Relativistic modeling for Gaia and BepiColombo

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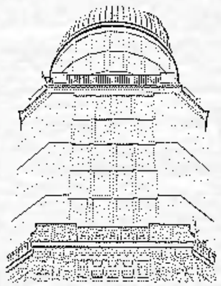
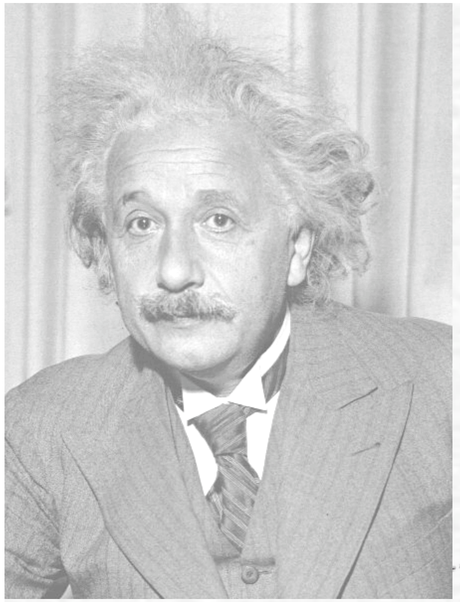
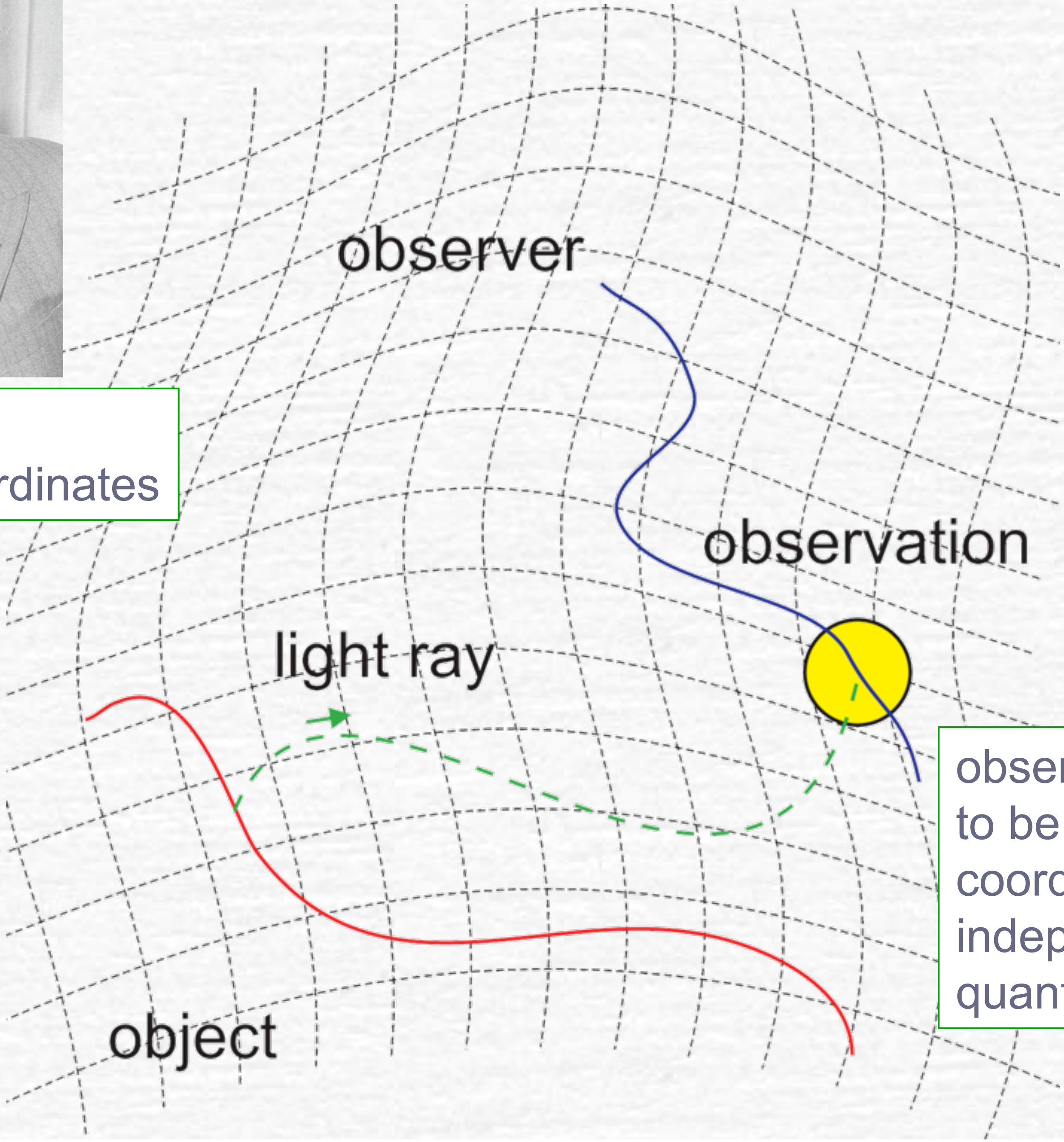


BepiColombo MORE Meeting, 16 February 2009

Relativistic modeling for Gaia



Astronomical observation



no physically preferred coordinates

observables have to be computed as coordinate independent quantities

General relativity for space astrometry

Relativistic reference systems

Relativistic equations of motion

Equations of signal propagation

Definition of observables

Relativistic models of observables

Observational data

Astronomical reference frames

Astrometric catalogs are just realizations of the BCRS

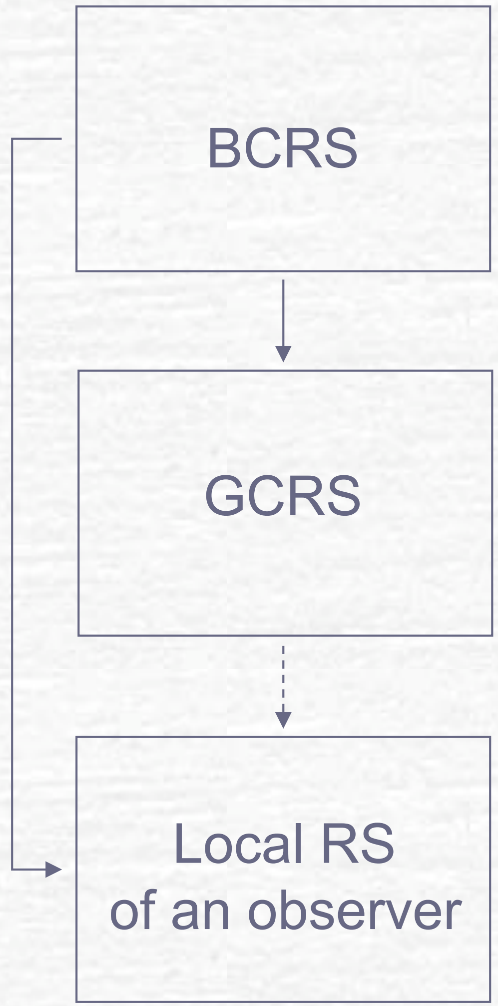
The IAU 2000 framework

- Three standard astronomical reference systems were defined

  - BCRS (Barycentric Celestial Reference System)
  - GCRS (Geocentric Celestial Reference System)
  - Local reference system of an observer

- All these reference systems are defined by

  the form of the corresponding metric tensors.



Technical details: Brumberg, Kopeikin, 1988-1992
Damour, Soffel, Xu, 1991-1994
Klioner, Voinov, 1993
Soffel, Klioner, Petit et al., 2003
Klioner,Soffel, 2000; Kopeikin, Vlasov, 2004

Necessary condition: consistency of the whole data processing chain

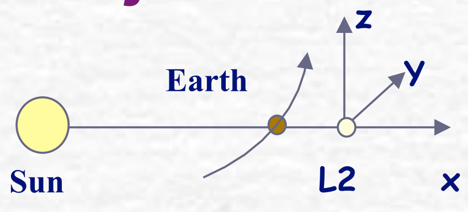
- Any kind of inconsistency is very dangerous for the quality and reliability of the results

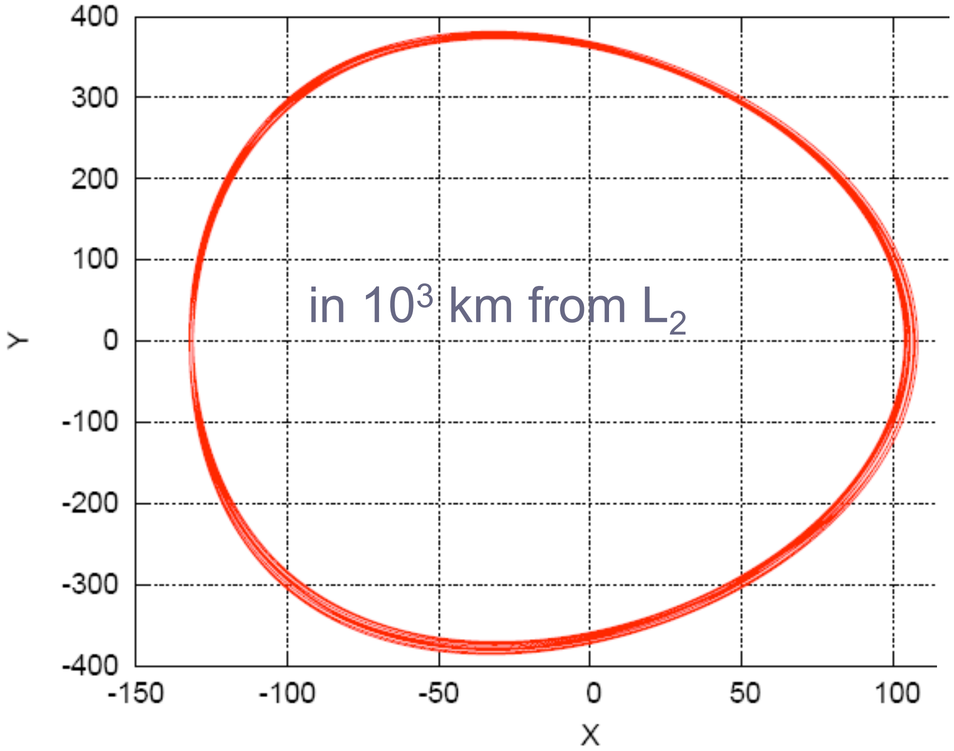
- The whole data processing and all the auxiliary information should be assured to be compatible with the PPN formalism (or at least GR)

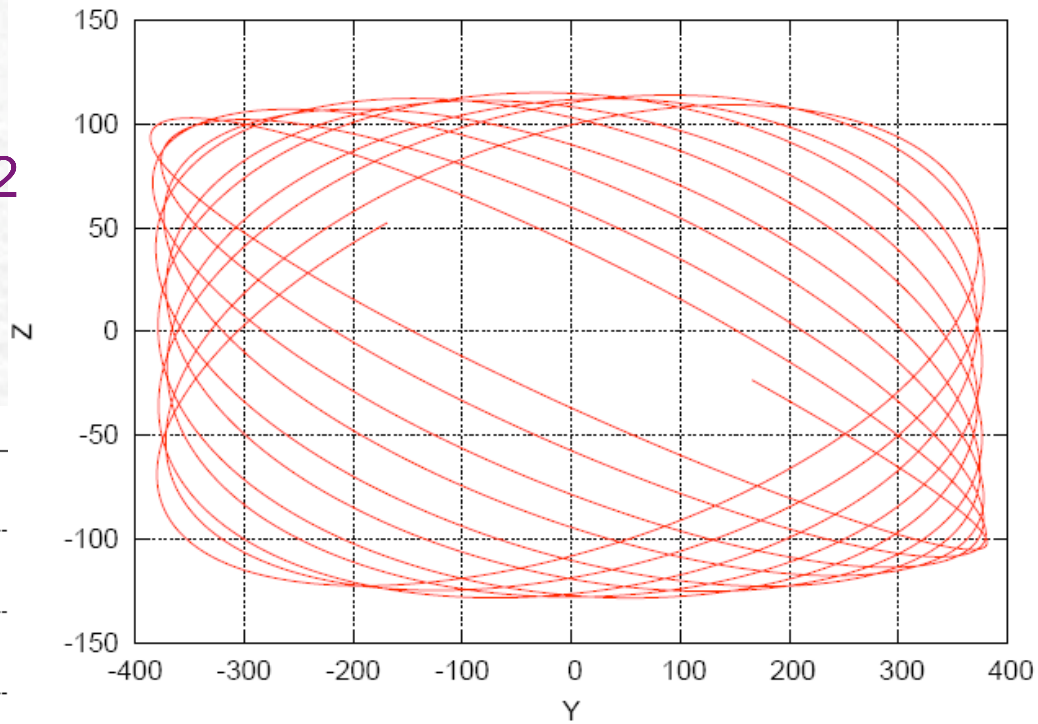
    - planetary ephemeris: coordinates, scaling, constants
    - Gaia orbit: coordinates, scaling, constants
    - astronomical constants
    - onboard clock monitoring (time synchronization)
    - ???

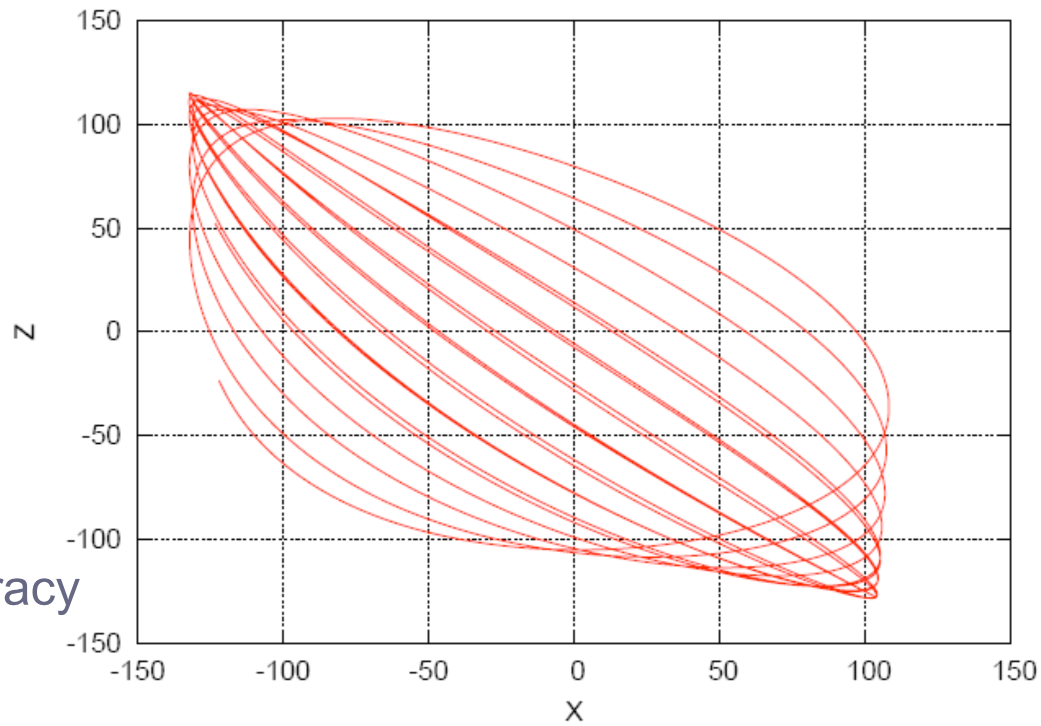
- Monitoring of the consistency during the whole project

Example 1: Lissajous orbit around L_2









Because of Newtonian aberration,
the velocity must be known with an accuracy
of 1 mm/s

Example 2:
Optical aberrations by a rotating instrument

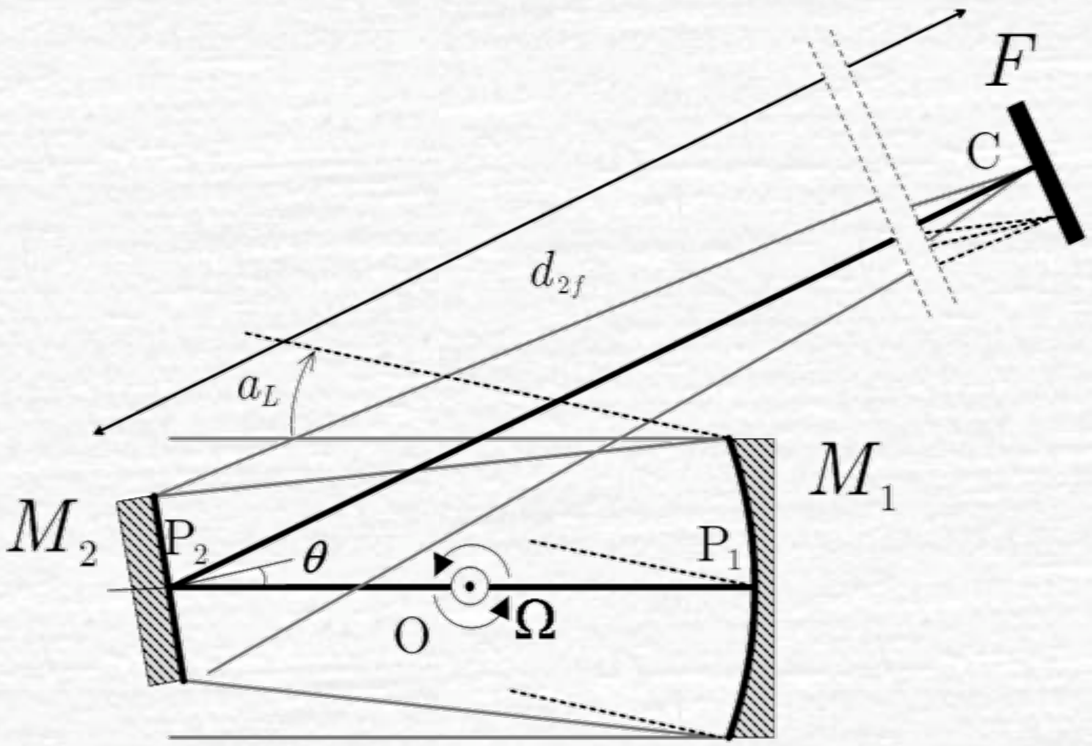
- Two special-relativistic effects modifying PSF of a rotating instrument:
    - Finite light velocity leads to propagation delays within telescope; these delays depend on the position in the field of view
    - Special-relativistic change of the reflection law (Einstein, 1905)

- Reassessment study for Gaia was necessary

(Anglada, Klioner, Soffel, Torra, 2007, Astronomy & Astrophysics, 462, 371)

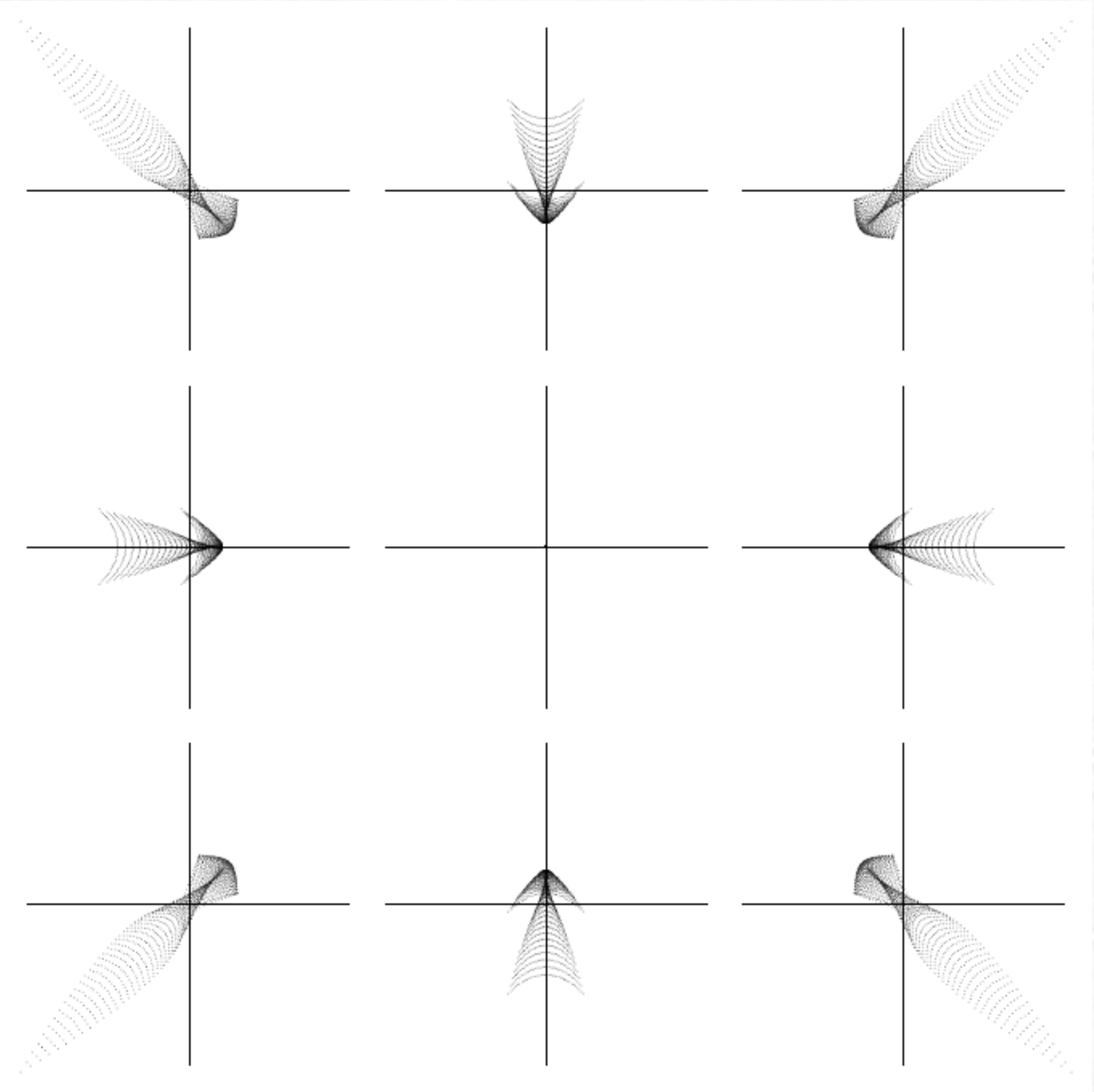
Optical aberrations by a rotating instrument

• Model instrument



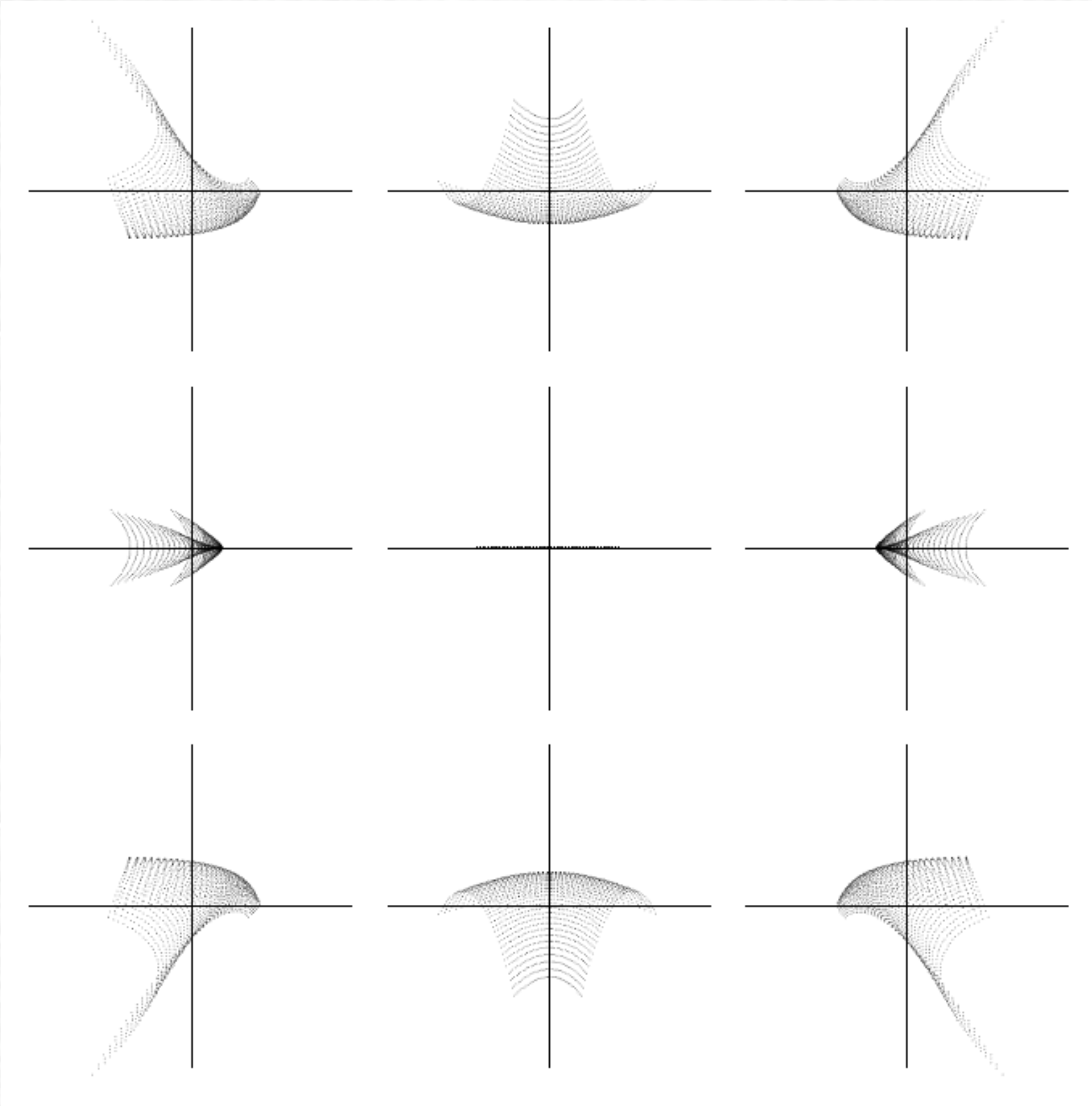
Optical aberrations by a rotating instrument

- Aberration patterns by the instrument at rest



Optical aberrations by a rotating instrument

- Aberration **patterns by the rotating instrument**



Data analysis models compatible with IAU 2000

- Ephemeris construction (JPL, IMCCE, IAA):
  - equations of motion OK
  - radar ranging, classic positional data, etc. OK
  - time scales: precompiled, lower accuracy partially OK
  - new generation of the ephemerides:

    computed together with the ephemeris, full accuracy

    (Fienga, et al. 2007) OK

conventional space ephemeris

+ time ephemeris (Fukushima, 1995) $\Rightarrow$ relativistic 4-dim ephemerides

Relativistic equations of motion used in practice

- In BCRS for planets:

  the so-called Einstein-Infeld-Hoffman equations

  first published in 1917
  used in JPL since 1971

$$
\begin{aligned}
\ddot{\boldsymbol{x}}_A = & -\sum_{B\neq A}\mu_B \frac{\boldsymbol{r}_{AB}}{|\boldsymbol{r}_{AB}|^3} \\
& +\frac{1}{c^2}\sum_{B\neq A}\mu_B \frac{\boldsymbol{r}_{AB}}{|\boldsymbol{r}_{AB}|^3}\left\{\sum_{C\neq B}\frac{\mu_C}{|\boldsymbol{r}_{BC}|} + 4\sum_{C\neq A}\frac{\mu_C}{|\boldsymbol{r}_{AC}|} + \frac{3}{2}\frac{(\boldsymbol{r}_{AB}\cdot\dot{\boldsymbol{x}}_B)^2}{|\boldsymbol{r}_{AB}|^2}\right. \\
& \qquad - \frac{1}{2}\sum_{C\neq A,B}\mu_C\frac{\boldsymbol{r}_{AB}\cdot\boldsymbol{r}_{BC}}{|\boldsymbol{r}_{BC}|^3} \\
& \qquad \left. - 2\,\dot{\boldsymbol{x}}_B\cdot\dot{\boldsymbol{x}}_B - \dot{\boldsymbol{x}}_A\cdot\dot{\boldsymbol{x}}_A + 4\,\dot{\boldsymbol{x}}_A\cdot\dot{\boldsymbol{x}}_B\right\} \\
& +\frac{1}{c^2}\sum_{B\neq A}\mu_B\frac{\dot{\boldsymbol{x}}_A-\dot{\boldsymbol{x}}_B}{|\boldsymbol{r}_{AB}|^3}\left\{4\,\dot{\boldsymbol{x}}_A\cdot\boldsymbol{r}_{AB} - 3\,\dot{\boldsymbol{x}}_B\cdot\boldsymbol{r}_{AB}\right\} \\
& -\frac{1}{c^2}\frac{7}{2}\sum_{B\neq A}\frac{\mu_B}{|\boldsymbol{r}_{AB}|}\sum_{C\neq A,B}\mu_C\frac{\boldsymbol{r}_{BC}}{|\boldsymbol{r}_{BC}|^3} + O(c^{-4}),
\end{aligned}
$$

- In the GCRS for Earth satellites: e.g., IERS Conventions, 2003

- In ALL theoretical works TCB and TCG are used to derive these equations

  $\Longrightarrow$ only linear functions of TCB and TCG are allowed

Relativistic Time Scales: TCB and TCG

- $t = TCB$ Barycentric Coordinate Time = coordinate time of the BCRS
- $T = TCG$ Geocentric Coordinate Time = coordinate time of the GCRS

These are part of 4-dimensional coordinate systems so that
the TCB-TCG transformations are 4-dimensional: $(r_E^i = x^i - x_E^i(t))$

$$T = t - \frac{1}{c^2}\left(A(t) + \underline{v_E^i r_E^i} \right) + \frac{1}{c^4}\left(B(t) + \underline{B^i(t) r_E^i + B^{ij}(t) r_E^i r_E^j + C\left(t, \boldsymbol{x}\right)} \right) + O\left(c^{-5}\right)$$

- Therefore: $TCG = TCG(TCB, \underline{x^i})$

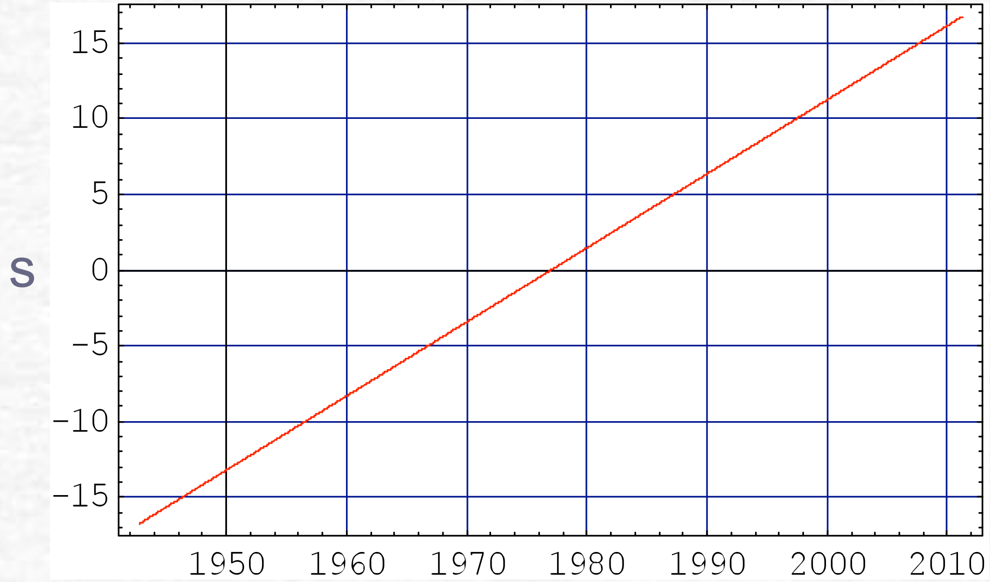
- Only if space-time position is fixed in the BCRS $x^i = x_{obs}^i(t)$
  TCG becomes a function of TCB:

$$TCG = TCG(TCB, x_{obs}^i(TCB)) = TCG(TCB)$$

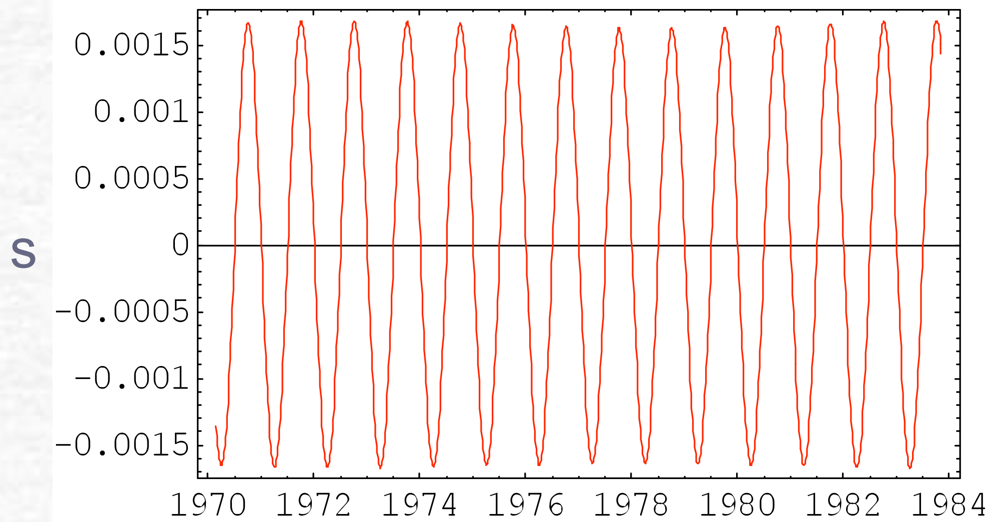
Relativistic Time Scales: TCB and TCG

- Important special case $x^i = x^i_E(t)$ gives the TCG-TCB relation at the geocenter:

main feature: linear drift 1.48×10^{-8}
zero point is defined to be Jan 1, 1977
difference now: 14.7 seconds



linear drift removed:



Relativistic Time Scales: proper time scales

- τ proper time of each observer: what an ideal clock moving with the observer measures…

- Proper time can be related to either TCB or TCG (or both) provided that the trajectory of the observer is given:

$$x^i_{obs}(t) \quad \text{and/or} \quad X^a_{obs}(T)$$

The formulas are provided by the relativity theory:

$$\frac{d\tau}{dt} = \left(-g_{00}\left(t, \mathbf{x}_{obs}(t)\right) - \frac{2}{c} g_{0i}\left(t, \mathbf{x}_{obs}(t)\right) \dot{x}^i_{obs}(t) - \frac{1}{c^2} g_{ij}\left(t, \mathbf{x}_{obs}(t)\right) \dot{x}^i_{obs}(t)\, \dot{x}^j_{obs}(t) \right)^{1/2}$$

$$\frac{d\tau}{dT} = \left(-G_{00}\left(T, \mathbf{X}_{obs}(T)\right) - \frac{2}{c} G_{0a}\left(T, \mathbf{X}_{obs}(T)\right) \dot{X}^a_{obs}(T) - \frac{1}{c^2} G_{ab}\left(T, \mathbf{X}_{obs}(T)\right) \dot{X}^a_{obs}(T)\, \dot{X}^b_{obs}(T) \right)^{1/2}$$

Relativistic Time Scales: proper time scales

- Specially interesting case: an observer close to the Earth surface:

$$\frac{d\tau}{dT} = 1 - \frac{1}{c^2}\left(\frac{1}{2}\dot{X}^2_{obs}(T) + W_E\left(T, \mathbf{X}_{obs}\right) + \text{"tidal terms"}\right) + O\left(c^{-4}\right)$$

$$\sim 10^{-17}$$

- Idea: let us define a time scale linearly related to T=TCG, but which is numerically close to the proper time of an observer on the geoid:

$$\boxed{TT = (1 - L_G)\, TCG, \quad L_G \equiv 6.969290134 \times 10^{-10}}$$

$$\frac{d\tau}{d\,TT} = 1 - \frac{1}{c^2}\left(\text{"terms } \sim h, v^i\text{"} + \text{"tidal terms"} + \ldots\right) + \ldots$$

can be neglected in many cases

h is the height above the geoid

v^i is the velocity relative to the rotating geoid

Relativistic Time Scales: TT

- To avoid errors and changes in TT implied by changes/improvements in the geoid, the IAU (2000) has made L_G to be a defined constant:

$$L_G \equiv 6.969290134 \cdot 10^{-10}$$

- TAI is a practical realization of TT (up to a constant shift of 32.184 s)

- Older name TDT (introduced by IAU 1976): fully equivalent to TT

Relativistic Time Scales: TDB-1

- Idea: to scale TCB in such a way that the “scaled TCB” remains close to TT

- IAU 1976: TDB is a time scale for the use for dynamical modelling of the Solar system motion which differs from TT only by periodic terms.

- This definition taken literally is flawed:

  such a TDB cannot be a linear function of TCB!

  But the relativistic dynamical model (EIH equations) used by e.g. JPL is valid only with TCB and linear functions of TCB…

Relativistic Time Scales: TDB-2

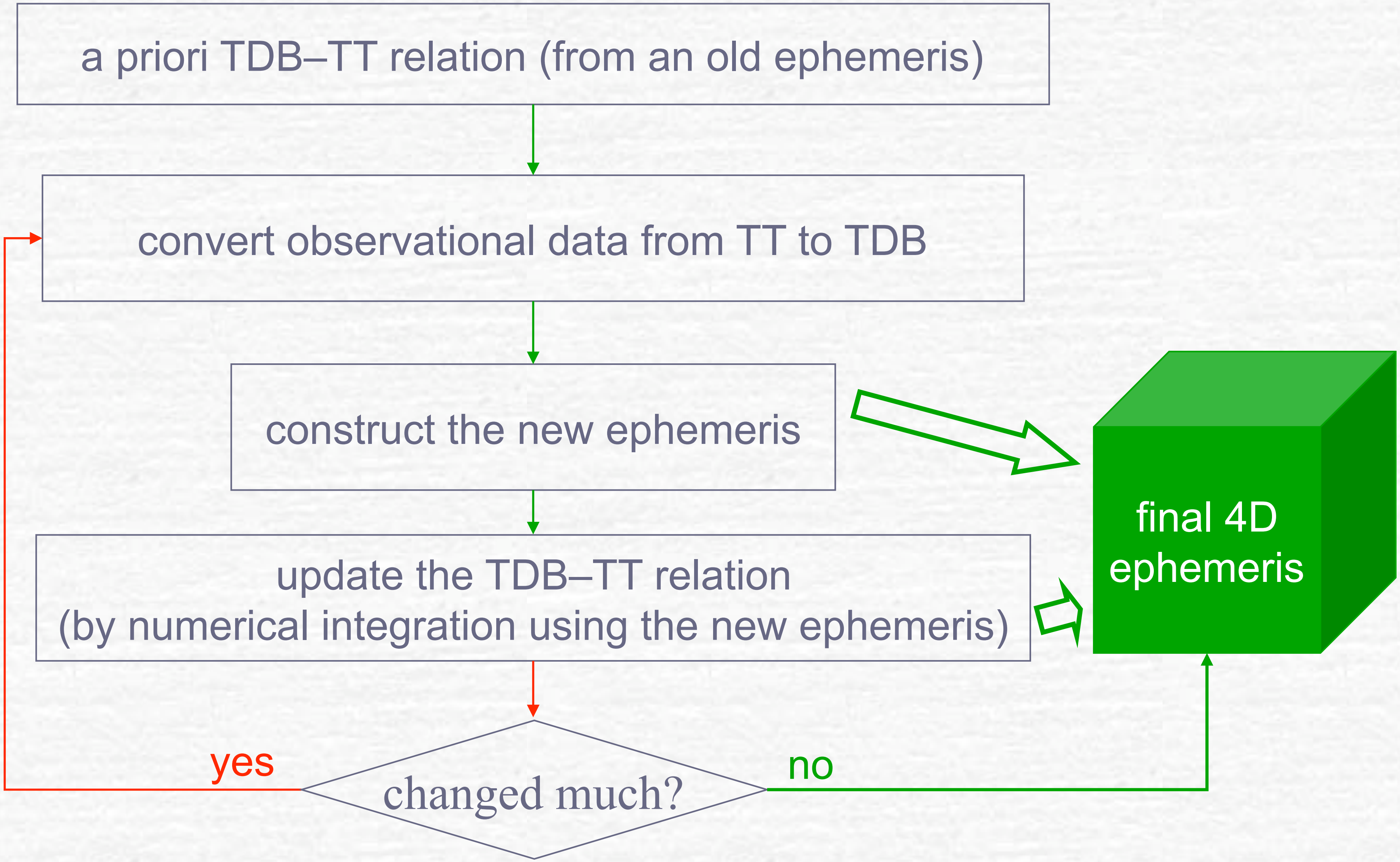
The IAU (2006) has re-defined TDB to be a fixed linear function of TCB:

- TDB is defined through a conventional relationship with TCB:

$$\boxed{TDB = TCB - L_B \times \left(JD_{TCB} - T_0\right) \times 86400 + TDB_0}$$

- T_0 = 2443144.5003725 exactly,
- $JD_{TCB} = T_0$ for the event 1977 Jan 1.0 TAI at the geocenter and increases by 1.0 for each 86400s of TCB,
- $L_B \equiv 1.550519768 \times 10^{-8}$,
- $TDB_0 \equiv -6.55 \times 10^{-5}$ s.

Iterative procedure to construct an ephemeris with TDB in a fully consistent way



How to compute TT(TDB) from an ephemeris

- Fundamental relativistic relation between TCG and TCB at the geocenter

$$
\begin{aligned}
T &= \mathsf{TCG} \\
t &= \mathsf{TCB}
\end{aligned}
$$

$$\frac{dT}{dt} = 1 + \frac{1}{c^2}\,\alpha(t) + \frac{1}{c^4}\,\beta(t) + \mathcal{O}(c^{-5})$$

$$\alpha = -\frac{1}{2}\,v_E^2 - \sum_{A\neq E}\frac{GM_A}{r_{EA}},$$

$$
\begin{aligned}
\beta = -\frac{1}{8}\,v_E^4 &+ \left(\beta - \frac{1}{2}\right)\left(\sum_{A\neq E}\frac{GM_A}{r_{EA}}\right)^2 + (2\beta - 1)\sum_{A\neq E}\left(\frac{GM_A}{r_{EA}}\sum_{B\neq A}\frac{GM_B}{r_{AB}}\right) \\
&+ \sum_{A\neq E}\frac{GM_A}{r_{EA}}\left(2(1+\gamma)v_A^i v_E^i - \left(\gamma + \frac{1}{2}\right)\,v_E^2 - (1+\gamma)v_A^2 + \frac{1}{2}a_A^i r_{EA}^i + \frac{1}{2}(v_A^i r_{EA}^i / r_{EA})^2\right)
\end{aligned}
$$

How to compute TT(TDB) from an ephemeris

- definitions of TT and TDB

1) TT(TCG) :

$$\mathsf{TT} = \mathsf{TCG} - L_G \times (JD_{\mathsf{TCG}} - T_0) \times 86400$$

$$T_0 = 2443144.5003725,\ L_G = 6.969290134 \times 10^{-10}$$

2) TDB(TCB) :

$$\mathsf{TDB} = \mathsf{TCB} - L_B \times (JD_{\mathsf{TCB}} - T_0) \times 86400 + \mathsf{TDB}_0$$

$$T_0 = 2443144.5003725,\ L_B = 1.550519768 \times 10^{-8},\ \mathsf{TDB}_0 = -6.55 \times 10^{-5}\ \mathrm{s}$$

How to compute TT(TDB) from an ephemeris

- two corrections

$$TT = TDB + \Delta TDB(TDB)$$

$$TDB = TT - \Delta TT(TT)$$

- two differential equations

$$\frac{d}{dTDB}\Delta TDB = \left(L_B + \frac{1}{c^2}\alpha(TDB)\right)\left(1 + L_B - L_G\right) - L_G + \frac{1}{c^4}\beta(TDB)$$

$$\frac{d}{dTT}\Delta TT = \frac{1}{c^2}\alpha(TT - \Delta TT)\left(1 - L_B + L_G\right) + \frac{1}{c^4}\left(\beta(TT - \Delta TT) - \alpha^2(TT - \Delta TT)\right) + \left(L_B - L_G\right)\left(1 + L_G\right)$$

Representation with Chebyshev polynomials

- Any of those small functions can be represented by a set of Chebyshev polynomials

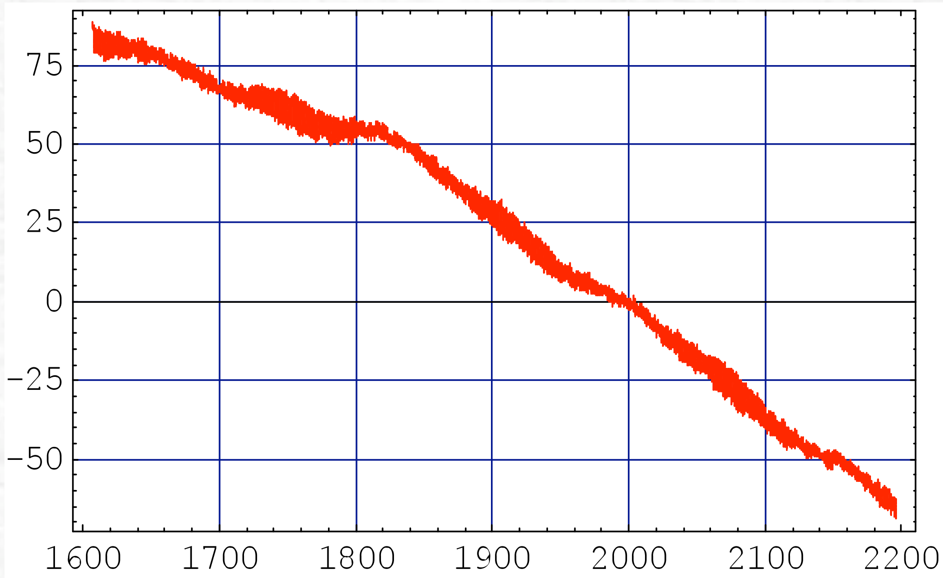
$$y(x) \approx \sum_{n=0}^{N} a_n \, T_n(x)$$

- The conversion of tabulated $y(x)$ into a_n is a well-known task…

TT-TDB: DE405 vs. SOFA for full range of DE405

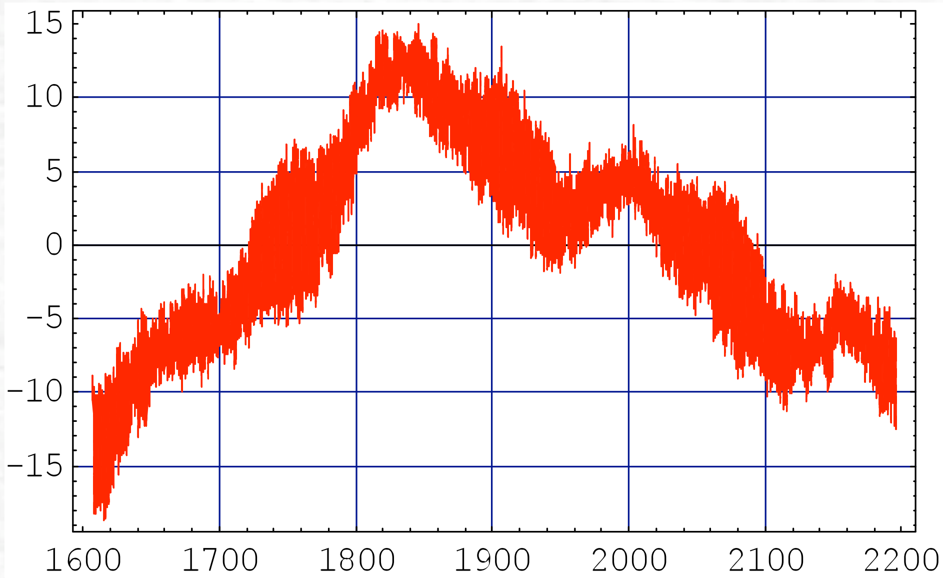
- SOFA implements the corrected Fairhead-Bretagnon analytical series based on VSOP-87 (about 1000 Poisson terms, also non-periodic terms)

ns



$$0.868\text{ ns} - 8.28\cdot\ 10^{-18} t +$$

ns



Other time scales

- The same procedure with numerical integration can be used to compute

  - proper time of a space craft

  - coordinate time of some other planetocentric reference system and TCB

Scaled time scales: the price to pay

- If one uses scaled version TCB – T_{eph} or TDB – one effectively uses three scaling:

  - time $$t^* = F \cdot TCB + t_0^*$$
  - spatial coordinates $$\mathbf{x}^* = F \cdot \mathbf{x}$$
  - masses (μ= GM) of each body $$\mu^* = F \cdot \mu$$

$$F = 1 - L_B$$

WHY THREE SCALINGS?

Equations to leave unchanged

- These three scalings together leave the dynamical equations unchanged:

  - for the motion of the solar system bodies:

$$\ddot{\boldsymbol{x}}_A = -\sum_{B\neq A}\mu_B \frac{\boldsymbol{r}_{AB}}{|\boldsymbol{r}_{AB}|^3}$$
$$+\frac{1}{c^2}\sum_{B\neq A}\mu_B\frac{\boldsymbol{r}_{AB}}{|\boldsymbol{r}_{AB}|^3}\left\{\sum_{C\neq B}\frac{\mu_C}{|\boldsymbol{r}_{BC}|}+4\sum_{C\neq A}\frac{\mu_C}{|\boldsymbol{r}_{AC}|}+\frac{3}{2}\frac{(\boldsymbol{r}_{AB}\cdot\dot{\boldsymbol{x}}_B)^2}{|\boldsymbol{r}_{AB}|^2}\right.$$
$$-\frac{1}{2}\sum_{C\neq A,B}\mu_C\frac{\boldsymbol{r}_{AB}\cdot\boldsymbol{r}_{BC}}{|\boldsymbol{r}_{BC}|^3}$$
$$\left.-2\,\dot{\boldsymbol{x}}_B\cdot\dot{\boldsymbol{x}}_B-\dot{\boldsymbol{x}}_A\cdot\dot{\boldsymbol{x}}_A+4\,\dot{\boldsymbol{x}}_A\cdot\dot{\boldsymbol{x}}_B\right\}$$
$$+\frac{1}{c^2}\sum_{B\neq A}\mu_B\frac{\dot{\boldsymbol{x}}_A-\dot{\boldsymbol{x}}_B}{|\boldsymbol{r}_{AB}|^3}\left\{4\,\dot{\boldsymbol{x}}_A\cdot\boldsymbol{r}_{AB}-3\,\dot{\boldsymbol{x}}_B\cdot\boldsymbol{r}_{AB}\right\}$$
$$-\frac{1}{c^2}\frac{7}{2}\sum_{B\neq A}\frac{\mu_B}{|\boldsymbol{r}_{AB}|}\sum_{C\neq A,B}\mu_C\frac{\boldsymbol{r}_{BC}}{|\boldsymbol{r}_{BC}|^3}+O(c^{-4}),$$

  - for light propagation:

$$c\,(t_2-t_1)=|\boldsymbol{x}_2-\boldsymbol{x}_1|$$
$$+\sum_A\frac{2\,\mu_A}{c^2}\ln\frac{|\boldsymbol{r}_{1A}|+|\boldsymbol{r}_{2A}|+|\boldsymbol{r}_{21}|}{|\boldsymbol{r}_{2A}|+|\boldsymbol{r}_{1A}|-|\boldsymbol{r}_{21}|}+O(c^{-4}),$$

TCG/TCB-, TT- and TDB-compatible planetary masses

- GM of the Earth (from SLR):
  - TT-compatible $\left\{\mu^{**}_{\mathrm{Earth}}\right\}_{SI} = \left(398600441.5 \pm 0.4\right) \times 10^{6}$
  - TCG/B-compatible $\left\{\mu_{\mathrm{Earth}}\right\}_{SI} = \dfrac{1}{1-L_G}\left\{\mu^{**}_{\mathrm{Earth}}\right\}_{SI} = \left(398600441.8 \pm 0.4\right) \times 10^{6}$
  - TDB-compatible $\left\{\mu^{*}_{\mathrm{Earth}}\right\}_{SI} = \left(1-L_B\right)\left\{\mu_{\mathrm{Earth}}\right\}_{SI} = \left(398600435.6 \pm 0.4\right) \times 10^{6}$

If one uses TCG and TCB one has only one mass…

Relativistic Model for Gaia

- GREM: Gaia Relativity Model

  Klioner, 2003, AJ, 125, 1580 ; Klioner, 2004, Phys.Rev.D, 69, 124001
  Klioner, Peip, 2003, A&A, 410, 1063
  Zschocke, Klioner, 2006, Gaia-CA-TN-LO-SZ-001
  Klioner, 2008, GAIA-CA-TN-LO-SK-006-2 (compact description)

- Two modes:

  - Predictor mode: predict the observed position for a known object

  - Corrector mode: restore parameters of an object from observable direction
    (not always possible & not always accurate)

- Two kinds of objects:

  - “stars”
  - solar system objects

- Accuracy in predictor mode: up to 0.1 μas

GREM: retarded moment?

- Kopeikin & Schäfer, 1999:
  deflected body's position & velocity should be evaluated at the retarded moment:

$$\delta\boldsymbol{\sigma}_{pN} = -\sum_A \frac{(1+\gamma)GM_A}{c^2} \frac{\hat{\boldsymbol{d}}_A}{|\boldsymbol{d}_A|} \left(1 + \boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}_{\mathrm{oA}}\right),$$

$$\boldsymbol{d}_A = \boldsymbol{\sigma} \times \left(\boldsymbol{r}_{\mathrm{oA}} \times \boldsymbol{\sigma}\right), \qquad \boldsymbol{r}_{\mathrm{oA}} = \boldsymbol{x}_{\mathrm{o}}(t_{\mathrm{o}}) - \boldsymbol{x}_A(t^*),$$

$$t^* = t_{\mathrm{o}} - \frac{1}{c}\left|\boldsymbol{x}_{\mathrm{o}}(t_{\mathrm{o}}) - \boldsymbol{x}_A(t^*)\right|.$$

- one iteration is not sufficient to compute *t** (Klioner & Peip, 2003): error 0.5 µas

- at least 3 evaluation of the ephemeris is required

GREM: no retarded moment

- Klioner, 1989: formal pN solution for deflectors with constant velocity

$$\delta\boldsymbol{\sigma}_{pN} = -\sum_A \frac{(1+\gamma)GM_A}{c^2}\frac{\boldsymbol{\sigma}\times(\boldsymbol{\rho}_{oA}\times\boldsymbol{g}_A)}{\delta_A^2}\,(1+\hat{\boldsymbol{g}}_A\cdot\hat{\boldsymbol{\rho}}_{\mathrm{oA}})$$

$$\boldsymbol{g}_A = \boldsymbol{\sigma} - \boldsymbol{k}_A, \quad \boldsymbol{\rho}_{\mathrm{oA}} = \boldsymbol{x}_{\mathrm{o}}(t_{\mathrm{o}}) - \boldsymbol{x}_A(t_{\mathrm{o}}), \quad \delta_A = |\hat{\boldsymbol{g}}_A\times\boldsymbol{\rho}_{oA}|.$$

$$\boldsymbol{k}_A = \frac{1}{c}\,\dot{\boldsymbol{x}}_A(t_{\mathrm{o}})$$

- The body's position and velocity only at the moment of observation t_0
- Klioner & Peip, 2003: accuracy 0.02 μas
- only 2 evaluation of the ephemeris is required

Gaia: data processing

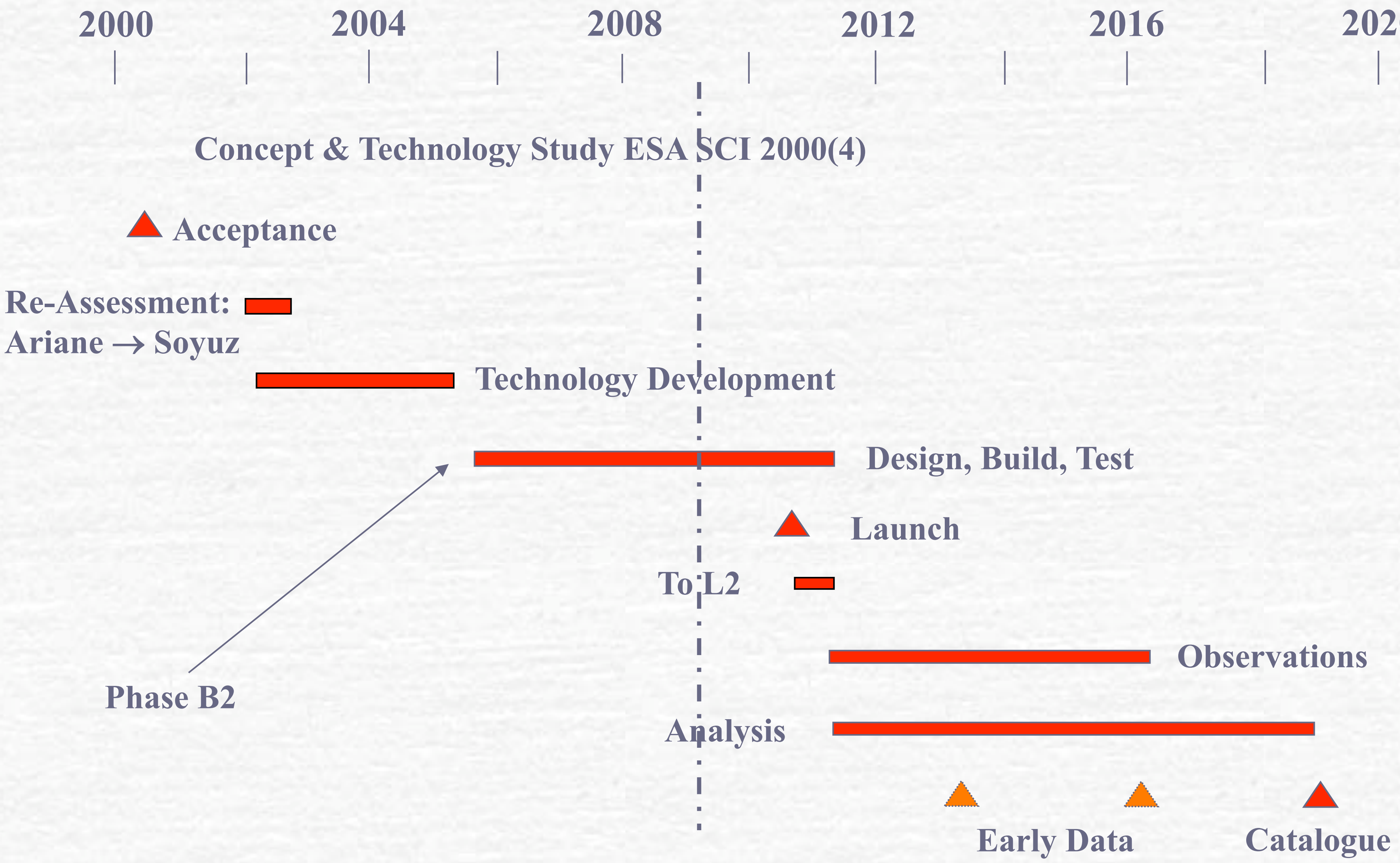
- Parameters
  - At least 5 parameters for each star: 5×10^9
  - 4 parameters of orientation each 15 seconds: 10^8
  - 2000 calibration parameters per day: 4×10^6
  - global parameters (e.g., PPN γ): 10^2

- Observations

  about 1000 raw images for each star: 10^{12}

- Data volume: 1 PB (iterative data processing)
- Computational efforts: ~10^{19} to 10^{21} flops

Gaia: timetable



Relativistic modeling for BepiColombo

What to model?

1. Motion of the observing station in the BCRS:

   a. Earth rotation in the GCRS: precession/nutation+polar motion
      a good model is necessary to have 1mm/s and a few cm accuracy

   b. Transformation in the BCRS: trivial

   c. Motion of the geocenter in the BCRS:
      a given(?) solar system ephemeris

2. Motion of the spacecraft in the BCRS:

   a. PPN form of the EIH equations (Will, 1993?)
   b. Rotational motion of Mercury in Mercurian Celestial RS
   c. Structure of the gravitational field of Mercury
      in a Mercurian "corotating" RS
   d. Influence of Mercurian gravitational field on the spacecraft
   e. Non-gravitational forces: the onboard accelerometer

What to model?

3. Light propagation (delay + frequency?)

    a. Shapiro delay in 1+2 pN
    b. Retarded moment or the moment of closest approach for the positions of the Sun, Mercury, Earth, …

Post-post-Newtonian terms necessary?

Yes, in principle, but

- only numerical magnitude is interesting for practical work
- in relativity analytical orders of magnitude are used

The situation is similar to “analytical expansions”
(e.g., in powers of eccentricities) in classical celestial mechanics…

$$c^{-2}, c^{-4}, e^{4}, \ldots$$

Post-post-Newtonian light propagation?

Full post-post-Newtonian expression for the Shapiro time delay with PPN parameters (Klioner, Zschocke, 2007):

$$
\begin{aligned}
c\,\tau = R \; &+ \; (1+\gamma)\, m \, \log \frac{x + x_0 + R}{x + x_0 - R} \\
&+ \frac{1}{8} \quad \epsilon \, \frac{m^2}{R} \left(\frac{x_0^2 - x^2 - R^2}{x^2} + \frac{x^2 - x_0^2 - R^2}{x_0^2} \right) \\
&+ \frac{1}{4} \quad \left(8(1+\gamma) - 4\beta + 3\epsilon\right) \; m^2 \, \frac{R}{|\boldsymbol{x} \times \boldsymbol{x}_0|} \; \arctan \frac{x^2 - x_0^2 + R^2}{2|\boldsymbol{x} \times \boldsymbol{x}_0|} \\
&- \frac{1}{4} \quad \left(8(1+\gamma) - 4\beta + 3\epsilon\right) \; m^2 \, \frac{R}{|\boldsymbol{x} \times \boldsymbol{x}_0|} \; \arctan \frac{x^2 - x_0^2 - R^2}{2|\boldsymbol{x} \times \boldsymbol{x}_0|} \\
&+ \frac{1}{2} \, (1+\gamma)^2 \, m^2 \, \frac{R}{|\boldsymbol{x} \times \boldsymbol{x}_0|^2} \, (x - x_0 - R)\,(x - x_0 + R)\;.
\end{aligned}
$$

$$m = \frac{GM}{c^2}$$

The higher-order terms give up to 10 meters. Are all these terms relevant?

Post-post-Newtonian light propagation?

Are all these terms relevant? NO!

$$
\begin{aligned}
c\,\tau = R \;&+\; (1+\gamma)\, m \,\log \frac{x + x_0 + R}{x + x_0 - R} \\
&+ \frac{1}{8}\, \epsilon\, \frac{m^2}{R} \left(\frac{x_0^2 - x^2 - R^2}{x^2} + \frac{x^2 - x_0^2 - R^2}{x_0^2} \right) \\
&+ \frac{1}{4}\, \big(8(1+\gamma) - 4\beta + 3\epsilon\big)\, m^2\, \frac{R}{|\boldsymbol{x} \times \boldsymbol{x}_0|}\, \arctan \frac{x^2 - x_0^2 + R^2}{2|\boldsymbol{x} \times \boldsymbol{x}_0|} \\
&- \frac{1}{4}\, \big(8(1+\gamma) - 4\beta + 3\epsilon\big)\, m^2\, \frac{R}{|\boldsymbol{x} \times \boldsymbol{x}_0|}\, \arctan \frac{x^2 - x_0^2 - R^2}{2|\boldsymbol{x} \times \boldsymbol{x}_0|} \\
&+ \frac{1}{2}\, (1+\gamma)^2\, m^2\, \frac{R}{|\boldsymbol{x} \times \boldsymbol{x}_0|^2}\, (x - x_0 - R)\,(x - x_0 + R)\;.
\end{aligned}
$$

$$m = \frac{GM}{c^2}$$

Post-post-Newtonian light propagation?

NO!

The only numerically relevant term can be written as

$$c\,\tau = R + (1+\gamma)\,m\,\log\frac{x + x_0 + R + (1+\gamma)\,m}{x + x_0 - R + (1+\gamma)\,m}$$

This has already been derived by Moyer (2003) in a different way.

All other terms can be estimated as

$$c\,\delta\,\tau \le \frac{m^2}{d}\left(\frac{3}{4} + \frac{15}{4}\,\pi\right)$$

This gives maximally 4 cm for Sun-grazing ray,
and much less in typical cases…

Similar situation with light deflection,
post-post-Newtonian equations of motion, etc.

Other terms in the light propagation?

Kopeikin, Schäfer, 1999; Klioner, 2003; Klioner, Peip, 2003:

The position of the gravitating body in the shown formula must be evaluated either at the moment of closest approach

$$t^* = t_\mathrm{o} - \frac{1}{c}\,|\boldsymbol{x}_\mathrm{o}(t_\mathrm{o}) - \boldsymbol{x}_A(t^*)|.$$

or at the moment of closest approach between the gravitating body and the light ray:

$$t^{\mathrm{ca}} = \max\left(t_\mathrm{e}, t_\mathrm{o} - \max\left(0, \frac{\boldsymbol{g}\cdot(\boldsymbol{x}_\mathrm{p}(t_\mathrm{o}) - \boldsymbol{x}_\mathrm{A}\ (t_\mathrm{o}))}{c\,|\boldsymbol{g}|^2}\right)\right),$$

$$\boldsymbol{g} = \boldsymbol{\mu} - \frac{1}{c}\,\dot{\boldsymbol{x}}_\mathrm{A}\ (t_\mathrm{o}),$$

Other terms in the light propagation?

Klioner, 1992:

The Shapiro effect due to the quadrupole moment can be estimated as

$$c\,\delta\tau_{J_2} \le 3.18 \frac{GM}{c^2} J_2$$

The Shapiro effect due to the angular momentum S can be estimated as (P is the radius of the body)

$$c\,\delta\tau_S \le 4 \frac{G \cdot S}{P}$$

For the Sun this gives 16 ps and 8 ps, respectively (0.48 cm and 0.24 cm).

Some suggestions

1. Be very consistent in the model;
   better too consistent at the price of slower data processing

   This should be of course at reasonable level!

2. Do not use time scales TT and TDB:

   use TCG and TCB and avoid artificial scaling of e.g. masses *GM*

   Mercury Coordinate Time must be used to model Mercury rotation.
   Why to bother with all the scalings?

3. Use 128 bit arithmetic for orbit propagation and forget about round-off

4. Integrate the Sun, do not use the center of mass to reduce the system

   Use an approximation to the center of mass integrals only to fix the initial conditions; define the parameters only for the orbit w.r.t. the Sun

Backup slides

Data analysis models compatible with IAU 2000

- Ephemeris construction (JPL, IMCCE, IAA) OK
- VLBI OK
- Lunar Laser Ranging partially OK

relativistically consistent model for the figure-figure interaction for the Earth-Moon system must be applied

relativistically consistent model for Moon rotation

Data analysis models compatible with IAU 2000

• Ephemeris construction (JPL, IMCCE, IAA)	OK
• VLBI	OK
• Lunar Laser Ranging	partially OK
• Satellite Laser Ranging	OK
• Hipparcos, Gaia, …	OK
• time keeping and time transfer algorithms	OK
• pulsar timing	OK
• Earth/Moon rotation	
up to 2006: a purely Newtonian model was used	not OK
first relativistic theory: Klioner et al. (2007)	partially OK

Minor problems in the models still exist: higher-order terms, scaling, etc.