MORE

Mercury Orbiter Radio-science Experiment

# The measurements of Mercury obliquity and physical librations with BepiColombo


L. Iess[*], M. Mercolino[†], P. Persi[*], P.Tortora[‡]

** Dipartimento di Ingegneria Aerospaziale ed Astronautica*

*Università La Sapienza, Roma, Italy*

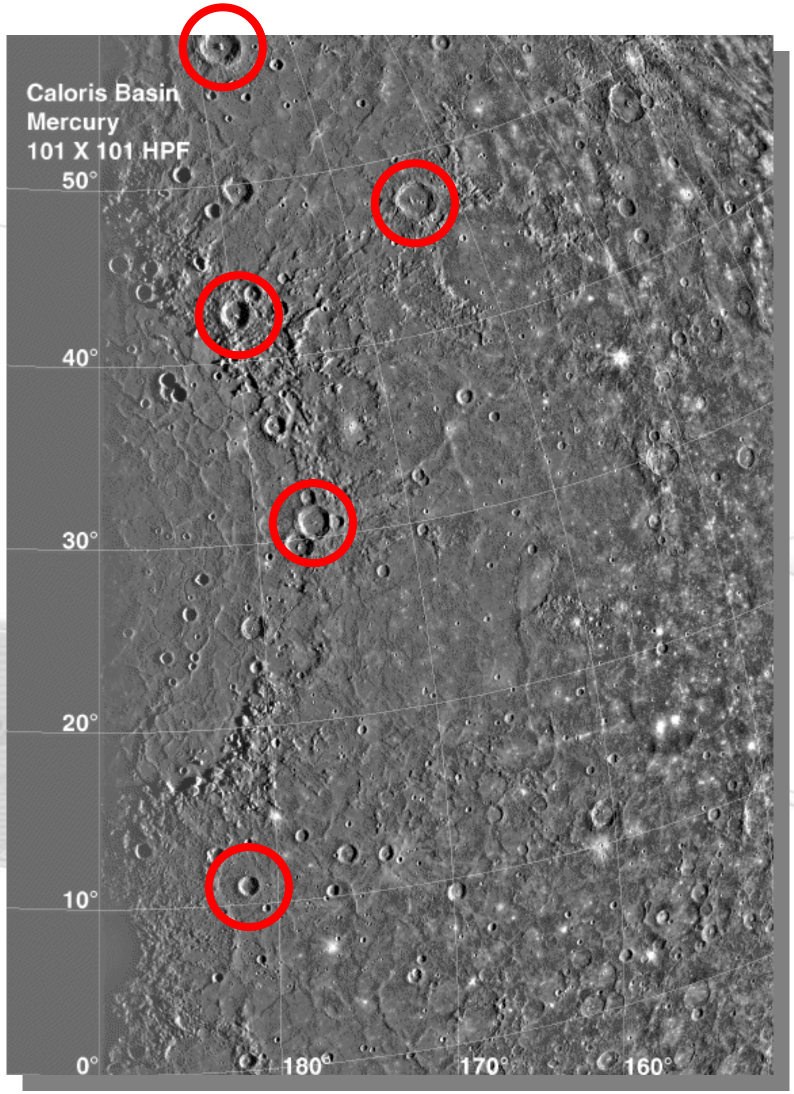
*† European Space Operation Centre – European Space Agency, Darmstadt, Germany*

*‡ DIEM – II Facoltà di Ingegneria, Università di Bologna, Forlì, Italy*

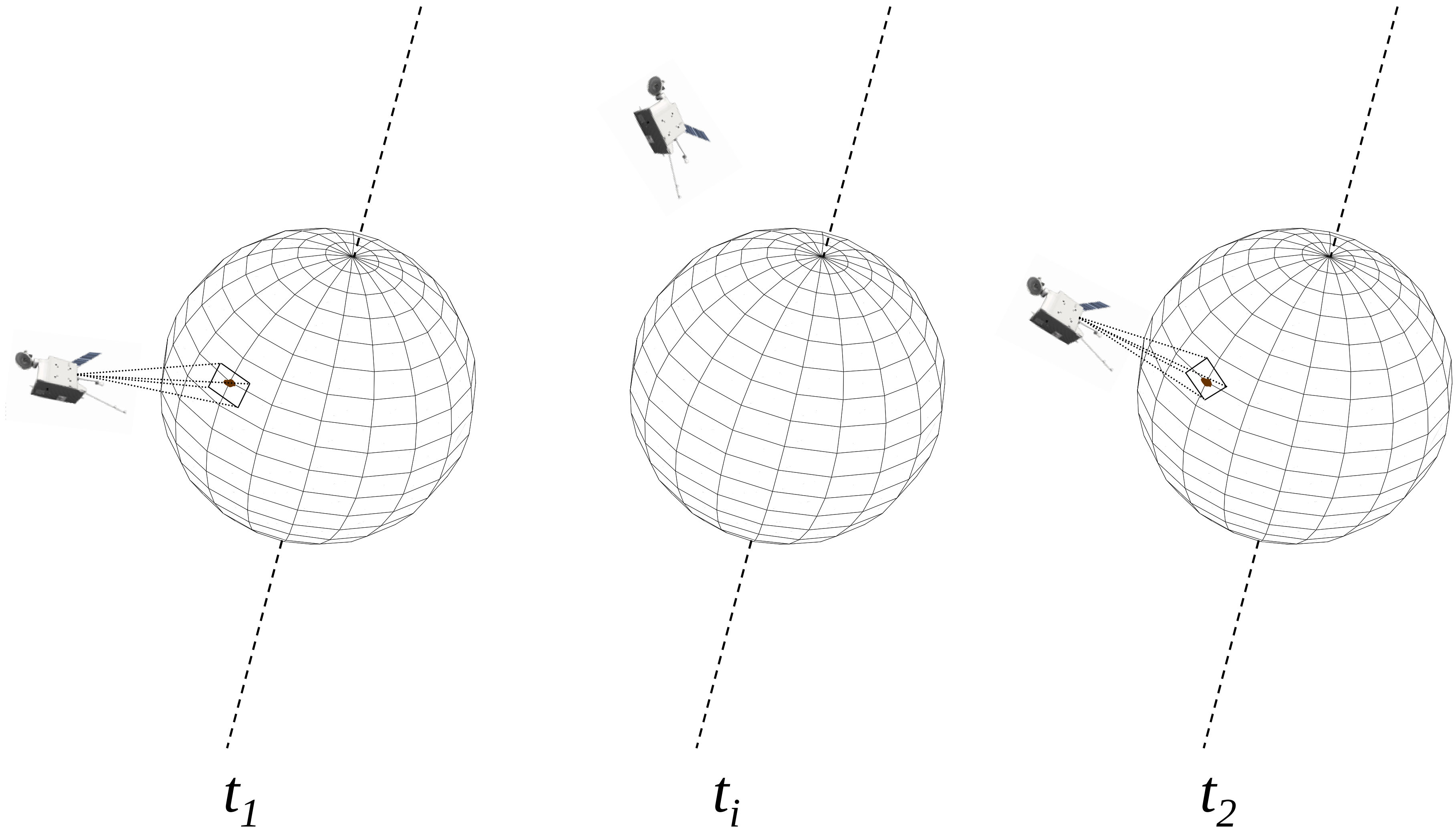
# Outline of the experiment

- The estimation of Mercury’s polar axis direction (obliquity) and librations amplitude is based on multiple observations of some bright spots on its surface, by MPO’s Narrow Angle Camera (for a higher resolution)
- An image correlation process is needed to get the basic observables used in the estimation procedures:
  - ✓ first the polar axis direction is estimated, by identifying a new reference frame, rotating with a constant angular velocity
  - ✓ then, the residual information contained in the observables, concerning Mercury’s librations, is used to estimate their amplitude (observed as a varying angular velocity of the Mercury body fixed reference frame)

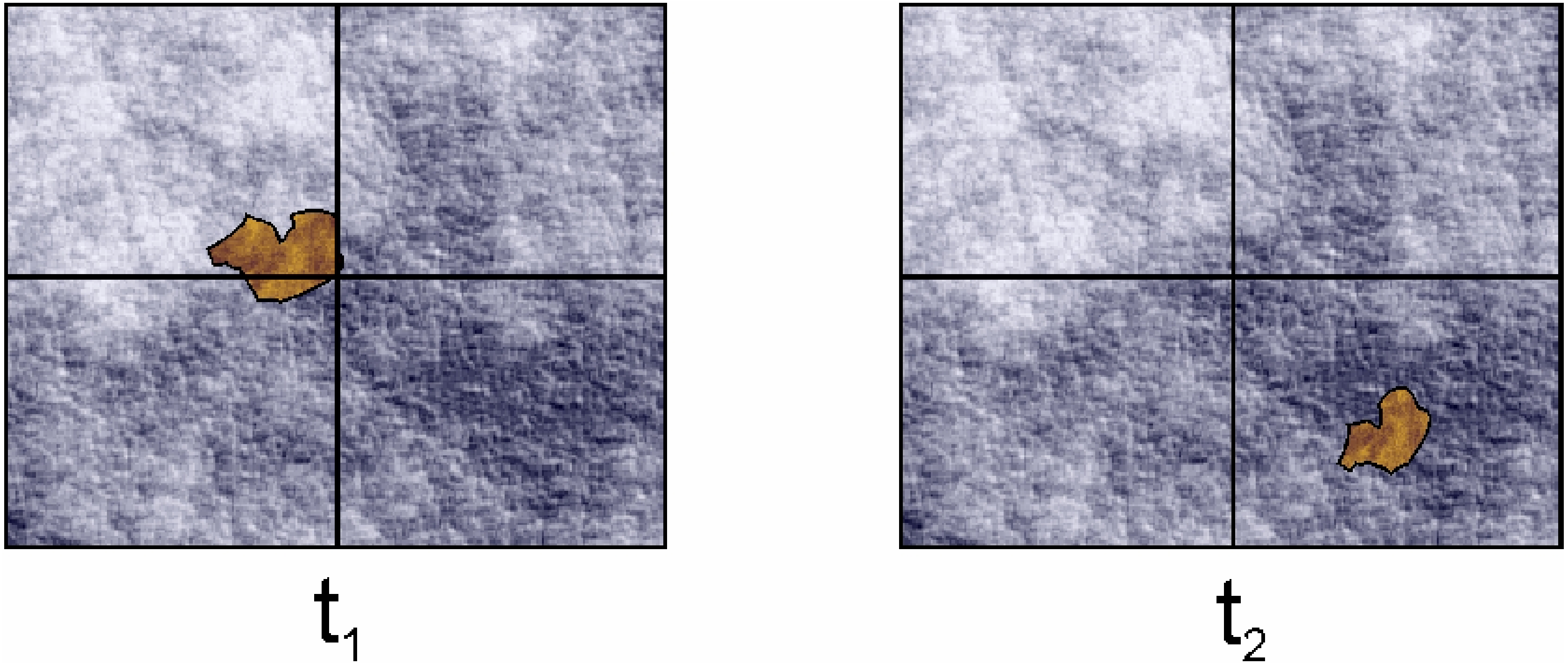
# An example of Mercury's surface with some "bright spots"



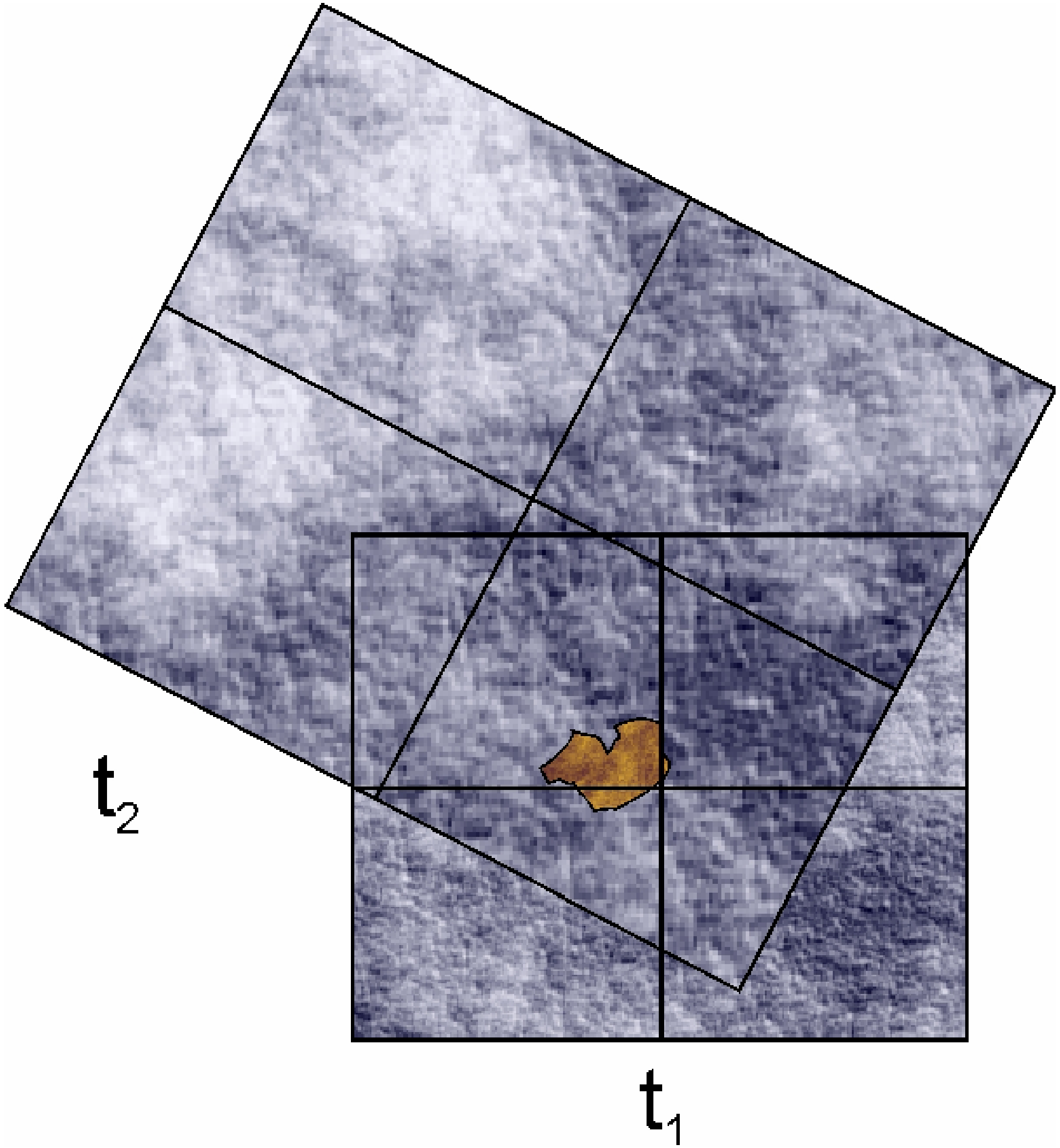
# Generation of multiple observations of Mercury's surface



# Two images of the same bright spot on Mercury's surface



# Correlation of the two images to get the bright spot position

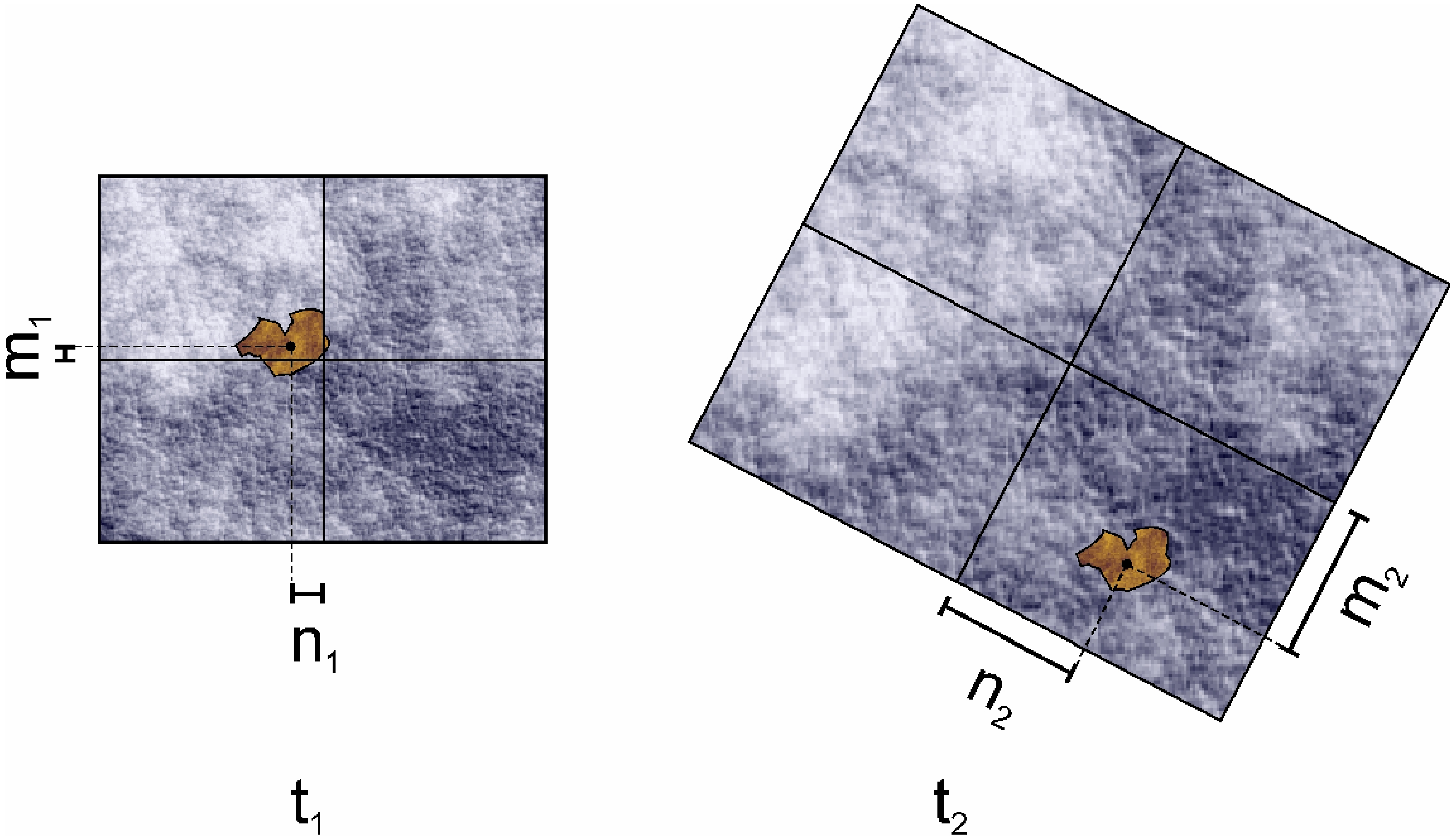



In general:

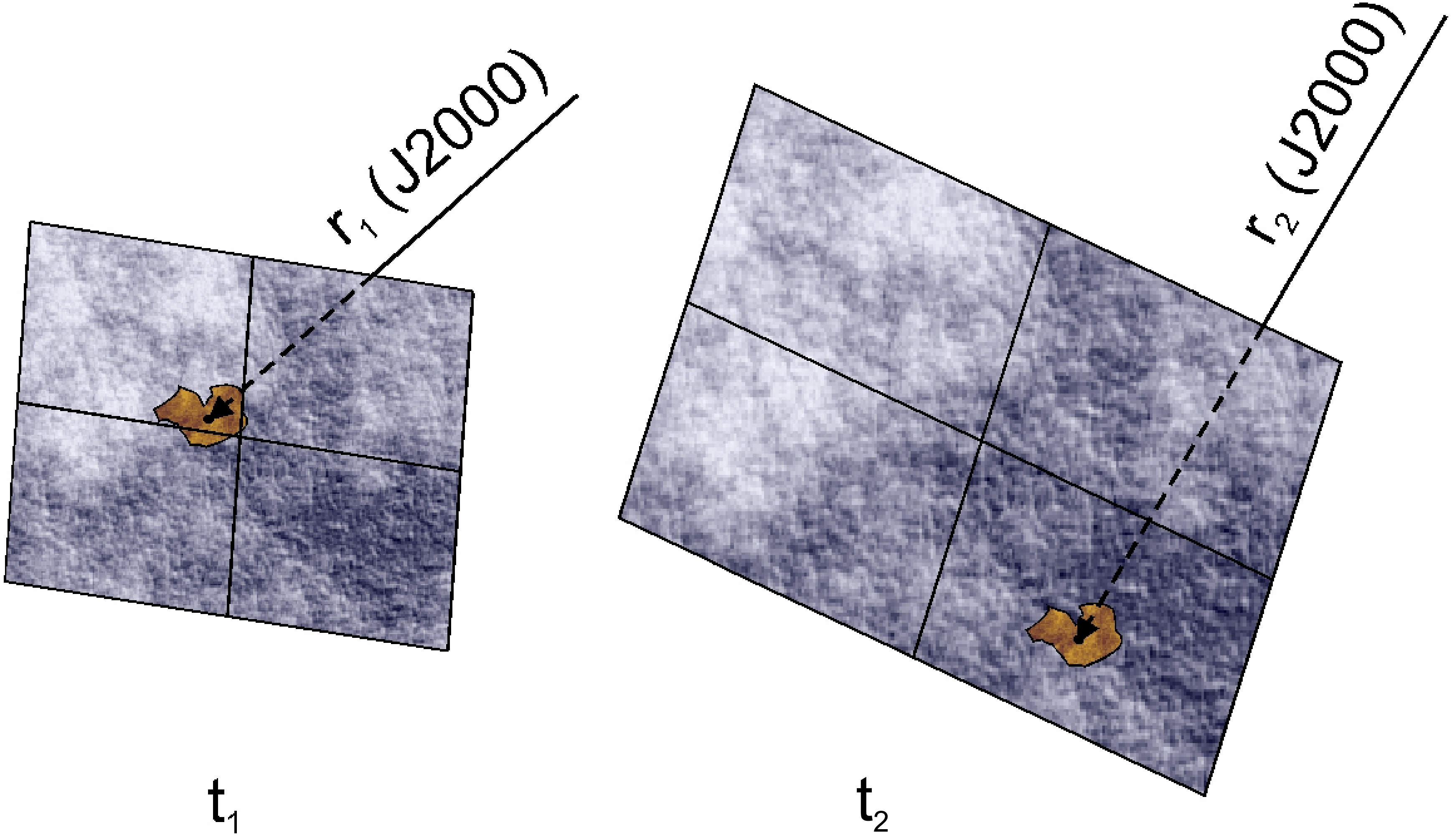
- a rotation
- a translation
- a stretching

of the 2nd image is necessary to get the maximum correlation

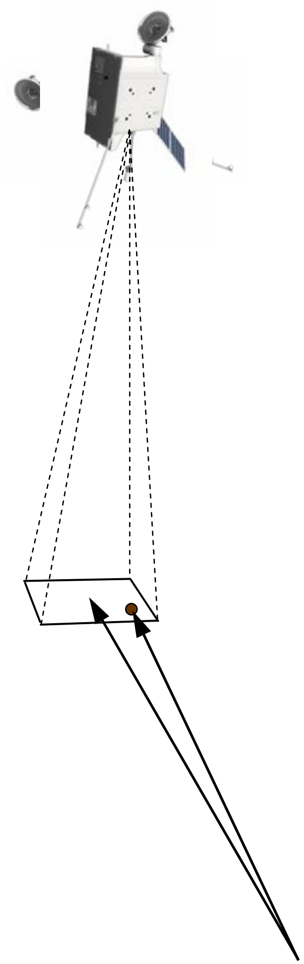
# Computing the bright spot relative position on each image



# Computing the J2000 (Mercury-centric) referenced vectors representing the bright spot position at different timetags

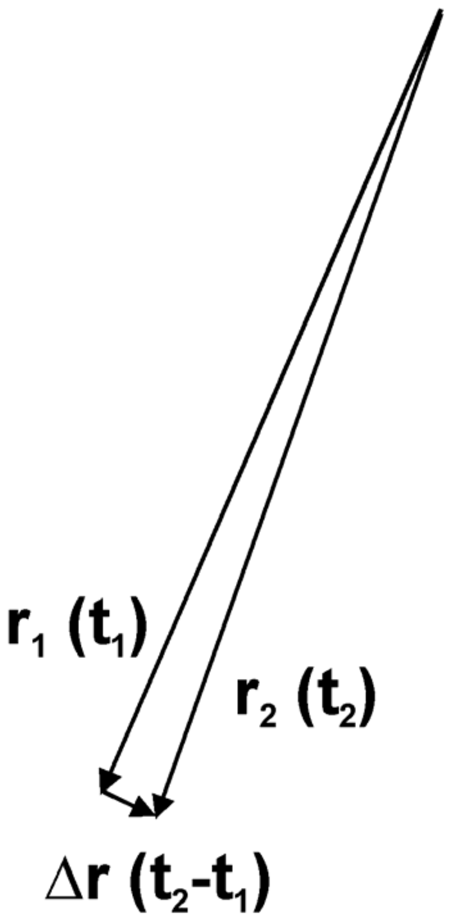


# Summary of vectors computation



- Determine MPO's position at both timetags through the most accurate OD process;
- Obtain MPO's attitude matrix at epochs;
- Compute the J2000 Mercury-centric vectors of image centers;
- Correlate pair of images to obtain bright spot's position on each image and compute the displacement of the bright spot wrt the center of each image;
- Rotate the J2000 Mercury-centric vectors of image centers in the direction of the bright spot position to obtain the J2000 Mercury-centric vectors of the bright spot positions at epochs;

# Observables for obliquity and libration amplitude estimation




- The hypothesis is that all observables ($\Delta r_i$), generated at each successive pass on the same bright spot, are perpendicular to Mercury's polar axis;
- Being all these vectors referenced to the Mercury centered J2000 inertial frame, they can be used to compute its rotation axis direction

# Estimation of Mercury's polar axis direction

The polar axis direction is represented by the unit vector

$$\boldsymbol{\omega} = \left[ \sin\varphi \sin\varepsilon \quad -\cos\varphi \sin\varepsilon \quad \cos\varepsilon \right]$$

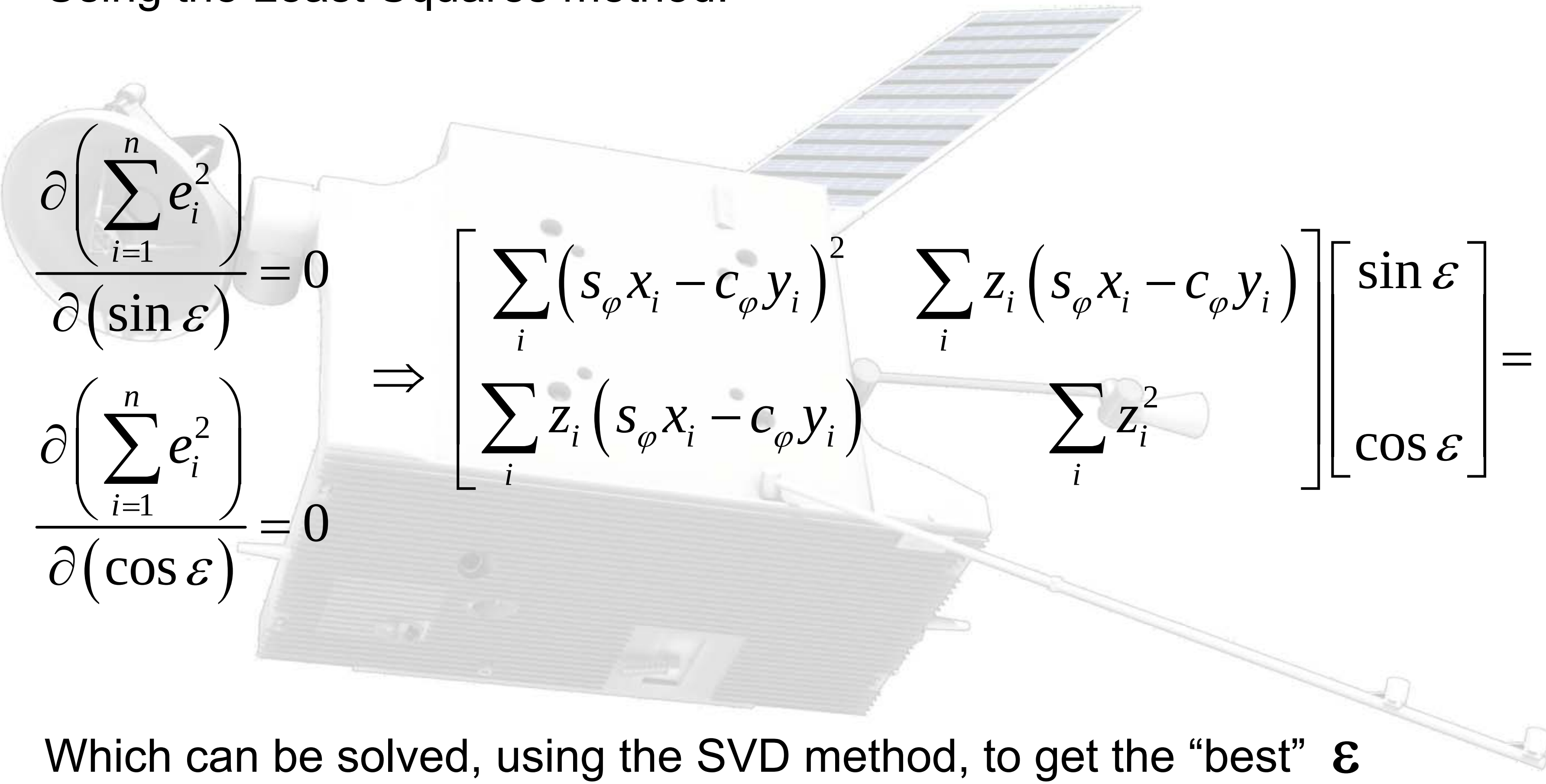
Where $\varphi$ (the phase of the obliquity) is known (Cassini's state is assumed)

$$\Delta \mathbf{r}_i \cdot \boldsymbol{\omega} \cong 0 \qquad i = 1, n \qquad \Rightarrow \qquad \begin{aligned} x_1\omega_1 + y_1\omega_2 + z_1\omega_3 &= e_1 \\ x_2\omega_1 + y_2\omega_2 + z_2\omega_3 &= e_2 \\ \ldots\ldots & \\ x_n\omega_1 + y_n\omega_2 + z_n\omega_3 &= e_n \end{aligned} \qquad \Rightarrow \qquad \sum_{i=1}^{n} e_i^2 = \min$$

Where $x_i$, $y_i$ and $z_i$ represent the coordinates of the differences vectors

# Estimation of Mercury's polar axis direction (cont'd)

Using the Least Squares method:



$$\frac{\partial\left(\sum_{i=1}^{n} e_i^2\right)}{\partial\left(\sin\varepsilon\right)} = 0$$

$$\frac{\partial\left(\sum_{i=1}^{n} e_i^2\right)}{\partial\left(\cos\varepsilon\right)} = 0$$

$$\Rightarrow \begin{bmatrix} \sum_i \left(s_\varphi x_i - c_\varphi y_i\right)^2 & \sum_i z_i \left(s_\varphi x_i - c_\varphi y_i\right) \\ \sum_i z_i \left(s_\varphi x_i - c_\varphi y_i\right) & \sum_i z_i^2 \end{bmatrix} \begin{bmatrix} \sin\varepsilon \\ \cos\varepsilon \end{bmatrix} = 0$$

Which can be solved, using the SVD method, to get the "best" $\varepsilon$

# Estimation of the libration amplitude

For the libration of Mercury a simple model was assumed in the simulations, where the rotation angle about the polar axis is:

$$\varphi = \omega_{ROT} t + \varphi_L \sin\left[\omega_{REV}\left(t - t_0\right)\right]$$

Each observation, once rotated back to the estimated Mercury body-fixed reference frame, gives a **residual longitude difference**, which is a function of the **libration phase** and **amplitude**:

$$\Delta\varphi_i = \varphi_{2i} - \varphi_{1i} = \varphi_L \left\{\sin\left[\omega_{REV}\left(t_{2i} - t_0\right)\right] - \sin\left[\omega_{REV}\left(t_{1i} - t_0\right)\right]\right\}$$

Using the Least Squares method:

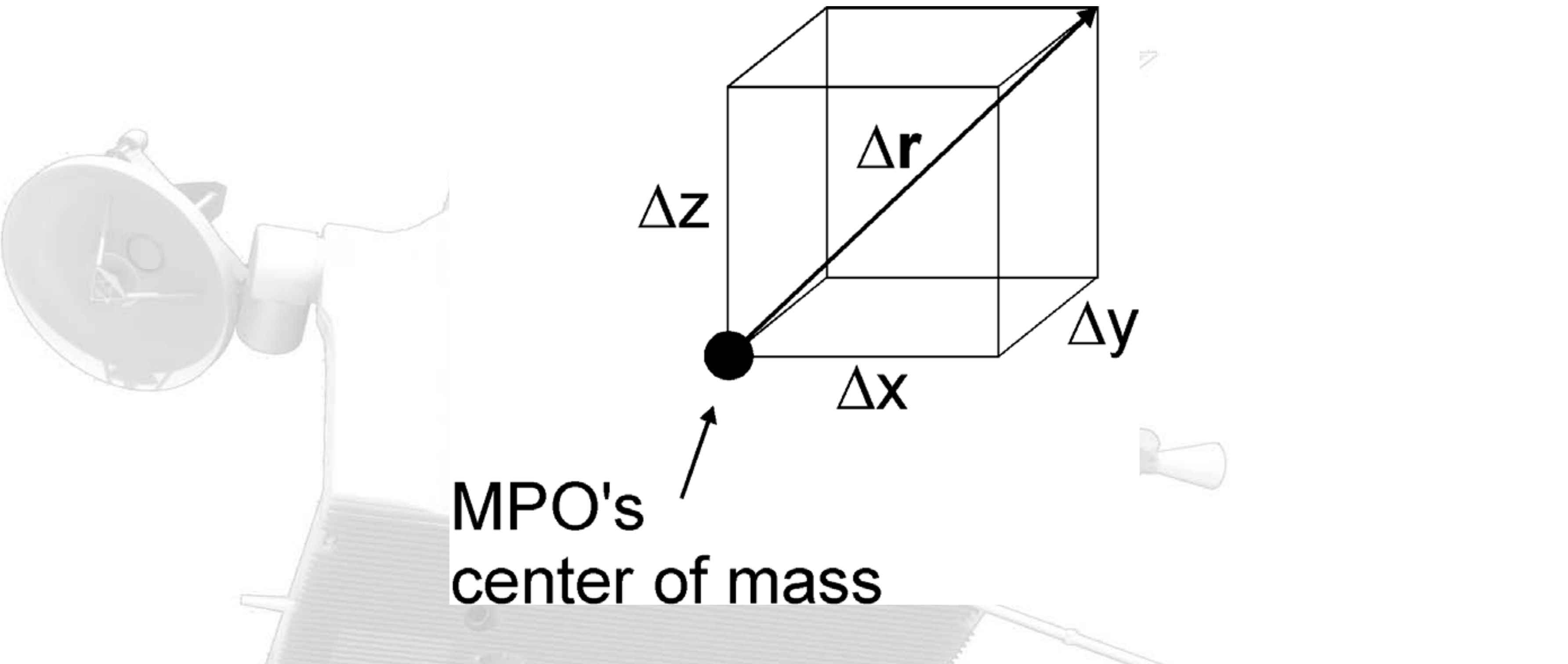
$$\varphi_L = \frac{\sum_i \left\{\Delta\varphi_i \left[\sin\left(\omega_{REV}\left(t_{2i} - t_0\right)\right) - \sin\left(\omega_{REV}\left(t_{1i} - t_0\right)\right)\right]\right\}}{\sum_i \left\{\sin\left[\omega_{REV}\left(t_{2i} - t_0\right)\right] - \sin\left[\omega_{REV}\left(t_{1i} - t_0\right)\right]\right\}^2}$$

# Introduction of observation errors

Observation errors have been introduced in the numerical simulations, to characterise the sensitivity of the estimation algorithms to non-nominal conditions. The following errors have been considered:

- errors in the determination of MPO’s centre of mass
- errors in the attitude determination and control system
- errors in the camera timing
- errors in image cross-correlation process
- errors in SIMBIO-SYS camera inertial pointing due to MPO thermal deformations

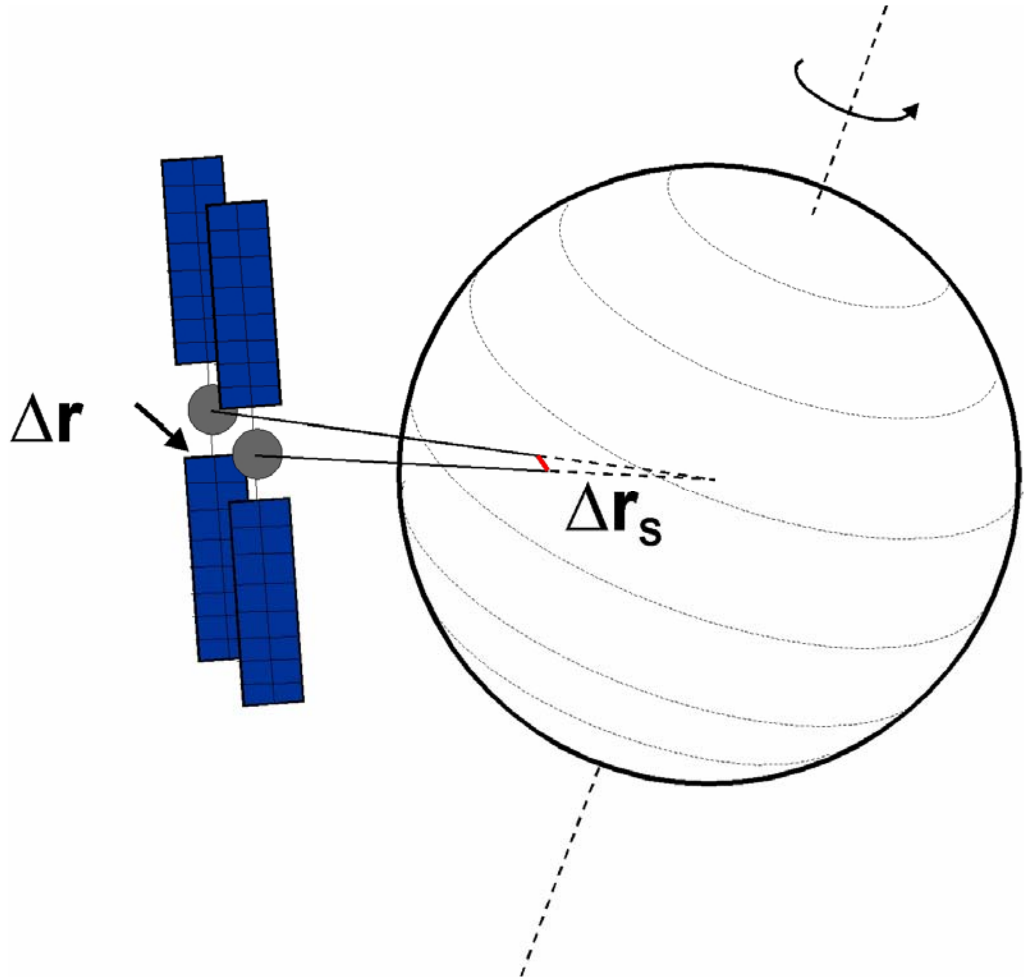
# Errors in the determination of MPO's centre of mass (1/2)




$$\Delta x \sim \frac{N\left(0,\sigma_p^2\right)}{\sqrt{3}} \qquad \Delta y \sim \frac{N\left(0,\sigma_p^2\right)}{\sqrt{3}} \qquad \Delta z \sim \frac{N\left(0,\sigma_p^2\right)}{\sqrt{3}}$$

$\sigma_p = 10$ m (a conservative value)

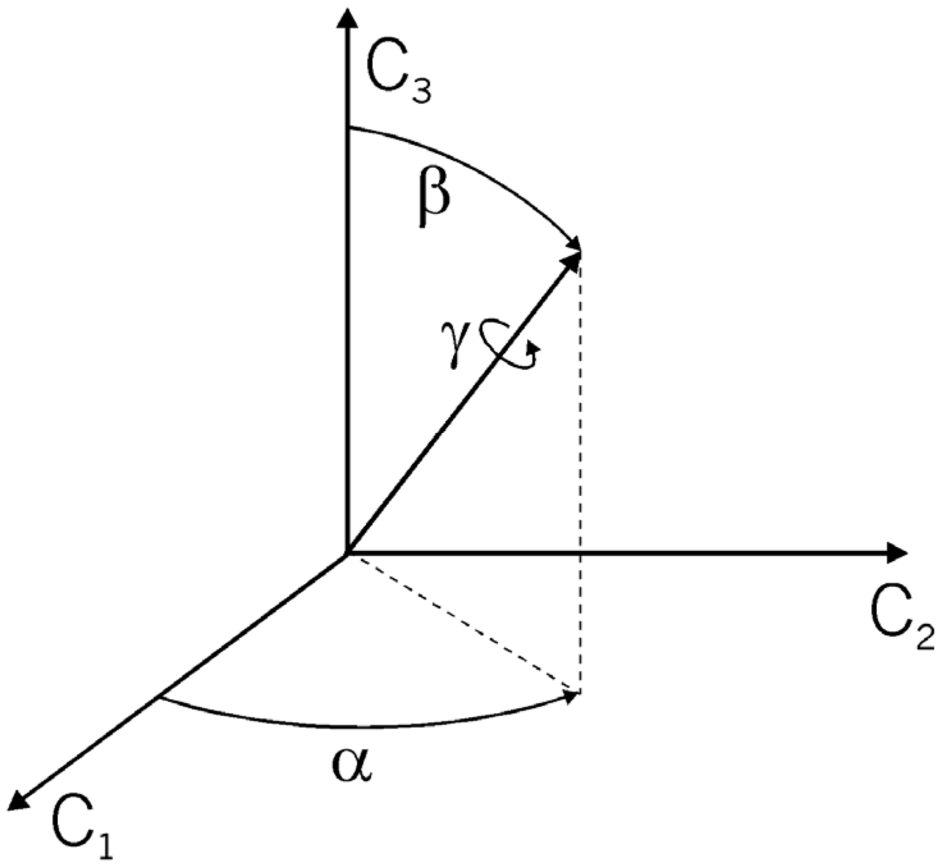
# Errors in the determination of MPO’s centre of mass (2/2)




The error $\Delta \mathbf{r}_s$ on Mercury’s surface ranges from about 8 m to about 13 m

# Errors in the attitude determination and control system

A randomly sampled rotation matrix is computed through the Euler axis/angle parameterization




$$\alpha = [0, 360] \text{ deg}$$

$$\beta = [0, 180] \text{ deg}$$

$$e(1) = \sin(\beta)\cos(\alpha)$$
$$e(2) = \sin(\beta)\sin(a)$$
$$e(3) = \cos(\beta)$$

$$\gamma \sim N(0, \sigma^2_\gamma)$$

$$\sigma_\gamma = 7{*}10^{-4} \text{ deg} \quad (2.5'')$$

# Errors in the camera timing

- Timing errors of 1.E-3 s have been simulated, adding a positional error to the nominal vector, representing the position of MPO's center of mass
- The positional errors are directed along the local MPO's velocity vector and quantized to +/-1.E-3*|v|
- Given MPO's orbital velocities in the order of about 2 km/s the resulting error is in the order of  2 m

# Numerical simulations - parameter space

- Maximum altitude for landmarks imaging (600 Km, 900 Km)
- Minimum solar elevation angle for landmarks imaging (30°, 45°)
- Beta angle: 0°, 20°, 35°, 50°, 70°, 90°
- Number of surface landmarks (50, 100, 200)
- Magnitude of systematic error (1”, 2”, 4”)

(300 Monte Carlo runs for each combination of parameters)

# Mean and Std Dev - Obliquity and Libration for Different Error Models (β=0° H=600 Km E=30°)

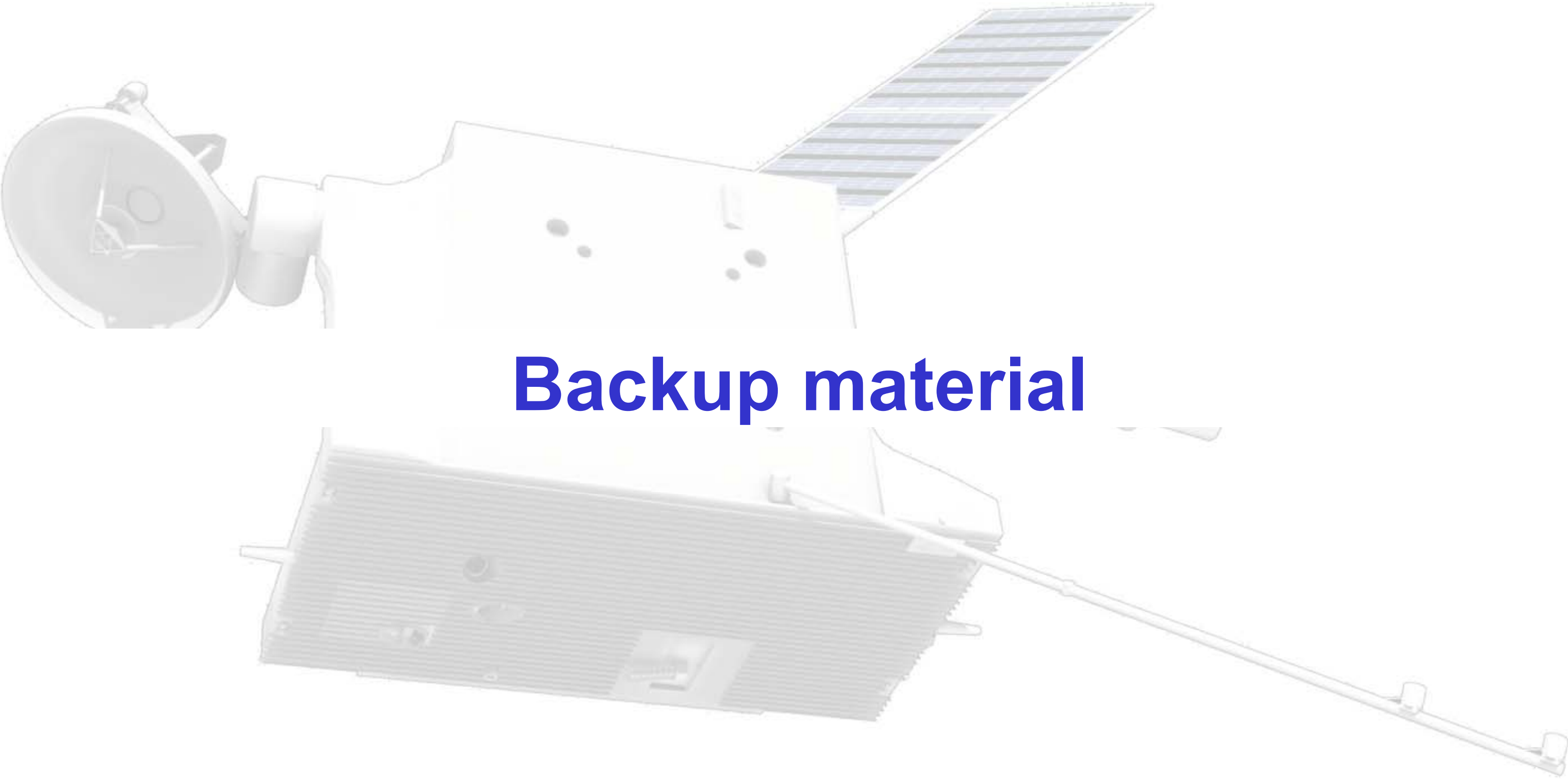
## (actual values: ε=420'', φ=42'')

Mean and Standard Deviation of Obliquity and Libration (best case - β=0°)

| | $\bar{\varepsilon}$ | $\sigma_{\varepsilon}$ | $\bar{\phi}$ | $\sigma_{\phi}$ |
|---|---|---|---|---|
| Δα=1" | 419.9" | 0.49" | 41.9" | 0.89" |
| Δα=2" | 420.0" | 0.46" | 41.9" | 0.89" |
| Δα=4" | 420.0" | 0.51" | 42.0" | 0.92" |

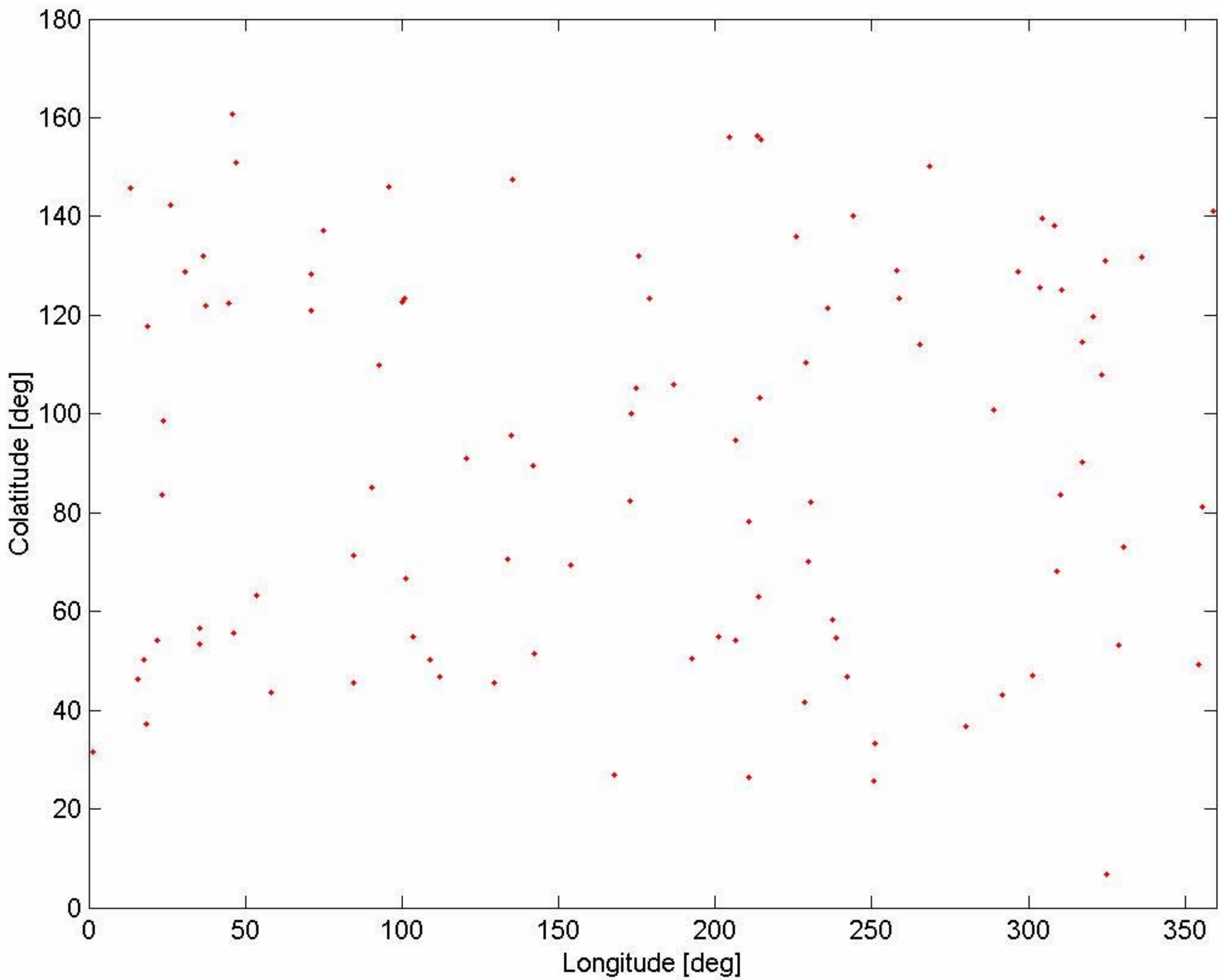
# Conclusions and Future Work

- The rotational state (obliquity angle and libration amplitude) of Mercury can be estimated using the performance of three instruments plus S/C telemetry:
  - ✓ MORE and ISA for precise orbit determination
  - ✓ SIMBIO-SYS for hi-res image acquisition
  - ✓ S/C telemetry for the attitude matrix
- A dedicated S/W for the elaboration of the information listed above has to be designed (and needs to be implemented); a breadboard was developed for the simulations shown here;
- Comparison with the results obtained within the OD S/W (to be developed by group of the University of Pisa) is crucial to cross-check the conclusions;
- Results obtained so far by means of simulations show the experiment is feasible and offers a high estimation accuracy;
- An accurate re-visitation of the simulations shown here is in order, using the new, updated information on the S/C from the contractor;

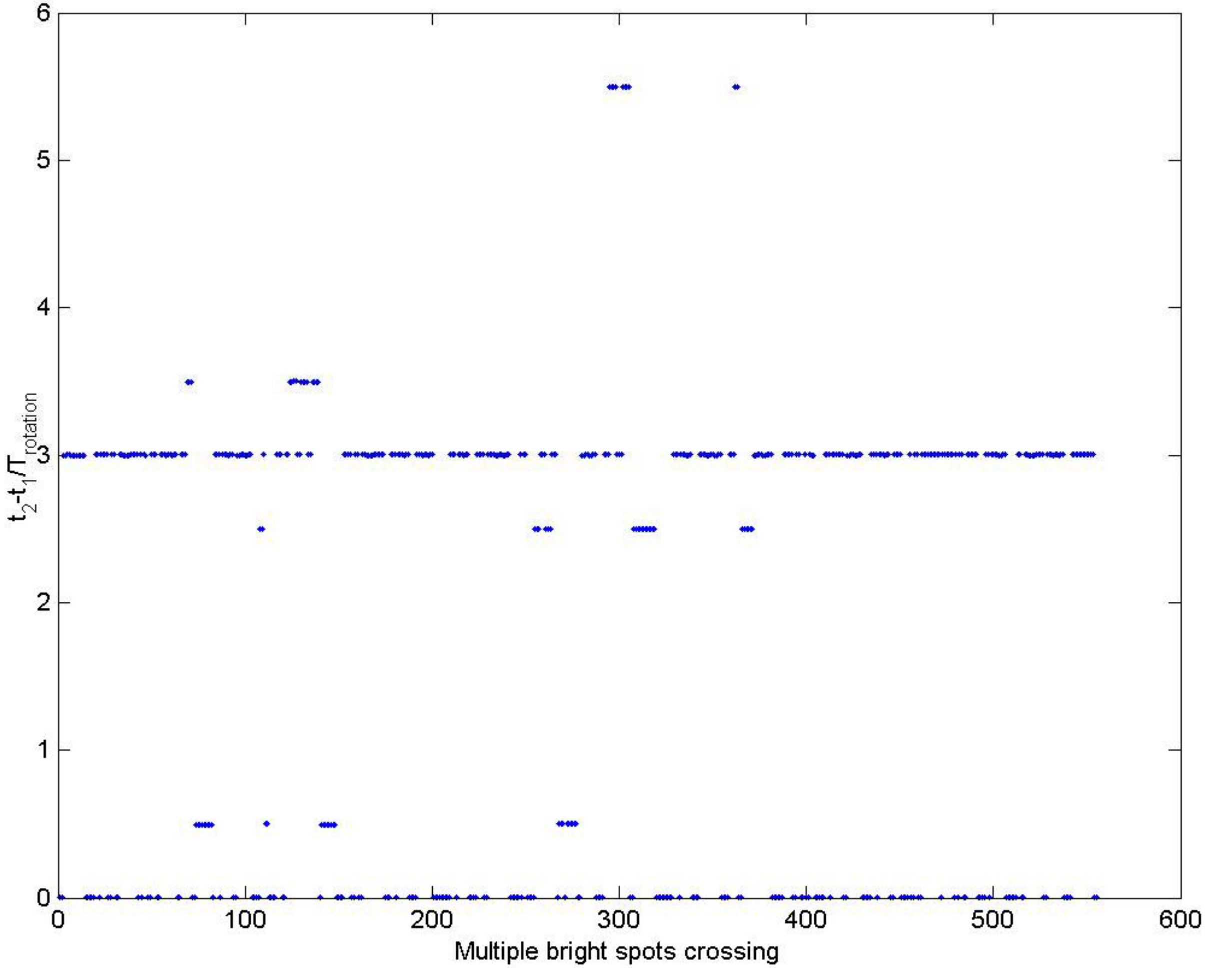


# Backup material

# 100 Randomly sampled bright spots on Mercury surface



# Distribution of the multiple observations time difference ($t_2$-$t_1$) generated over a 1-year numerical simulation ($\beta$=0°)



# Time distribution of observations of Mercury's libration ($\beta$=0°)