Geodesy and interior structure of Mercury

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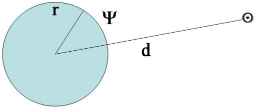
OUTLINE

TIDAL POTENTIAL (I)

- tidal force=differential gravitational force
- gradient of a tidal potential
    - direct effect of the Sun

$$V_T = -\frac{GM_\odot}{d}\sum_{l=2}^{\infty}\left(\frac{r}{d}\right)^l P_l(\cos\Psi)$$

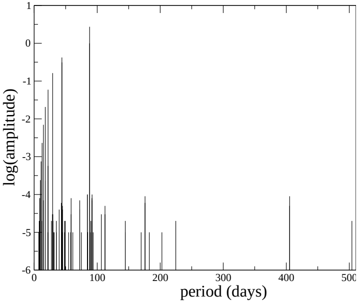


- orbital motion (Kepler's laws)
- rotational motion
- restrict to degree 2

- Venus: 4×10^{-6} smaller
- indirect effect due to planetary effects on orbital motion
- VSOP87 ephemerides (Bretagnon & Francou 1988) valid for several thousand years around J2000.0

TIDAL POTENTIAL (II)

- tidal deformation and potential can mathematically be described with three spherical harmonics
- main period: half a Mercury solar day = one Mercury year (3:2 resonance)
- no simple division as for the Earth: typical periods of zonal, tesseral and sectorial waves are long period, diurnal, and semidiurnal



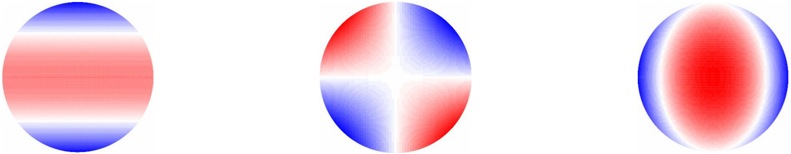


FIGURE: sectorial waves

TIDAL POTENTIAL (III)

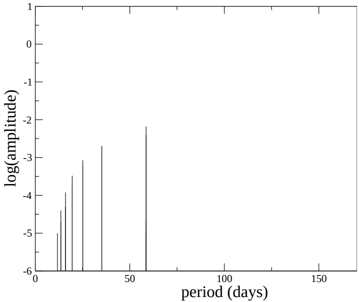


FIGURE: tesseral waves

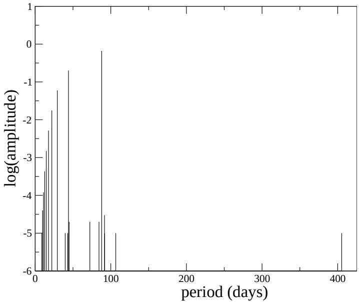


FIGURE: zonal waves

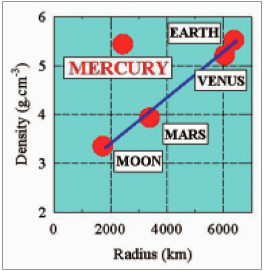
TIDAL REACTION

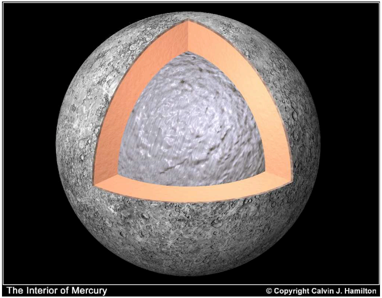
Tidal potential causes

- periodically varying surface displacements (Love numbers h and l)
    - estimate for equipotential surface: $\frac{V_T}{g} \approx 1\text{m}$
    - $\delta r = h\frac{V_T}{g}$
- variations in the external potential field (Love number k)
    - estimate: $\frac{V_T}{V} \approx \frac{M_\odot}{M} \left(\frac{R}{a}\right)^3 \approx 5 \times 10^{-7}$
    - $\delta V = (1 + k)V_T \qquad \text{(at surface)}$
- surface gravity variations (Love numbers h and k)
    - estimate: gradient of potential: $2\frac{V_T}{R} = 3 \times 10^{-6}\text{ms}^{-2}$
    - g=3.7ms^{-2}

BASIC FACTS

- mass $M = 3.302 \times 10^{23}$ kg (Anderson et al. 1987)
- radius $R = 2439 \pm 1$ km
- density $\rho = 5430 \pm 10$ kg/m^3
- large core





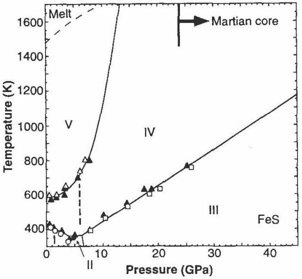
COMPOSITION (I)

- data
  - large Fe/Si ratio (large core)
  - low surface FeO content (spectral observations)
- mantle
  - mantle mineralogy: assume olivine, pyroxene, garnet
  - chemical composition: strongly dependent on formation history (Taylor and Scott 2005)
  - density, rigidity and incompressibility: relatively small differences
  - density variation $\simeq 100$ kg/m^3 (few %)
- mantle model
  - homogeneous density $\rho = 3500$ kg/m^3
  - rigidity and incompressibility: same pressure dependence as in upper mantle of the Earth

COMPOSITION (II)

core:

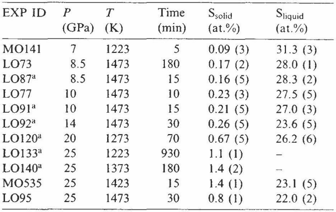
- Fe + S (abundant + soluble at Mercury pressures)
- x_S between 0.1 wt% and 14 wt%
- density, rheological parameters corrected for P and T
- γ-iron: fcc phase



FeS phase diagram (Fei et al. 1995)

CORE EVOLUTION (I)

- models ranging from entirely liquid to entirely solid core
- $\delta\rho$ between solid and liquid $\approx 3.5\%$ (Anderson 2003)
- relatively low pressure compared to Earth: sulfur almost doesn't solidify with iron ($x_S < x_S^{\mathrm{eut}}$)
- pure iron inner core

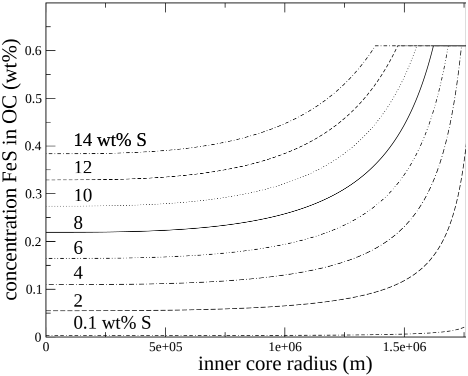


EXP ID	P (GPa)	T (K)	Time (min)	S_{solid} (at.%)	S_{liquid} (at.%)
MO141	7	1223	5	0.09 (3)	31.3 (3)
LO73	8.5	1473	180	0.17 (2)	28.0 (1)
LO87[a]	8.5	1473	15	0.16 (5)	28.3 (2)
LO77	10	1473	10	0.23 (3)	27.5 (5)
LO91[a]	10	1473	15	0.21 (5)	27.0 (3)
LO92[a]	14	1473	30	0.26 (5)	23.6 (5)
LO120[a]	20	1273	70	0.67 (5)	26.2 (6)
LO133[a]	25	1223	930	1.1 (1)	–
LO140[a]	25	1373	180	1.4 (2)	–
MO535	25	1423	15	1.4 (1)	23.1 (5)
LO95	25	1473	30	0.8 (1)	22.0 (2)

FIGURE: sulfur solidification (Li et al. 2001)

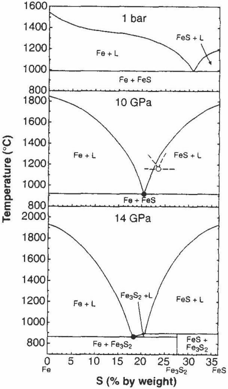
CORE MODELING (II)

- almost pure iron core, increasing sulfur concentration in outer core



PHASE DIAGRAMS

- maximum S concentration: eutectic composition ($x_S \approx 22\%$)



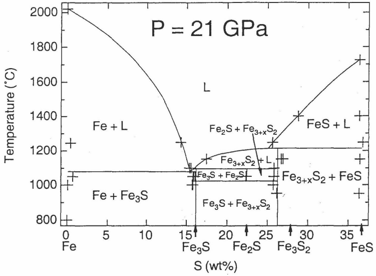
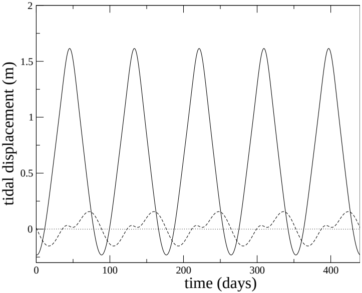


FIGURE: melting in Fe-FeS (Fei et al. 1997, 2000)

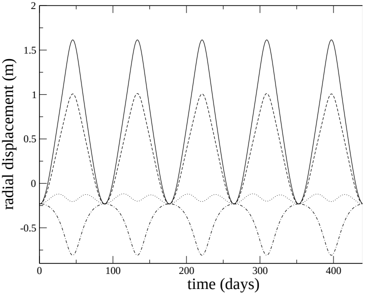
- eutectic reached for large inner core, low pressure
- Ni increases sulfur content of eutectic composition

TIDAL DISPLACEMENTS

4 wt% sulfur, inner core radius=1000 km. Starting from J2000.0



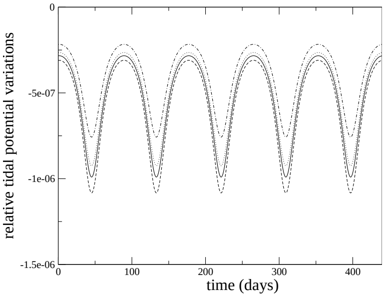
radial (solid line), East-West (dashed) and North-South (dotted) displacements at the equator

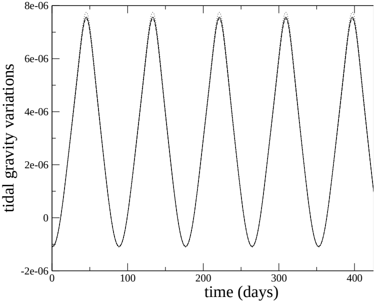


solid line: equator (sect.+zonal),
dashed line: 30° latitude,
dotted line: 60°,
dashed-dotted line: 90° (zonal)

External potential and gravity variations

Starting from J2000.0



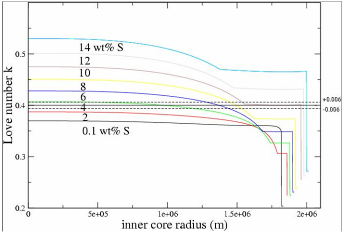


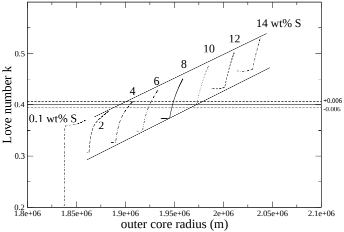
at 85° latitude

MORE accuracy for degree-two: 10^{-9} (Milani et al. 2001)

at equator

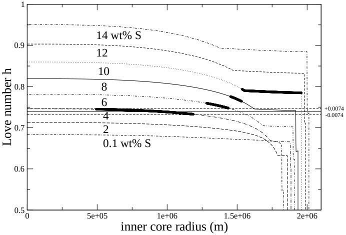
LOVE NUMBERS k

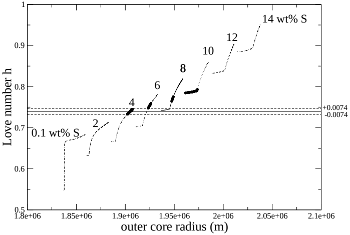




- solid cores: 5 times smaller
- Love numbers increase with
  - increasing core radius
  - decreasing sulfur concentration in outer core
- MORE accuracy: $\lesssim 1\%$ (Milani et al. 2001)
- important constraint on core: strong reduction of possible models

LOVE NUMBERS h

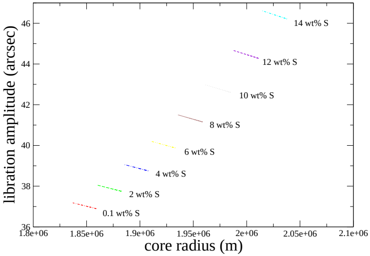




- measurements: laser altimeter + radio tracking of orbiter
- BELA simulations (Christensen et al. EGU2006): accuracy of a few % on h_2
- much stronger constraint on interior form combination of both Love numbers: error on sulfur concentration of a few percent, and on core radius some ten of kilometers

FORCED LIBRATION

$$\Delta\varphi = \frac{3}{2}\frac{B-A}{C_m}(1 - 11e^2 + 959/48e^4 + ...)$$



- amplitude liquid core $\approx$ 2x(amplitude solid core)
- for our models: core size is determining factor
- effect of core-mantle coupling $< 1\%$
- Expected accuracy: few arcsec (Wu et al. 1995, Milani et al. 2001, Jehn et al. 2004, Yseboodt et al. 2004)
- constraint on models
- even larger range: fixed $B - A$ and fixed mantle assumed

GRAVITY FIELD DETERMINATION

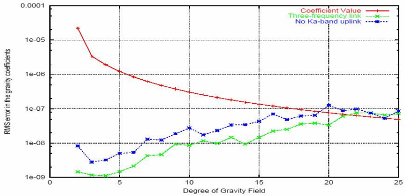
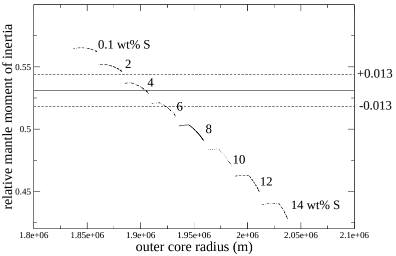


Figure 3: Root mean square error in the estimation of the coefficients of the gravity field for the three-frequency link case and the two-frequency link case without Ka-band uplink ("worst case" assumptions).

- $\left(\frac{B-A}{MR^2}\right) = 4C_{22}$
- presently badly known: $C_{22} = (1.0 \pm 0.5) \times 10^{-5}$ (Anderson et al. 1987)
- MESSENGER: precision below 1% (Solomon et al. 2001)
- MORE: 0.01% (Milani et al. 2001)

LIBRATION



constraint mainly on core size or sulfur content (1 as error, or 2.5%)

- Alternatively: free libration, but damped

$$P_{\text{free}} = \frac{2\pi}{n}\left[\frac{1}{3}\frac{C^m}{B-A}\frac{1}{e\left(\frac{7}{2}-\frac{123}{16}e^2\right)}\right]^{1/2}$$

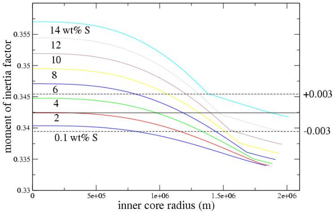
- solid core: 15.830 years
- liquid core: 10.5 years to 12 years (Rambaux et al. 2007)
- other forced libration periods easily separable (Peale et al. 2007)

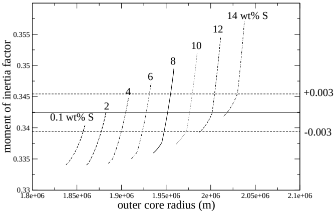
Obliquity

$$\frac{C}{MR^2} = \frac{\left[\frac{J_2}{(1-e^2)^{3/2}} + eC_{22}\left(7 - \frac{123}{8}e^2\right)\right]}{\frac{\sin I}{\epsilon_C} - \cos I}\,\frac{n}{\mu}$$

- The polar moment of inertia C can be determined from measuring the obliquity ϵ_C.
- Relation valid for Mercury occupying its Cassini state
- Theoretical range: about [1,2.5] arcmin
- caveat: spin axis does not occupy Cassini state
    - free precession: expected damped
    - spin does not follow Cassini state due to planetary perturbations. Yseboodt and Margot (2006), Peale (2006): spin axis within 1 arcsec from Cassini state
    - Margot's measurements (unpublished) seem to agree with the theoretical values and to confirm that Mercury occupies the Cassini state (Peale 2006, Yseboodt and Margot 2006).

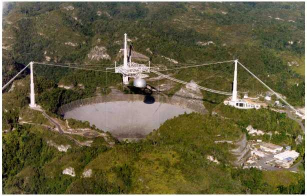
MOMENTS OF INERTIA

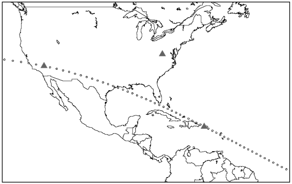




- expected accuracy: 1 as, or 1% (RSDI, space missions)
- nominal BepiColombo minimum needed
- libration (C_m) less sensitive to inner core
- Peale 1976: $\left(\frac{C_m}{B-A}\right)\left(\frac{B-A}{MR^2}\right)\left(\frac{MR^2}{C}\right) = \frac{C_m}{C} \leq 1$

GROUND-BASED OBSERVATIONS





- Radar Speckle Displacement Interferometry (RSDI)
  - one-shot precision: 2 arcsec
  - long observation campaigns: 0.2 arcsec
- Margot et al. 2004 (AGU): $\Delta\varphi \approx 60 \pm 6$as, $\epsilon_C = 2.1 \pm 0.1$amin (but new values)
- Although there are still large uncertainties on $B - A$ and obliquity, this value shows that, with very high probability (95%), the core is liquid
- J_2 at the low end of the Mariner 10 values

CONCLUSIONS

- main geodetic constraints on core:
    - tides: k_2, h_2, 1% precision, 25% uncertainty (0.4±0.1)
    - obliquity: C, 1% precision (1as), 4% uncertainty (0.345±0.015)
    - libration: C_m/C, 2.5% precision (1as), 15% uncertainty (0.5± 0.075 for our models)
- tidal measurements maybe most important for core radius
- different sensitivities to the interior structure
- combination of measurements of the low-degree gravitational field, the rotation, and the tides of Mercury will improve our knowledge of Mercury's interior