Abstract. Usually, the mid-transit times of an exoplanet are non-Keplerian, that is, they are not periodic. These variations in the timing of transits can sometimes be attributed to perturbations by other exoplanets present in the system, which may or may not transit the star. We have developed an algorithm that allows to compute the mass and the six orbital elements of an invisible (non-transiting) exoplanet, given only the central times of transit of a transiting body. Also, the mass of the star and the mass and orbital elements of the transiting exoplanet are recovered. We have applied our method, based on a genetic algorithm, to the Kepler 419 system. We were able to compute all fourteen free parameters of the system, which, when integrated in time, give the observed transits with a precision comparable with the observational errors. We also studied the dynamics and the long term orbital evolution of the Kepler 419 planetary system as defined by the orbital elements computed by us, in order to determine its stability.

Joint work with M. D. Melita.

Author: V. CHOPOVDA
Massey University (New Zealand)
Title: *"A family of periodic orbits in the Caledonian symmetric four-body problem"*
TRADITIONAL POSTER
Abstract. The general planar four-body problem can be simplified by considering the special case of symmetric motion with collinear initial positions and transverse initial velocities. The simple models that occur may aid our understanding of the general problem. In this study, we find a family of periodic orbits, and subsequently analyse their stability. The progenitor of the family of orbits performs an interplay motion similar to that of the periodic three-body orbit discovered by Schubart in 1956.

Joint work with Winston Sweatman and Robert McKibbin, Institute of Natural and Mathematical Sciences, Massey University, Auckland

Author: C. COLOMBO
Politecnico di Milano (Italy)
Title: *"Design of disposal of spacecraft at libration point orbits in the elliptical restricted three body problem"*
TRADITIONAL POSTER

Abstract. Libration Point Orbits (LPOs) in the Sun-Earth system have been selected for astronomy missions, such as Herschel, Planck, SOHO, and Gaia and future missions, such as Euclid, ATHENA, PLATO will also use this type of orbits. Indeed, LPOs have a stable geometry with respect to the Sun and the Earth, thus offering a vantage point for the observation of the Sun and the Universe together with optimal operating condition in terms of radiation environment, telecoms and thermal design. In addition, the amount of propellant to target orbits around L1 and L2 is low compared with alternative orbits. Recent ESA studies have highlighted the importance of considering the end-of-life disposal for LPOs since the early stages of the mission design to define a sustainable strategy for the disposal with the objective to avoid interference with the protected regions and to minimise the possibility of an uncontrolled re-entry at the Earth many years after the mission end. A transfer into a graveyard heliocentric orbit was implemented for ISEE-3/ICE, Planck and Herschel. In this work the Elliptical Restricted Three Body Problem (ER3BP) is used as a simplified, yet powerful tool to design the end-of-life of LPO spacecraft through an energetic approach. A first manoeuvre is given to leave the LPO, while a second manoeuvre is used to decrease the three-body problem energy of the spacecraft. The disposal design is optimised to maximise the distance from Earth for a period of 100 years and to minimise the possibility of gravitational interaction with the Earth, due to perturbation induced by other planets. Moreover, the ER3BP formulation is used to show the dependence on the sensitivity of the manoeuvre to the angular position of the Earth-Moon barycentre with respect to the Sun. A high-fidelity dynamical simulation is applied to validate the results. As a mission application the end-of-life of Gaia and Lisa pathfinder mission are presented.

Joint work with Davide Menzio, Greta De Marco, Stefania Soldini.

Author: P. DI CINTIO
CNR-IFAC & INFN, Firenze (Italy)
Title: *"Numerical simulations of space debris with symplectic integrators"*
TRADITIONAL POSTER

Abstract. In the study of the long term evolution of the space debris population it is of paramount importance to be able to achieve a good compromise between accuracy and computational speed, in order to propagate the orbit of thousands of objects at the same time.

We present a novel implementation of a symplectic orbital propagator in Cartesian coordinates for the dynamics of space debris from the Low to High Earth orbit range.

Test simulations including luni-solar perturbations suggest that our optimized numerical code can attain a significant reduction in computational times with respect to previous orbit propagators based on averaged dynamics in orbital elements.

Moreover, we show preliminarily results for case studies where the effects of non-gravitational perturbations such as solar radiation pressure and atmospheric drag, often not accounted for in symplectic integrators, are considered.

Finally, the comparison between the results of a few long term evolution scenarios simulated with the SDM model, using both the original propagator (working on averaged dynamics in orbital elements) and the new one, are presented.

Author: P. DI CINTIO
CNR-IFAC & INFN, Firenze (Italy)
Title: *"Noise, discreteness effects and the continuum limit in N-body systems, revisited"*
TRADITIONAL POSTER

Abstract. By means of active and frozen N-Body simulations we revise the role of discreteness effects and external noise in the dynamics of self gravitating systems and non-neutral plasmas. In particular, we show that the use of frozen N-body realization may, in certain cases, lead to misleading conclusions.

Author: V. DI PIERRI
Università di Pisa (Italy)
Title: *"Testing alternative theories of gravity with the BepiColombo Radio Science Experiment"*
TRADITIONAL POSTER

Abstract. The parameterized post-Newtonian (PPN) formalism is a general metric theory of gravity depending upon 10 parameters, called post-Newtonian parameters. In particular, experiments within the Solar System can be described within the PN limit, that is the regime of slow motion and weak fields, which includes all the symmetrical metric theories of gravity, with vanishing torsion tensor. In this work we introduce torsion theories, which are metric theories with non-vanishing torsion tensor, in their PPN expansion. In this framework we derived the equations of motion of a massive body including torsion, describing the dynamical effects by means of some torsion parameters to be added to the standard PN parameters. We will show how the torsion parameters can be estimated with the ESA-BepiColombo radio science experiment (RSE). To this aim, we implemented the torsion contribution to the dynamics in the ORBIT14 software, which is an orbit determination code specifically developed for the Bepi Colombo RSE by the Celestial Mechanics Group of the University of Pisa. The software allows for both the simulation of radio observables (range and range-rate) and for the determination of a large number of parameters concerning fundamental physics and Mercury geophysics by means of a global least squares fit. We will show the results of a set of simulations performed with ORBIT14, with the aim of estimating the torsion parameters. We will also discuss the issues arising from correlations between PN and torsion parameters.

Joint work with Giacomo Tommei, Giulia Schettino, Andrea Milani.

Author: G. DUARTE FERREIRA
University of Barcelona (Spain)
Title: *"Invariant Manifolds near L1 and L2 in the Planar Restricted Three-Body Problem"*
TRADITIONAL POSTER

Abstract. In this work we investigate the connections between the stable and unstable manifolds of periodic orbits around the points L1 and L2 of the Restricted Three-Body Problem (RTBP). In the Planar Circular RTBP (PCRTBP) we can see, based on [1] (and its references), that there is a connection of these manifolds on the phase space of this problem, which means that there is a mecanism of an orbit that goes outside the orbit of one primary (for instance, that describes an orbit close to an ellipse with greater semi-major axis), to go through it and start to describe an orbit inside of it (close to an ellipse with smaller semi-major axis) and/or vice-versa; in other words, this mecanism is a bridge connecting orbits outside with orbits inside. Using the tools in [2] we start by computing the normal forms around L1 and L2 to describe periodic orbits around each of these points. After that, with the same tools, we focus on the computation of the stable and unstable manifolds of these orbits and on the searching for connections between them.

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Author: F.L. DUBEIBE
Universidad de los Llanos / Universidad
Industrial de Santander (Colombia)
Title: *"Effect of the event horizon in the planar circular restricted three-body problem"*
E-POSTER

Abstract. We study the effects on the dynamics of the planar circular restricted three body problem introduced by the existence of event horizons in the primaries. We used Paczyński-Wiita potentials to model the gravitational potentials generated by each primary, and the dynamics is studied by means of the Poincaré surface of section and the Largest Lyapunov exponents.

Author: M. GALIAZZO
University of Vienna (Austria)
Title: *"Dynamical disruption of binary TNOs: Typhon-Echidna"*
TRADITIONAL POSTER

Abstract. Currently we know more than 70 binary/multiple systems which are roughly 5 % of the total population of TNOs. Many of these objected are scattered inward in the Centaurs zone and even further, after close encounters with Neptune. In this paper we analyse the orbital evolution of the binary system Typhon and Echidna, asteroids which contain crystalline water ice. This binary has an orbit which can cross Uranus and Neptune together and therefore via close encounters its orbit can be perturbed in a way it can reach the inner solar system; therefore there is the possibility that its binarity feature can be destroyed (the couple can be disassociated), having more close encounters with all giant planets, but we can not esclude also with terrestrial planets. We performed numerical forward and backward integrations of clones of Typhon-Echidna (as ones single orbit) orbit for 200 Myrs. The results confirms that in case of close encounters the binary can be heavily affected, especially after deep close encounters with Jupiter and Neptune, however the chances to have such close encounters is low, only 1.8 % of the clones have the binary separated. In addition we find that Thyphon-Echidna (formerly 2002 CR46) orbit evolution has a low probability to cross terrestrial planets orbit, but close encounters with the Earth and Venus show that binaries can get disassociated. Because it is not unusual that large (more than 100 km in diameter) Trans-Neptunian Objects evolve in to the inner solar system, they can have binaries dynamically disassociated during their orbital evolution both by giant planets and terrestrial planets. Performing backward integration, e find that Typhon-Echidna was very likely a former Kuiper Belt Objects (KBOs), because its orbital behaviour is similar to other KBOs orbital evolutions, found in past works, we can preliminary generalize its orbit to the ones of all the KBOs. This is a first study on these kind of objects, in the future we will study more cases.

Author: A. GIERZKIEWICZ
Agriculture University in Krakow (Poland)
Title: *"Chaotic rotation of Hyperion - computer-assisted proofs"*
E-POSTER

Abstract. The rotation of Hyperion is often modelled by equations of motion of an ellipsoidal satellite. The model is expected to be chaotic for large range of parameters. I will present some computer-assisted proofs in its dynamics using CAPD C++ library.

Author: C. KAUR
S.G.T.B. Khalsa College - University of Delhi (India)
Title: *"Effects of Poynting-Robertson Drag on Resonance in a Geo-Centric Satellite due to Earth's Equatorial Ellipticity"*
TRADITIONAL POSTER

Abstract. This paper focuses on resonance of the motion of a geocentric synchronous satellite under the effect of gravitational forces of the Sun and the Earth including its equatorial ellipticity. With the assumption that the two bodies (the Earth and the Sun) lie in an ecliptic plane and the third body (Satellite) lie in the orbital plane, four resonance points results from commensurability between the mean motion of the Satellite and Γ (angle measured from the minor axis of the earth's equatorial ellipse to the projection of the satellite on the plane of the equator). Following the procedure of Brown and Shook the amplitude and the time period have been evaluated. The effects of Γ on amplitude and time period of the resonance oscillation in different cases at the resonance points have been shown.

Joint work with Bijay Kumar Sharma and Sushil Yadav.

Keywords: Three-body problem, Ecliptic Plane, Resonances, Poynting-Robertson Drag.

Author: Z. MAKO
Sapientia University (Romania)
Title: *"The distribution of velocities on weak stability boundary"*
TRADITIONAL POSTER

Abstract. This presentation provides a study of the stable and unstable regions around the smaller primary in the framework of the spatial elliptic restricted three-body problem. First, we investigate the rate of symmetry of the stable region and we determine the correlation between the initial inclination and the measures of stable and unstable regions for different systems. After that, we study the distribution of velocities on the boundary of stable region.

Joint work with Ferenc Szenkovits (Babas-Bolyai University, Cluj-Napoca, Romania).

Author: M. MINGLIBAYEV
al-Farabi Kazakh National University (Kazakhstan)
Title: *"New solutions of restricted three-body problem in the form of isosceles triangle"*
TRADITIONAL POSTER

Abstract. We investigate the spatial restricted three-body problem in the barycentric reference frame, when three body form a triangle during motion. The bodies are treated as material points. The special case of the problem has been revealed in barycentric coordinate system, when the total Newtonian gravity force of two primary bodies act on a massless particle as a central force. The relative orbit of two primary bodies can be any: circular, elliptic, parabolic or hyperbolic. We have proved that the total Newtonian gravity force of two primary bodies to be a central force relative third, massless body, it is necessary and sufficient them to move on an isosceles triangle on vertex of which is a massless body. The considered isosceles restricted three-body problem was been highlighted and formulated. The masses of the primary two bodies are arbitrary. Based on the general differential equations of restricted three-body problem in barycentric reference frame, obtained simplified differential equations of the isosceles restricted three-body problem. It was been proved that the orbit of an isosceles restricted three-body problem is planar. We have analyzed the possible motions of bodies and discussed the analytical solutions of this problem. In the barycentric reference system for the planar restricted three-body problem we found, analytically, the initial values of the positions and velocities, in which three bodies, during their motion form an isosceles triangle on vertex of which will be a massless body. We have solved numerically the not simplified general differential equations of the planar circular restricted three-body problem (Earth + Moon + massless body) with the initial conditions found analytically. The results of numerical solutions show that our initial conditions lead three bodies to form an isosceles triangle on the vertex of which is massless body.

Joint work with Zhumabek T.M.

Author: M. MURAWIECKA
`University of Namur (Belgium)`
Title: *"Simulation issues in chaotic medium Earth orbits"*
`TRADITIONAL POSTER`

Abstract. In many fields of science, the primary objective of macroscopic simulations is the extraction of physical and dynamical information on a coarse level, a task generally achieved using numerical solvers. In the form we consider here, these solvers are ordinary differential equation solvers called to solve Newton's equation for the motion of a point mass particle in the Earth gravity field perturbed by a certain potential. Within this perturbing potential, the presence of the tesseral 2 : 1 resonance as well as the net of lunisolar resonances produce a rather chaotic environment [Celletti and Gales, 2014, Daquin et al., 2015, Daquin et al., 2016, Gkolias et al., 2016].

Regarding the timescales of some simulations (about 105 particles' revolutions) and the noted sensitivity to initial conditions, the use of numerical solvers and the reliability of the computed quantities need to be supported: the exponential growth of error with time may indeed seriously denature the numerical trajectories and corrupt their post treatments. In the medium Earth orbits context, our experiments performed with the NIMASTEP software [Delsate and Compère, 2012] show that for individual trajectories originating close to the 2 : 1 tesseral chaotic region, the semi-major axis, the physical observable of interest, is not accurately computed depending on the time step. The orbits close to the lunisolar resonances are also subjected to this problem (in terms of their eccentricity and inclination), leading in some cases to inaccuracies regarding the time of re-entry. We show that these problems may disappear if the convergence is thought in a weak sense. For the sake of reproducibility, we provide a reservoir of numerical simulations that may serve as a starting point for coworkers. It has to be noted that some of our simulations performed with the Megno chaos indicator [Cincotta et al., 2003] confirm previous results obtained through the numerical treatment of the simplified averaged system. This work should provide a path forward to the relevant dynamical conclusions that we can draw even if single trajectories differ.

Joint work with J. Daquin and A. Lemaître.

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Author: J.C. MUZZIO
IALP (CONICET - UNLP) and Fac. Ccs. Astron.
Geof. (UNLP), La Plata (Argentina)
Title: *"Partially Chaotic Orbits Are Alive and Well"*
TRADITIONAL POSTER

Abstract. In principle, autonomous Hamiltonian systems with three degrees of freedom support three types of orbits: regular (they obey two additional integrals besides energy), partially chaotic (they obey one additional integral only) and fully chaotic (they just obey the energy integral). Nevertheless, the existence of partially chaotic orbits, although theoretically possible, has been denied by different authors. The main arguments are that there is a sudden transition from the regular to the fully chaotic regime and that, if the integration of partially chaotic orbits were pursued for a long enough time, their fully chaotic nature would be revealed. Here we show that partially chaotic orbits, although rare, actually exist, we provide examples of such orbits and we describe their nature.

Author: D. PAVLOV
Institute of Applied Astronomy RAS (Russia)
Title: *"Are numerical theories irreplaceable? A computational complexity analysis"*
TRADITIONAL POSTER

Abstract. It is widely known that numerically integrated orbits are more precise than analytical theories for celestial bodies. However, the time complexity of the integration is proportional to the required point in time, while analytical series usually require asymptotically less time.

The following question arises: can the precision of numerical theories be combined with the computational speed of analytical ones? This work gives a negative answer to that question for three-body problem, considering the Sitnikov problem as the case for the proof.

A formal statement is given for the initial value problem (IVP) for a system of ordinary dynamical equations in the context of computational complexity theory. The computational complexity of this problem is analyzed. The proof for the lower complexity bound is based on the complex behavior of trajectories. In the case of the three-body problem, the existence of oscillatory solutions in the Sitnikov problem, found by Alexeyev (1981), is sufficient to show that there can not exist an algorithm that solves the IVP in polynomial time of the length of the input. That proves the lower time complexity bound for an algorithm to compute the system state in the Sitnikov problem: the bound is linear with respect to the required point in time.

Joint work with Nikolay Vasiliev (PDMI RAS, LETI).

Author: W. POLYCARPE
IMCCE, Observatoire de Paris (France)
Title: *"Is Titan responsible for Iapetus' orbit?"*
TRADITIONAL POSTER
Abstract. Latest astrometrical results on Saturnian moons show evidence for high migration rates (Lainey et al. 2012; 2016). The tidal dissipation in the planet is responsible for those large orbital expansions and should therefore be much stronger than usually expected. The idea of significant tidal dissipation in the massive core (Remus et al. 2012) and the convective envelope (Le Guenel et al. 2015; Fuller et al. 2016) of the planet has been brought forward as a likely explanation. Nevertheless, high changes in semi-major axes of the moons shuffle up the ideas we had on the past evolution of the system of Saturn. In particular, depending on tidal mechanisms at play, several Mean Motion Resonances could have been crossed just a few million years ago.

In the frame of high tidal migration, we investigate the consequences of the past 5 : 1 mean motion resonance between Titan and Iapetus, which could have happened between 5 and 500 million years ago. Numerical simulations show that the most common outcome for Iapetus is to be ejected, as Titan migrates through the resonance. However, if Titan has a very high recession ($Q < 2000$), Iapetus may survive the resonance and come out of it with an eccentricity consistent with today's value. The effect on Iapetus' inclination is still under investigation.

Author: Ò. RODRÍGUEZ DEL RÍO
Universitat Politécnica de Catalunya (Spain)
Title: *"Ejection-collision orbits in the RTBP"*
TRADITIONAL POSTER

Abstract. As it is well known, for any value of the mass parameter $\mu \in (0\,,\,1/2]$ and sufficiently restricted Hill regions, there are exactly four ejection–collison orbits. We check their existence and extend numerically these four orbits for $\mu \in (0\,,\,1/2]$ and for less restrictive values of the Jacobi constant.

We consider n ejection-collision orbits, we explore them numerically for $\mu \in (0\,,\,1/2]$ and suitable values of the Jacobi constant and we discuss the appearing bifurcations.

Joint work with Mercè Ollé and Jaume Soler.

Author: L. RUIZ DOS SANTOS
Universidade Federal de Itajubá (Brazil)
Title: *"Modelling of viscoelastic tides with the pseudo-rigid body theory"*
TRADITIONAL POSTER

Abstract. In this work, we present ordinary differential equations for the motion of linear viscoelastic bodies interacting under gravity. The equations are fully three dimensional and allow for the integration of the spin, the orbit, and the deformation of each body. Using such a formulation, we can present good models for the tidal forces that take into account the possibly different rheology of each body. The equations are obtained within a finite dimension Lagrangian framework with dissipation function. The main contribution is a procedure to associate to each spring-dashpot model, which defines the rheology of a body, a potential and a dissipation function for the body deformation variables. The theory is applied to the Earth (solid part plus oceans) and a comparison between model and observation of the following quantities is made: norm of the Love numbers, rate of tidal energy dissipation, Chandler period, and Earth-Moon distance increase.

Joint work with Ragazzo, C.

Author: G. SCHETTINO
IFAC-CNR (Italy)
Title: *"Frequency Analysis of space debris orbits in the LEO region"*
TRADITIONAL POSTER

Abstract. As part of the deep dynamical analysis of debris orbits carried out within the H2020 ReDSHIFT project, we present a characterization of the orbital elements of low-altitude objects in terms of their periodic components. Considering a representative sample of possible initial orbital conditions in the Low Earth Orbit (LEO) region, we propagated the dynamics of the objects over a suitable time span. The dynamical model includes the effects of geopotential up to degree and order 5, lunisolar perturbations, solar radiation pressure and atmospheric drag. We further accounted for different values of the area-to-mass ratio of the debris. Then, we decomposed the resulting quasi-periodic series in their spectral components by a numerical computation of Fourier transforms, accounting for the finite duration of the signals. The aim of this spectral analysis is to clearly link each frequency signature to the dynamical effect which originates it in order to build a frequency chart of the LEO region.

Indeed, the detailed analysis of the principal spectral components turns out to be a powerful tool to enable a better understanding of the relative importance of each specific gravititational and non-gravitational perturbation in the LEO region as a function of the initial semi-major axis, eccentricity and inclination of the debris.

Ultimately, the analysis will be used, in conjunction with the cartography of the LEO phase space, to identify the most suitable perturbations to be exploited to facilitate the passive "dynamical de-orbiting" of a spacecraft at the end of life.

This work is funded through the European Commission Horizon 2020, Framework Programme for Research and Innovation (2014-2020), under the ReDSHIFT project (grant agreement n. 687500).

Joint work with Elisa Maria Alessi, Alessandro Rossi, Giacomo Tommei and Giovanni Valsecchi.

Author: B.K. SHARMA
National Institute of Technology - Patna (India)
Title: *"Kinematic Model of Earth-Moon System validates the observed system parameter but constrains Moon to be 3Gy age and suggests possible precursors for earthquake/sudden volcanic eruptions."*
E-POSTER

Abstract. NASAs press release of "Moon having receded by 1 m from Earth in a quarter of century" on the Silver Jubilee Anniversary (20 July 1994) of Man's landing on Moon led to the development of the Kinematic Model(KM) of evolving Earth-Moon System. Best fit KM parameters is adopted for the analysis of evolving E-M system assuming the present length of day of Earth to be 24 hours, orbital period of Moon divided by spin period of Earth = LOM(length of month)/LOD(length of day)=27.322 and the age of Moon=4.46Gy. Using the best fit model parameters the velocity of recession of Moon is derived as 2.3cm/y as compared to 3.7cm/y by Lunar Laser Ranging experiment. The tension between the predicted and the observed velocity of recession could not be removed even after taking evolving moment of inertia of Earth as ascertained from the curve fitting of observed length of day curve over the eons. This tension constrains Moon to be younger at 3Gy age. This is technically feasible with multiple-impact scenario of Moon formation. Rufu et al.(2017) conclude that, complete formation of Moon from efficient merger of moon-lets, can account for the formation of E-M system with its present properties. KM is used to plot lengthening of day curve a time span of 900My because reliable LODs are available for this period only. This theoretical plot is superimposed on the observed lengthening of day curve. The observed plot is generated from the observed length of day(LOD) in different geologic epochs by John West Wells and Charles P.Sonnet by the study of coral fossils and ancient tidalites. Kaula-Harris deduced synodic months from marine creatures. Walker & Zahnla and Williams studied the Australian banded Formation and from the estimate of saros cycle arrived at LOD. Kaula-Harris and Walker-Zahnle-Williams LODs are discarded for their inconsistency. The observed lengthening of day curve and theoretical lengthening of day curve have close fit for 900My span. Stratification of Early Earth into onion like-internal stucture implies evolving rotational mass of inertia C. The effect of evolving C is included in lengthening of day curve. This has imperceptible improvement in the match of the two curves. Long term studies of world-wide seismic activities and variation of Earths spin rate clearly establishes that in winter the spin rate comes down leading to a stressed Earth and increased seismic activity in December-January. This suggests that that measurement of Length of Day (LOD) could be a reliable indicator of the changes of moment of inertia of Earth taking place and hence it could contain Earthquake and impending sudden volcanic eruptions precursors. LOD fluctuations may become an effective Early Warning System for natural disasters of this category.

Key words: geo-synchronous orbits, tidal drag, evolving earth-moon system, recession velocity, gravitational potential well, energy maxima, stability, seismic activity.

Author: B.K. SHARMA
National Institute of Technology - Patna (India)
Title: *"Kinematic Model of Earth-Moon System validates the observed system parameter but constrains Moon to be 3Gy age and suggests possible precursors for earthquake/sudden volcanic eruptions"*
E-POSTER

Abstract. NASAs press release of Moon having receded by 1 m from Earth in a quarter of century on the Silver Jubilee Anniversary (20 July 1994) of Mans landing on Moon led to the development of the Kinematic Model (KM) of evolving Earth-Moon System. Best fit KM parameters is adopted for the analysis of evolving E-M system assuming the present length of day of Earth to be 24 hours, orbital period of Moon divided by spin period of Earth = LOM(length of month)/LOD(length of day) = 27.322 and the age of Moon = 4.46 Gy. Using the best fit model parameters the velocity of recession of Moon is derived as 2.3 cm/y as compared to 3.7 cm/y by Lunar Laser Ranging experiment. The tension between the predicted and the observed velocity of recession could not be removed even after taking evolving moment of inertia of Earth as ascertained from the curve fitting of observed length of day curve over the eons. This tension constrains Moon to be younger at 3 Gy age. This is technically feasible with multiple-impact scenario of Moon formation. Rufu et al. (2017) conclude that, complete formation of Moon from efficient merger of moon-lets, can account for the formation of E-M system with its present properties. KM is used to plot lengthening of day curve from the birth of Moon to the present time. This theoretical plot is superimposed on the observed lengthening of day curve. The observed plot is generated from the observed length of day (LOD) in different geologic epochs by John West Wells, Charles P. Sonnet and Kaula-Harris by the study of coral fossils, ancient tidalites and marine creatures respectively. The two have close fit for most of the geologic epochs studied except at the time of birth of Moon and for the period earlier to 1200 Ma. Observed value of LOD at birth is larger than expected at the time of birth and this implies larger rotational mass of inertia. Longer LOD from the inception to 1200 Ma could not be due to Ice Age as there was no Ice Age recorded earlier than 900 Ma. Hence it is postulated that it could be due to plate tectonic movement and continents redistribution. Long term studies of world-wide seismic activities and variation of Earths spin rate clearly establishes that in winter the spin rate comes down leading to a stressed Earth and increased seismic activity in December-January. This suggests that that measurement of Length of Day (LOD) could be a reliable indicator of the changes of moment of inertia of Earth taking place and hence it could contain Earth-quake and impending sudden volcanic eruptions precursors. LOD fluctuations may become an effective Early Warning System for natural disasters of this category.

Key words: geo-synchronous orbits, tidal drag, evolving earth-moon system, recession velocity, gravitational potential well, energy maxima, stability, gravitational sling-shot.

Author: V. SIDORENKO
Keldysh Institute of Applied Mathematics RAS (Russia)
Title: *"Attitude motion of space debris in sun-synchronous orbits: simulation of long-term evolution"*
E-POSTER

Abstract. Sun-synchronous orbits (SSO) are characterized by the highest density of space debris population. To reduce the risk of impact Active Debris Removal (ADR) has been recommended. However, most of the proposed ADR methods have difficulties with capturing fast rotating and tumbling objects. Therefore, it is crucial to be able to predict the rotational state of different types of debris.

The aim of our study is to simulate the rotational motion of large space debris objects in SSO and to reveal general laws of its evolution. Gravity gradient torque and eddy currents torque induced by geomagnetic field are taken into account as well as internal energy dissipation due to deformable elements and residual propellant. It is shown, that rotational motion evolution consists of three principal stages: transient process, exponential decay, and gravity capture. During the transient phase, the motion is primarily determined by internal dissipation, which transforms arbitrary initial spin to the rotation about the axis with the greatest moment of inertia. In the second stage angular velocity decays exponentially due to eddy currents. When it reaches the value comparable to the objects mean motion in orbit, the stage of the gravity capture takes place. This stage is chaotic and results in one of two possible final stationary regimes: gravitational stabilization or rotation about orbital plane normal with angular velocity equal to 1.8 of orbital angular velocity.

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Author: N. SOLOVAYA
Lomonosov Moscow University (Russia)
Title: *"The dynamical instability of the triple hierarchical stellar systems"*
TRADITIONAL POSTER

Abstract. The dynamical evolution of triple hierarchical stellar systems is studied with using of the perturbations of third and forth orders. It is supposed that such systems are stable contrary to stellar systems with of comparable distances.

We considered the motion in the frame of the general three-body problem. Masses of the components are comparable and the ratio of the semi-major axis of their orbits is small parameter. We used the intermediate orbits, obtained by approximated solution of differential equations before transformation by the Zeipel's method.

The intermediated solution was obtained in hyperbolic integrals with the Hamiltonian until the terms of the second order by the method of the Hamilton-Jacobi. The orbits are non-Keplerian ellipses. The secular motion of the nodes, periasters, and essential periodic perturbations were taken into account.

We found that due to the mutual perturbations of the distant star the value of the mean motion of the close pair is slowed and vice versa. The mean motion of distant star is increased.

When we took into account the perturbations of the hight orders we obtained the solution in which the mean motions of the both components have the secular acceleration.

These accelerations are little. But on the cosmological time interval the hierarchical systems will convert to stellar systems, which components have comparable distances. Such systems are unstable. They will either rapidly dissipated or stay to persist as binaries.

Joint work with Eduard Pittich, Slovak Astronomical Society, Slovak Academy of Sciences, Bratislava, Slovak Republic.

Author: Á. SÜLI
Eötvös University (Hungary)
Title: *"Statistics of collisional parameters computed from numerical simulations"*
TRADITIONAL POSTER

Abstract. There are two popular ways to speed up N-body simulations of planet formation: (i) confine motion to 2 spatial dimensions (2D), or (ii) to artificially enhance the physical radii of the bodies (this factor is denoted by A_{ref}). These short cuts have the same effect of increasing the collision probability between objects. Here, we have computed the basic collisional parameters for $A_{\mathrm{ref}} = 1, 2, 3, 5, 10$. Since N-body systems are highly stochastic we have performed 10 simulations for each value of A_{ref} to obtain reliable results. Each of the 50 simulations was containing 10^4 fully interacting bodies which were confined to 2D. We show that one simulation of a specific system provides reliable statistics of the impact parameters therefore in 3D we have simulated only one run for each A_{ref}, i.e. 5 simulations in 3D. Our main goal was to find out the probability distribution functions (pdf) of the basic collisional parameters such as the impact parameter, velocity and specific impact energy. Using a simple method we have improved the determination of the impact angle (and parameter) and we have shown that 99% of the impacts have an angle less than 75 degree and the distribution of the impact parameter is uniform. We show that in most cases the impact velocity is greater than the mutual escape velocity and for $A_{\mathrm{ref}} = 1$. The pdf of the impact velocity can be remarkably well fitted by a power-law function. We present a scaling law of the impact velocity on the A_{ref}. Using our scaling law future simulations can accurately compute the real impact velocity when $A_{\mathrm{ref}} > 1$ and making use of more sophisticated collision scenarios.

Joint work with Zsolt Regály, Konkoly Observatory.

Author: V. TITOV
Saint-Petersburg State University (Russia)
Title: *"Three Body Choreographies and Other Trajectories in Form Space"*
TRADITIONAL POSTER

Abstract. The form space is used in many remarkable papers, mainly for theoretical aspects of Three Body Problem. Nevertheless, for practical purposes, for example numerical simulations, explicit equations in reduced space and various explicit expressions are very useful. We explore the formulae for transformation of Euclidean coordinates into coordinates in form space and vise versa. To this end we consider the three body choreographies and other orbits such as isosceles and rectilinear trajectories in form space. The various properties are studied, including spatial choreographies. It is worthwhile to use Hill surface to represent the region of possible initial conditions for free-fall three-body problem.

Author: J.R.M. XAVIER
Applied Mathematics Division - VSSC (India)
Title: *"Effectiveness of KS elements in orbit predictions using Earth's Gravity, Drag and Solar Radiation Pressure"*
E-POSTER

Abstract. Predicting the orbit of a satellite is a fundamental requirement in many areas of aerospace, such as mission planning, satellite geodesy, re-entry prediction, collision avoidance, and formation flying. The major perturbations which affect the orbit of a satellite are the non-spherical gravitational field of the earth, atmospheric drag, solar radiation pressure, third-body gravitational effects, etc. For near-Earth orbits, the forces due to the non spherical nature of the Earth and atmospheric drag play an important role. Whenever, the satellite is in high altitude (above 600 km) the solar radiation pressure is more important than the atmospheric drag. Thus inclusion of the effect of these perturbing forces becomes important for precise orbit computation of near-Earth orbits. To predict the motion of the orbit precisely a mathematical representation for these forces must be selected properly for integrating in the equations of motion.

The classical Newtonian equations of motion, which are non linear are not suitable for long-term integration for computing accurate orbit. Many transformations have emerged in the literature to stabilize the equations of motion either to reduce the accumulation of local numerical errors or allowing the use of large integration step sizes, or both in the transformed space. One such transformation is known as KS transformation by Kustaanheimo and Stiefel, who regularized the nonlinear Kepler equation of motion and reduced it into linear differential equations of a harmonic oscillator of constant frequency. The method of KS elements has been found to be a very powerful method for obtaining numerical solution with respect to any type of perturbing forces, as the equations are less sensitive to round off and truncation errors. The equations are everywhere regular comparing to the classical Newtonian equations, which are singular at the collision of two bodies. Numerical studies with these equations were carried out using Earth's zonal harmonic terms, drag and solar radiation pressure.

In this paper a detailed study is carried out for orbit prediction using KS differential equations by including the non spherical gravitational potential (zonal and tesseral harmonic terms) of the Earth, atmospheric drag and solar radiation pressure as perturbing forces. Higher order Earth's gravity (zonal and tesseral) terms are included by utilizing the recurrence relations of associated Legendre polynomial and its derivatives. To know the effectiveness of the theory, the results are compared with some of the existing theories in literature and real satellite data.

Joint work with Mrs. T.R. Saritha Kumari